LOWER SECONDARY ARITHMETIC

For Junior Forms of Secondary Schools

CONTAINING EXERCISES

ORAL, PRACTICAL AND GRAPHIC

BY THE LATE

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REVISED EDITION

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PREFACE TO THE REVISED EDITION

THE book has been re-written so as to meet modern requirements ; and the changes introduced are in accordance with the recommendations contained in the 'Hints and Suggestions' of the Educational Department.

The special features of this edition are—

1. The Four Simple Rules are graphically illustrated, and the student is from the beginning introduced to the use of 'squared paper.'

2. Decimals are treated before Vulgar Fractions ; and metric weights and measures are used to illustrate decimal notation and the Four Rules.

3. Vulgar Fractions and the Four Rules are copiously illustrated by diagrams.

4. After the Compound Rules, ideas and methods of approximation and errors of measurement are introduced and the idea of working within certain limits of error is developed.

5. A chapter is devoted to the treatment of Easy Graphs and Statistical, Conversion, Proportion and Simple Interest Graphs find a place.

6. Greater importance is attached to Oral Examples ; they precede written work, and are such as can easily be worked mentally.

7. Graphic and Practical exercises are given throughout the book.

8. An early attempt is made to accustom the pupils to the use of symbols for generalized numbers.

9. Methods for shortening labour and securing rapidity receive special attention.

10. Drawing to scale and Decimalization of English and Indian money are introduced.

11. The pupils are made to construct for themselves tables of square and cubic measures and the multiplication tables of fractions.

12. At an early stage the Unitary Method is employed; but subsequently such examples are treated under Proportion; and since fractions represent ratios, they are worked by determining the ‘Multiplying Fraction’.

13. A chapter on Elementary Mensuration has been added.

The materials necessary for the Practical exercises are such as are within the easy reach of the pupils; and the Graphic exercises can be done by using Macmillan's Beginner's Set of Mathematical Instruments. Coloured crayons may be of immense value in the hands of the practical teacher.

The teacher should carefully lead his pupils to construct for themselves the various rules or processes, and not allow them to be blindly committed to memory.

It is hoped that the improved methods of teaching introduced will serve to create an intelligent interest in the study of the subject.

Any suggestions for the further improvement of the book, or any corrections, will be thankfully received.

MADRAS, J.S.D.

February, 1910.

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Further the numbers must be of the same order; for we cannot *at once* add together 15 tens and 15 units; but we must decompose 15 units into 1 ten and 5 units, and add these to 15 tens and obtain 16 tens and 5 units; or express 15 tens as 150 units and adding 15 units, get 165 units.

The numbers to be added are called **addends,** and the single number representing the result of addition is called the **sum, total, sum-total,** or **amount** of the given numbers.

The sign + (plus) is the sign of addition; thus $4+9$ means that 9 is to be added to 4.

14. The sign of equality (=) is used to denote the result of an operation.

Thus $4+9=13$ means that the result of adding 4 and 9 is 13; and is read 4 plus 9 equals 13.

15. The addends taken in any order give the same total.

Take three pieces of paper, 4 inches, 9 inches and 6 inches long respectively and mark them A, B and C respectively. Arrange them in a straight line in as many different ways as you can and find the totals in each case. What is it? What is the inference?

$4+9+6=4+6+9=9+6+4=6+4+9=6+9+4=19.$

This truth may be stated generally thus:

If a, b and c denote any three numbers,

$a+b+c=a+c+b=b+c+a=c+b+a=c+a+b.$

16. Squared paper* is paper that contains a net work of small squares, all equal to one another.

It will be seen that each line on the squared paper is divided into inches and each inch is divided into ten equal parts, called tenths of an inch or into eight equal parts called eighths of an inch.

17. Graphic representation of simple addition on squared paper.

If a side of one of the small squares in Fig. 1 denotes 1 unit, a length of 17 units will be represented by AB and a length of 9 units by CD.

Place CD in the same line and continuous with AB as in the figure. Then the sum of the two lines is AD, which contains 26 units. Therefore $17+9=26$.

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* The teacher should explain the meaning of a square and how a sheet of paper is ruled so as to make it a squared paper.

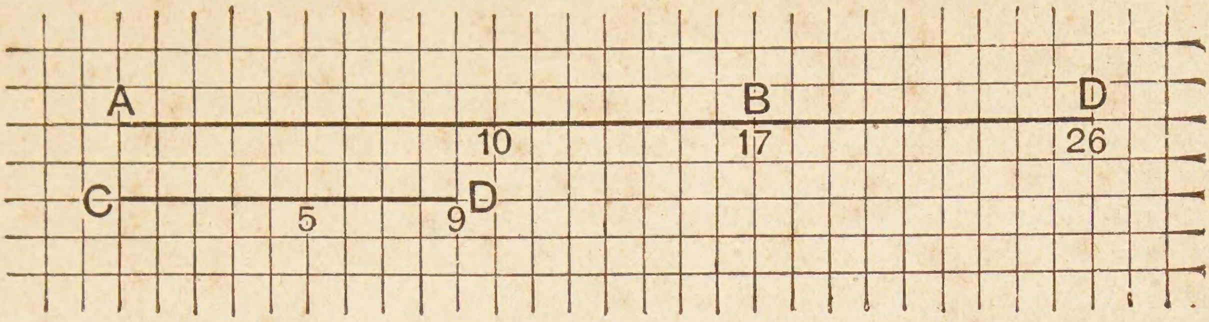

FIG. 1

18. Illustration by Rectangles.*

Let each square denote 1 unit. Take the rectangle ABCD containing 9 squares. Then ABCD represents 9 units; and the rectangle EFGH which contains 8 squares represents 8 units. Place the rectangle EFGH alongside with ABCD so that EH falls on BC as shewn in Fig. 2; and count the number of squares in the rectangle AFGD formed by the two rectangles. It contains 17 squares; and therefore 17 units. Therefore 9 + 8 = 17.

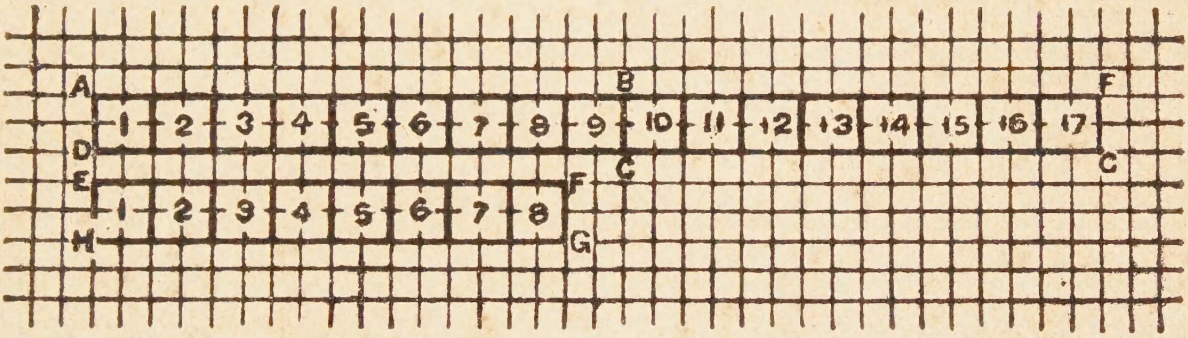

FIG. 2

Practical Exercise.

1. Let the pupils cut pieces of paper of different lengths and let them use them to learn addition of two or more numbers.

2. Let the pupils use coloured crayons on squared blackboard to find the sum of (1) 12+18, (2) 21+19, (3) 9+12+10, (4) 12+13+14.

3. Use squared paper to find the sum of the following by drawing.

(*a*) straight lines, (*b*) squares, (*c*) rectangles :—

(1) 13 + 10. (2) 17 + 14. (3) 25 + 15. (4) 27 + 19.
(5) 7 + 9 + 8. (6) 12 + 6 + 9. (7) 14 + 12 + 7. (8) 19 + 11 + 10.

---

* The teacher must explain what a rectangle is and that a square is a particular kind of rectangle.

19. Explanation of the ‘**Carrying**’ in addition.

EXAMPLE. Add together Rs. 30,759, 8,976, 56,789 and 675.

Ten Th.	Th.	H.	T.	U.
3	0	7	5	9
	8	9	7	6
5	6	7	8	9
		6	7	5
1	3	2	2	
9	7	1	9	9

Adding the units, the sum is 29 units ; decompose 29 units into 2 tens and 9 units ; place 9 in the units' column and 2 in the tens' column to be added with the tens.

The sum of the tens is 29 tens ; decompose 29 tens into 2 hundreds and 9 tens ; place 9 in the tens column and 2 in the hundreds column to be added with the hundreds. The sum of the hundreds is 31 hundreds ; that is 3 thousands and 1 hundred. Write 1 in the hundreds' column and 3 in the thousands' column ; and so on.

The sum is Rs. 97,199

Note.—The numbers in small type are intended only to shew the figure ‘ carried ’ and not to be written in actual working.

The addition of these numbers may also be done horizontally without writing them in columns, i.e. without placing the units vertically under units, the tens under tens and so on. But care should be taken to add units to units, tens to tens, hundreds to hundreds and so on.

20. To test the accuracy of the result of addition, add the columns from bottom to top and then from top to bottom and see if the two totals agree. If the numbers are written horizontally, add from left to right, and then from right to left ; and see if the two results are the same.

21. To secure speed in Addition, the following is a good method :—

Take any number and to it add any other number ; and go on adding the same number till you get to a given total.

EXAMPLE. Take 9, add 6, and continue to add 6 ; thus 9, 15, 21, 27, 33, up to 105. Or take 13, add 9 and continue to add 9 ; thus 13, 22, 31, 40, etc.

The following is another method :—

Write in a line any numbers at random ; and practise adding them up.

Write 7, 9, 11, 13, 8, 9, 6, 15, 12, etc., and add thus:
16, 27, 40, 48, 57, 63, 78, 90, etc.

The second line is intended to shew the mental process and should not be written down.

EXAMPLES.—ORAL.

Continue the addition of the following numbers, till you are asked by the teacher to stop.

1. 7, 12, 17, 22 . . . . . .
2. 6, 14, 22, 30 . . . . . .
3. 9, 16, 23, 30 . . . . . .
4. 11, 20, 29, 38 . . . . . .
5. 13, 21, 29, 37 . . . . . .
6. 15, 21, 27, 33 . . . . . .
7. 17, 26, 35, 44 . . . . . .
8. 16, 23, 30, 37 . . . . . .

EXAMPLE. Find the value of 47 + 29.

Proceed mentally thus: 40 + 20 = 60; 7 + 9 = 16; 60 + 16 = 76.

EXAMPLES.—ORAL.

Find the value of—

1. 19 + 13. 2. 21 + 19. 3. 27 + 17. 4. 35 + 27.
5. 57 + 25. 6. 89 + 21. 7. 98 + 32. 8. 97 + 43.
9. 35 + 27. 10. 49 + 36. 11. 37 + 39. 12. 47 + 52.
13. 112 + 48. 14. 201 + 59. 15. 317 + 123. 16. 329 + 81.

22. Subtraction is the process of finding how much one number is larger or smaller than another number of the same kind.

This may be done by finding how much must be *taken* away *from* a given larger number to be equal to another smaller number, or by finding what number has to be *added* to a given smaller number to make up another larger number.

23. The number that has to be subtracted, the smaller number is called the **subtrahend**; the number from which it is subtracted, the larger number is called the **minuend**; and the result of the operation is called the **difference** or **remainder.**

The sign — (minus) is the sign of subtraction and shews that the quantity following it should be subtracted from the quantity preceding it.

Thus 15 — 9 means that 9 is to be subtracted from 15.

24. Graphic representation of simple subtraction.

Let each square consisting of four small squares in Fig. 3 represent 1 unit. Then the rectangle ABCD which contains 15 such squares represents 15 units and the rectangle EFGH which contains 9 such squares represents 9 units.

The shaded part of the rectangle ABCD contains 9 squares and is therefore equal to the rectangle EFGH.

The remaining part of the rectangle ABCD contains 6 squares and therefore represents 6 units.

$\therefore$ 15 units $-$ 9 units $=$ 6 units.

$\therefore 15 - 9 = 6.$

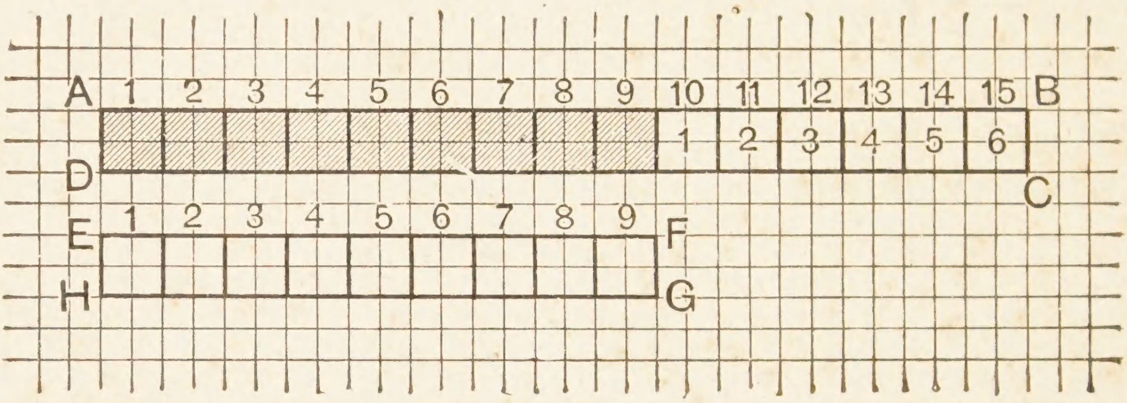

FIG. 3

Practical Exercise.

1. Measure a piece of paper 12 inches long and cut off a piece equal to 7 inches, and find the remainder.

2. Mark out 17 squares on the blackboard. Rub out 9 of them. What is the remainder ?

3. Draw on squared paper a length of 25 divisions ; cut off a part equal to 13 divisions. How many remain.

4. Use squared paper to find the difference between—

(1) 19 and 13. (2) 27 and 15. (3) 50 and 21. (4) 52 and 35.

5. Use squared paper to find the value of

(1) $15 - 6 - 5$. (2) $21 - 8 - 7$. (3) $32 - 13 - 12$. (4) $35 - 9 - 15$.

25. Explanation of the process of **Subtraction by Decomposition.**

EXAMPLE. What sum has to be taken from Rs. 875 to be equal to Rs. 686 ?

In other words, from Rs. 875 subtract Rs. 686.

$$\begin{array}{ll} \text{From } 875 = 800 + 70 + 5 = 800 + 60 + 15 = & 700 + 160 + 15 \\ \quad\text{take } 686 & = 600 + 80 + 6 \\ \hline \qquad\text{Ans. Rs. 189.} & 100 + 80 + 9 \end{array}$$

This may be written thus :—

	H.	T.	U
Rs.	~~8~~	~~7~~	~~5~~
=	7	16	15
Rs.	6	8	6
Rs.	1	8	9

5 units cannot be taken from 6 units ; so decompose 7 tens and 5 units into 6 tens and 15 units ; 15 units — 6 units = 9 units ; write 9 in the units' column.

8 tens cannot be taken from 6 tens ; decompose 8 hundreds and 6 tens into 7 hundreds and 16 tens ; 16 tens — 8 tens = 8 tens ; write 8 tens in the tens' column. 7 hundreds — 6 hundreds = 1 hundred ; write 1 in the hundreds' column.

26. The addition of equal numbers to the minuend and subtrahend does not affect the difference.

Mark out 15 squares on the blackboard and rub out 8 squares. What is the remainder ?

Next mark out 15 + 5 or 20 squares ; and then rub out 8 + 5 or 13 squares. What is the remainder now ?

Are the two remainders the same ? What is the inference ?

EXAMPLE. 15 — 8 = 7 ; also 20 — 13 = 7, where 5 is added both to the minuend, 15, and to the subtrahend, 8.

We shall now apply this truth to find the difference between 875 and 686.

H.	T.	U.
8	7	15
6	9	6

6 units cannot be taken from 5 units. Add 10 units to the 5 units in the minuend ; also add 10 units (or 1 ten) as 1 ten to the 8 tens in the subtrahend.

H.	T	U.
8	17	15
7	9	6
1	8	9

9 tens cannot be taken from 7 tens. Add 10 tens to the 7 tens in the minuend ; and 10 tens (or 1 hundred) as 1 hundred to the 6 hundreds in the subtrahend.

All this should be done mentally and only the remainder should be written.

This method of finding the difference is called the method of **Subtraction by equal additions.**

27. The **Complement** of a number is the difference between the number and a unit of the order next higher to that number.

Thus the complement of 8 is 2, and the complement of 79 is 21.

Also 8 with its complement 7, makes 15 ; and 6 tens with its complement 5 tens make 11 tens.

28. Subtraction has already been explained as the process of finding how much must be added to the subtrahend to make up the minuend (Art. 22).

For example, to find the difference between 8 and 5 is to find the number that has to be added to 5 to make up 8. We usually say 5 from 8 is 3 ; but we also know that 3 has to be added to 5 to make 8.

Finding the difference by this method is called **Subtraction by the method of complementary addition.** This is also known as the **Austrian** method : and is useful for rapid work in some of the subsequent rules.

We shall now illustrate this method by an example.

Take the example—What should be added to 686 to get 875 ?

875	6 and its complement 9 make 15 : write 9 and carry 1
686	(i.e. 1 ten from 15) ; 8 and 1, 9 ; 9 and its complement 8
——	make 17 ; write 8 and carry 1 (i.e. 1 hundred from 17 tens) ;
189 Ans.	7 and 1, 8 ; write 1.

29. To test the accuracy of the result of subtraction, add the remainder to the subtrahend and see if you get the minuend.

30. Addition and subtraction in **one operation.**

EXAMPLE. Subtract the sum of Rs. 4635, 9789, 8567, 4686 and 7654 from Rs. 76123.

Rs. 76123	*Mental Steps.*
——	10, 17, 26, 31, and *2*, 33 ; write 2 and carry 3
4635	8, 16, 22, 30, 33, and *9*, 42 ; ,, 9 ,, ,, 4
9789	10, 16, 21, 28, 34, and *7*, 41 ; ,, 7 ,, ,, 4
8567	11, 15, 23, 32, 36, and *0*, 36 ; ,, 0 ,, ,, 3
4686	3, and *4*, 7 ; ,, 4
7654	
Rs. 40792	

EXAMPLES.—ORAL.

1. How many rupees must be added to 12 rupees to make (1) 15, (2) 20, (3) 25, (4) 30 rupees?

2. What must be added to 9 to make (1) 15, (2) 21, (3) 27, (4) 32, (5) 37, (6) 40?

3. A purse contains 15 rupees; how many rupees must be taken away so that only 6 rupees may be left?

4. What must be subtracted from 21 to get (1) 8, (2) 15, (3) 12, (4) 17 as remainder?

5. What weight must be added to 13 palams to make up 40 palams?

6. A purse contains 100 rupees; how many rupees must be taken away so that there may be 60 rupees left?

EXAMPLES. II.

1. Two persons A and B have together a lakh of rupees; A has Rs. 49,796. What has B?

2. Govind has Rs. 8,799 and Rama has Rs. 5,021 less than Govind; what has Rama?

3. A is worth Rupees one lakh and fifty thousand; and B, C and D are worth respectively Rs. 25,250, 37,750, and 45,675. How much is A worth more than B, C and D together?

4. The total population of a certain country is 30 crores. The population of four of its provinces is respectively, 35 millions, 300 lakhs, $42\frac{1}{2}$ millions and $137\frac{1}{2}$ lakhs. What is the population of the rest of the country?

EXAMPLE. Find the value of 47 — 29.

Proceed mentally thus: 47 — 20 = 27; 27 — 9 = 18.

EXAMPLES.—ORAL.

1. I have Rs. 70 and give away Rs. 36. What have I left?

2. The greater of two sums is Rs. 81, and their difference Rs. 19; find the smaller.

3. Gopal and Krishna have together Rs. 129, Gopal has Rs. 58; what has Krishna?

4. Rama and Govind have together Rs. 235, Rama has Rs. 152; how much has Govind?

Example. A piano, table and carpet cost Rs. 750; the piano and table cost Rs. 625; and the table and carpet cost Rs. 175. Find the price of each.

The piano, table and carpet cost Rs. 750.
But the piano and table cost Rs. 625.
∴ The carpet costs Rs. 750 — Rs. 625 or Rs. 125.
But the table and carpet cost Rs. 175.
∴ The table costs Rs. 175 — Rs. 125 or Rs. 50.
∴ The piano costs Rs. 625 — Rs. 50 or Rs. 575.

EXAMPLES. III.

1. A house, the land on which it stands and its furniture together, cost Rs. 20,500; the lands and the house together cost Rs. 17,250; and the house and furniture together cost Rs. 12,875. Find the cost of each.

2. A, B and C together have Rs. 1,500; A and B together have Rs. 900; B and C together Rs. 1,000. How many rupees has C more than A?

3. A man has in his garden 20 fruit-trees consisting of mangoes, guavas and oranges. The number of orange and mango trees is 14; and the number of mango and guava trees is 13. How many more guava and orange trees has he than mango trees?

4. A man's trousers, coat and waist-coat cost Rs. 36 altogether. The coat costs Rs. 6 more than the waist-coat; and the trousers and waist-coat cost Rs. 24. What did each cost?

31. Positive and Negative Quantities.

Suppose a tradesman gains Rs. 10 in one transaction and loses Rs. 4 in another, his net gain on the two transactions is Rs. 6.

Here the gain may be reckoned as a positive quantity and the loss as a negative quantity.

If we use the + sign to denote gain and the — sign to denote loss, we may express the result of the two transactions thus: + Rs. 10 — Rs. 4 = + Rs. 6; or Rs. 10 — Rs. 4 = Rs. 6, without the plus sign before the positive quantity.

If he gains Rs. 6 in the one and loses Rs. 10 in the other, the result is a loss of Rs. 4; that is, Rs. 6 — Rs. 10 = — Rs. 4.

If he lost Rs. 10 in the one and lost Rs. 6 in the other, his total loss would be represented by — Rs. 10 — Rs. 6 or by — Rs. 16.

In the same way, in calculating population, births may be reckoned as a positive quantity and indicated by the sign + and deaths may be reckoned as a negative quantity and denoted by the sign —.

If in a certain month in a certain town, there are 1209 births and 789 deaths, there is an increase of 1209 — 789 or 420 in the population.

Again what a man has may be considered as a positive quantity, and what he owes as a negative quantity.

If Gopal has Rs. 10 and he owes Raman Rs. 4, he is worth Rs. 10 — Rs. 4 or Rs. 6. If Raman remits the debt, Gopal is worth Rs. 10; that is, he is worth Rs. 4 more.

Therefore taking away or subtracting a debt of Rs. 4 is the same as giving him or adding Rs. 4; that is, Rs. 6 — (— Rs. 4) = Rs. 6 + Rs. 4.

32. Graphic Representation of Positive and Negative quantities.

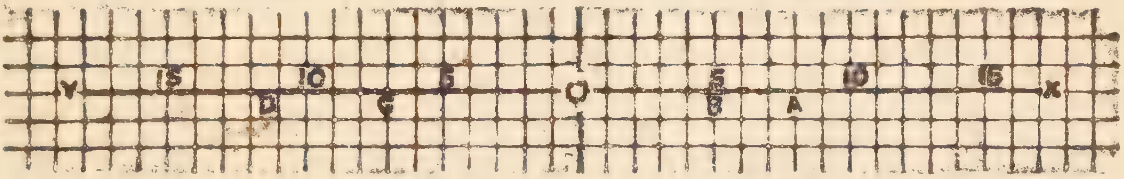

FIG. 4.

Suppose a boy starts from O to go to X. Then as long as he walks to the right and does not go beyond X he is making progress. We call this *positive* direction.

Let each division of the line YOX represent 1 yard. If he has reached A from O, he has gone 8 yards. Since OH is in the positive direction, 8 yards is a positive quantity. Any distance gone in the opposite direction, that is, to the left, takes the boy farther from his destination; and this direction is called *negative*; and any number of yards gone in this direction is called a negative quantity.

If after reaching A, he returns to B, he has walked in the negative direction a distance of 3 yards. Therefore OB = OA — AB = 8 yards — 3 yards = 5 yards.

If when he has to go from O to X, he walks from O towards Y, he goes to the left; and any distance he has gone is in the negative direction. Thus if he has reached COC will be represented by — 7 yards.

Suppose the boy goes first 5 yards from O to B and then goes 12 yards to the left, he is at C, 7 yards to the left of O, just as if he had gone direct from O to C.

Therefore $5 \text{ yards} - 12 \text{ yards} = -7 \text{ yards}$. Again if the boy after going 8 yards from O to A walks back 8 yards to the left, he is at the starting point again; that is, he has made no progress whatever. Therefore $+8 \text{ yards} - 8 \text{ yards} = 0$.

We thus see that $+8$ is the exact opposite of -8; that is, a number with a *minus* sign before it means the exact opposite of what it means without it.

If the boy goes from O to C and from C to D, he has in all gone 11 yards to the left; and $OD = OC + CD$.

But $OD = -11$ yards; therefore $OC + CD = -11$ yards, that is, $-7 \text{ yards} + (-4 \text{ yards}) = -11 \text{ yards}$.

We thus learn that—

(1) *A number with the sign + placed before it means that it should be taken as it stands. Thus* $+(+8) = 8$; *and* $+(-8) = -8$.

(2) *A number with the sign — before it means exactly the opposite of what it meant without it. Thus* $-(+8) = -8$; *and* $-(-8) = 8$.

Practical Exercise.

Use squared paper to illustrate the following graphically :—

1. $6 - 4 = 2$.
2. $12 - 8 = 4$.
3. $-6 + 7 = 1$.
4. $-9 + 9 = 0$.
5. $-12 + 8 = -4$.
6. $-15 + 8 = -7$.
7. $-5 - 7 = -12$.
8. $-11 - 11 = -22$.
9. $-5 + 7 - 9 = -7$.
10. $7 - 8 - 2 = -3$.
11. $-4 + 9 - 10 = -5$.
12. $-6 + 8 - 10 = -8$.

EXAMPLES.—ORAL.

What is a man worth, who has—

1. Rs. 12 but owes another Rs. 5.
2. ,, 9 ,, ,, ,, 9.
3 ,, 10 ,, ,, ,, 12.
4. ,, 15 ,, ,, ,, 20.

If the debt in each case is cancelled, how much better off is he?

What is the value of—

1 -9 Rs. + Rs. 3.
2. Rs. 9 − Rs. 12.
3. -10 Rs. + Rs. 7.
4 -19 Rs. − (-11 Rs.).
5. − (+ Rs. 7) + Rs. 6.
6. − (+ Rs. 4) + (− Rs. 3).
7. − (+ Rs. 8) − (− Rs. 12).
8. $+(-9) - (+9)$.
9. $-(-6) - (-9)$.
10. $7 - 10 + 2$.
11. $8 + 4 - 15$.
12. $12 - 25 + 10$.

CHAPTER II.

SIMPLE MULTIPLICATION.

33. **Multiplication** is the process of finding the sum of a given number of repetitions of the same number. It is addition in a shortened form where the addends are all equal.

Thus Rs. 15 + Rs. 15 + Rs. 15 + Rs. 15 = Rs. 60. Instead of repeating Rs. 15 four times and adding them together, we multiply Rs. 15 by 4.

The sign of multiplication is ×.

The number which is to be repeated or multiplied is called the **multiplicand**, the number which expresses how often the multiplicand is to be repeated is called the **multiplier**, and the sum of the repetitions is called the **product**. Thus in 15 × 4 = 60, the multiplicand is 15, the multiplier is 4 and the product is 60.

34. Graphic Illustration of Simple Multiplication.

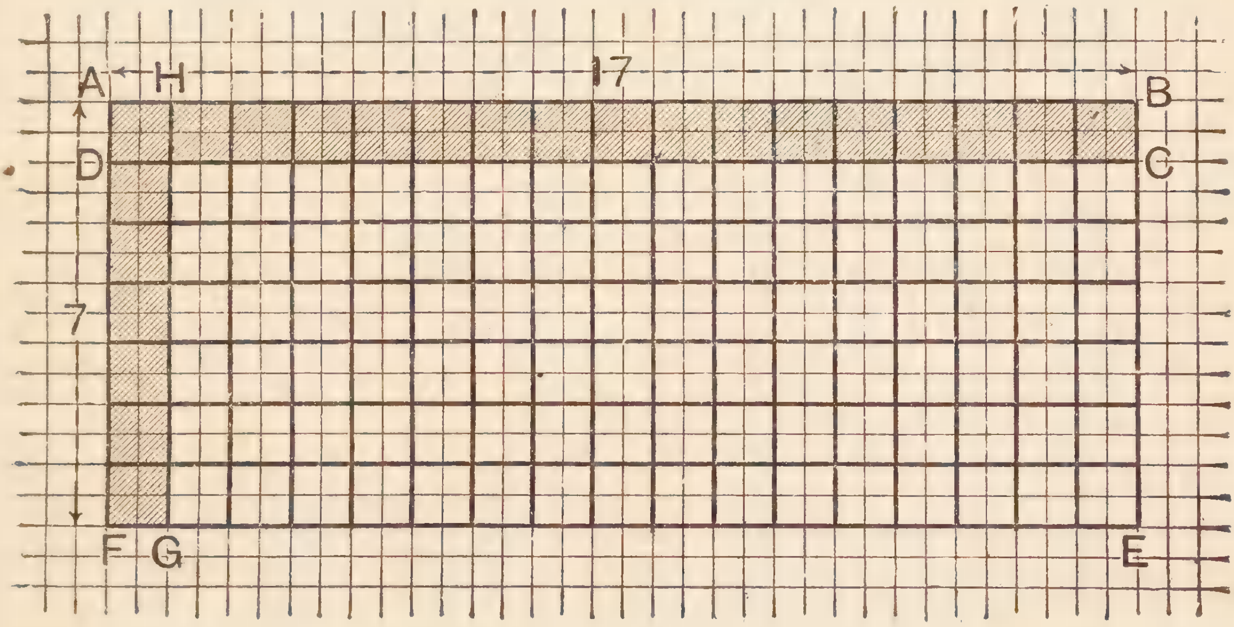

FIG. 5

Let four small squares represent the unit. Take the rectangle ABCD which contains 17 such squares. It represents 17 units. Take a row of 7 such rectangles and the rectangle ABEF is formed which contains 119 squares and therefore 119 units. Hence 17 × 7 = 119.

The product of 17 × 7 can therefore be represented by a rectangle whose sides are 17 and 7 respectively.

35. Interchange of multiplicand and multiplier does not affect the product.

In Fig. 5, the smaller rectangle AFGH contains 7 squares, i.e. 7 units; and ABEF is made up of 17 such rectangles. For if we consider AFGH as one *column* of 7 units, 17 such columns make up the whole rectangle ABEF.

Therefore the rectangle ABEF represents the product of 7 × 17.

But the same rectangle represents also the product of 17 × 7 (Art. 34) ∴ 17 × 7 = 7 × 17.

Therefore if a and b represent any two numbers, $a \times b = b \times a$.

Practical Exercise.

1. Use squared paper to prove the multiplication table from 12 times 16 up to 16 times 16.

2. Represent graphically on squared paper the product of the following; and find the product in each case.

1. 17 × 5. 2. 20 × 6. 3. 25 × 12. 4. 30 × 20.
5. 18 × 17. 6. 29 × 14. 7. 32 × 24. 8. 35 × 25.

3. Use squared paper to shew that—

1. 4 × 5 = 5 × 4. 2. 7 × 6 = 6 × 7.
3. 8 × 9 = 9 × 8. 4. 13 × 6 = 6 × 13.
5. 9 × 15 = 15 × 9. 6. 16 × 8 = 8 × 16.
7. 19 × 11 = 11 × 19. 8. 20 × 16 = 16 × 20.

36. When more than two numbers are multiplied together the result is called the **Continued Product** of the numbers.

In Fig. 5, we took 7 rows of 17 rectangles and found the product of 17 × 7 to be 119. But each of the 119 rectangles consists of 4 small squares; and the number of small squares is 476.

∴ 17 × 4 × 7 = 17 × 7 × 4 = 119 × 4 = 476.

Thus the continued product of 8, 7 and 5 = 8 × 5 × 7 = 40 × 7 = 280.

15 × 18 × 8 × 5 = 15 × 8 × 5 × 18 = 120 × 90 = 10800.

Observe here the change of order to lessen the labour of multiplication.

EXAMPLES.—ORAL.

Find the continued product of—

1. $15 \times 4 \times 6$.
2. $16 \times 3 \times 5$.
3. $15 \times 5 \times 8$.
4. $15 \times 5 \times 12$.
5. $12 \times 6 \times 15$.
6. $16 \times 7 \times 5$.
7. $16 \times 4 \times 15$.
8. $12 \times 6 \times 9$.
9. $14 \times 7 \times 5 \times 4$.
10. $15 \times 16 \times 8 \times 5$
11. $25 \times 13 \times 4 \times 7$.
12. $5 \times 5 \times 12 \times 12$.

---

37. When a number is multiplied by itself, the product is called the *square* of the number.

$6 \times 6 = 36$; 36 is the square of 6; the square of 6 is written 6^2; and is read 6 *square*.

Since 6^2 denotes 6×6, 6^2 is said to be the 2nd **power** of 6. $5 \times 5 \times 5 = 125$; and 125 is the *cube* of 5. The cube of 5 is written 5^3, and is read 5 *cube*.

Since 5^3 denotes $5 \times 5 \times 5$, 5^3 is said to be the 3rd power of 5. Similarly, 2^4 denoting $2 \times 2 \times 2 \times 2$ is said to be the 4th power of 2. The small number written to the right of and above a number is called the *index* of the power.

The square of any number $a = a \times a = a^2$.

The cube of any number $a = a \times a \times a = a^3$.

Note.—Any number may be considered as representing its *first power*. Thus 10 is 10^1, i.e. the first power of 10.

EXAMPLES.—ORAL.

1. Give the square of each of the natural numbers from 11 to 16.
2. Give the powers of each of the numbers, 2, 3, 4, 5 and 10 up to the 5th.

Practical Exercise.

1. Draw figures on squared paper to represent the square of each of the numbers from 17 to 25; and find the product in each case.
2. Construct a table of squares for the first 25 numbers.
3. Construct a table of cubes for the first 12 numbers.

---

38. Square root.

Since 36 is the square of 6, i.e. the product of 6×6, 6 is the square root of 36; also since $25 = 5 \times 5$, the square root of 25 is 5. Similarly the square root of 49 is 7; of 144 is 12. The square root of a^2 is a. Therefore the square root of a number is the number which when multiplied by itself will produce the given number.

EXAMPLES.—ORAL.

What is the square root of—

1. 16, 49, 64, 81, 121, 169, 225, 256.
2. 100, 900, 2,500, 3,600, 6,400.
3. Find from your table of squares, the numbers which when squared give numbers nearest to the following :—

24, 40, 50, 69, 80, 96, 120, 140, 170, 200, 300, 450, 500, 600.

39. To show graphically that $(20+5)\times 5 = 20\times 5+5\times 5$.

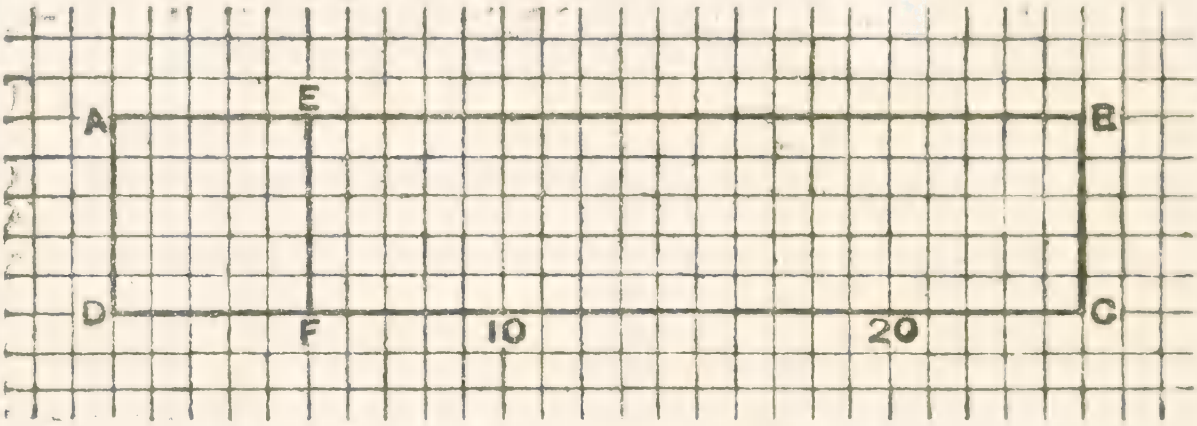

FIG. 6

In the figure, the rectangle ABCD has $(20+5)$ squares in each row ; and there are 5 such rows.

∴ The total number of squares in the rectangle ABCD$=(20\times 5)\times 5$.

The straight line EF divides the rectangle ABCD into two rectangles containing 20 squares and 5 squares in a row respectively; and in each rectangle there are five rows.

∴ The total number of squares in the rectangle ABCD$=20\times 5+5\times 5$.

∴ $(20+5)\times 5 = 20\times 5+5\times 5 = 20\times 5+5^2$.

∴ If a, b and c denote any three numbers $(a+b)\times c = ac+bc$.

Practical Exercise.

Use squared paper to shew that—

1. $(8+4)\times 9 = 8\times 9+4\times 9$.
2. $(11+5)\times 7 = 11\times 7+5\times 7$.
3. $(13+9)\times 8 = 13\times 8+9\times 8$.
4. $(17+15)\times 12 = 17\times 12+15\times 12$.
5. $23\times 5 = 20\times 5+3\times 5$.
6. $26\times 7 = 20\times 7+6\times 7$.
7. $27\times 7 = 20\times 7+7^2$.
8. $35\times 5 = 30\times 5+5^2$.

40. The multiplication of numbers of two or more digits depends on the principle explained above.

EXAMPLE 1. Thus $35 \times 7 = (30 + 5) \times 7 = 30 \times 7 + 5 \times 7$.

$$\begin{array}{r} 35 \\ 7 \\ \hline 245 \end{array}$$

We first find the product of 7×5 and add to it the product of 30×7.

7×5 units $= 35$ units $= 3$ tens $+ 5$ units; write 5 units in the units column. 7×3 tens $+ 3$ tens $= 24$ tens; write 2 hundreds and 4 tens.

EXAMPLE 2. $35 \times 27 = 35 \times (20 + 7) = 35 \times 20 + 35 \times 7$.

$$35 \times 20 = 30 \times 20 + 5 \times 20.$$

$$\begin{array}{l} 35 \\ 27 \\ \hline 700 = 35 \times 20 \\ 245 = 35 \times 7 \\ \hline 945 = 35 \times 27. \end{array}$$

5 units $\times 20 = 100$ units $= 10$ tens $+ 0$ units.
$= 1$ hundred $+ 0$ tens $+ 0$ units.
3 tens $\times 20 = 60$ tens $= 6$ hundreds.
6 hundreds $+ 1$ hundred $= 7$ hundreds.

EXAMPLE 3. Multiply 8,765 by 579.

$$579 = 500 + 70 + 9.$$

$$\begin{array}{r} 8765 \\ 579 \\ \hline 4382500 = 8765 \times 500 \\ 613550 = 8765 \times 70 \\ 78885 = 8765 \times 9 \\ \hline 5074935 = 8765 \times 579 \end{array}$$

Writing the partial products thus has the advantage of placing them in the order of their importance. For while the first line gives a rough approximation to the required result, the first and second lines added together give a nearer approximation than the first; but the addition of the three lines gives the correct result.

Or omitting the final zeros in the partial products, we may write thus:

$$\begin{array}{r} 8765 \\ 579 \\ \hline 43825 \\ 61355 \\ 78885 \\ \hline 5074935 \end{array}$$

In this case, care should be taken to place the first right hand digit of each partial product *directly under* the multiplying figure which produces it.

41. Verification of the result in Multiplication.

Since the interchange of multiplicand and multiplier does not affect the result (Art. 35) this affords a good method of testing the accuracy of the product obtained.

Use the multiplier as the multiplicand and the multiplicand as the multiplier, the two products must be equal.

Another method is by **casting out the nines**.

42. The process of casting out the nines from a given number is as follows :—

1. *Reject all nines (9's) and all digits whose sum makes 9.*

2. *Subtract 9 whenever the sum is greater than 9 ; and add the excess to the next digit.*

3. *Proceed thus till a single digit is left ; and this digit is the remainder.*

Example. Cast out nines from 536987.

First reject 9 ; next reject 3 and 6 whose sum is 9 ; and then add 5 and 8 = 13 ; reject 9 ; 4 and 7, 11 ; reject 9 ; remainder 2.

The addition and the casting out of 9's should be done mentally thus : 5, 13, 4, 11, 2.

EXAMPLES.—ORAL.

Cast out nines from—

1. 42735.
2. 509714.
3. 375478.
4. 895407.
5. 664566.
6. 544576.
7. 971342.
8. 508765.
9. 678532.
10. 468468.
11. 574737.
12. 857468.

43. The application of this process to test the accuracy of the product is as follows :—

1. *Cast out the 9's from (1) the multiplicand, (2) the multiplier, and (3) the product ; and find the remainder in each.*

2. *Multiply the first two remainders, and cast out the 9's again, if necessary.*

If this remainder is equal to the third remainder, the product may be correct.

When tested by this method, the product cannot be said to be *certainly* correct, but only *probably* correct. For this test will not disclose an error due to the misplacing of one of the partial products. If two or more of the digits be interchanged, e.g. 58 for 85, or if 0 be written for 9 or 9 for 0, the answer will, of course, be wrong ; but the test will give no hint that it is so.

Applying this test to Example 3 under Art. 40 after casting out the nines from the multiplicand 8765, multiplier 579, and product 5074935, the remainders are respectively 8, 3 and 6. The product of the first two remainders is 8 × 3 or 24. Casting out nines from 24, the remainder is 6 ; and this is also the third remainder. Therefore the product is probably correct.

EXAMPLES. I.

1. A fruit seller orders 45 baskets of mangoes, each basket containing 125. What is the total number ordered?

2. A book contains 348 pages; and each page has 37 lines. Find the total number of lines in the book.

3. A person sells 88 cawnies of land at the rate of Rs. 2,575 for a cawnie. How much does he realize?

4. An acre contains 4840 square yards; and 640 acres make one square mile. Find the number of square yards in a square mile.

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44. To shew **graphically** that $25^2 = 20^2 + \text{twice } 20 \times 5 + 5^2$.

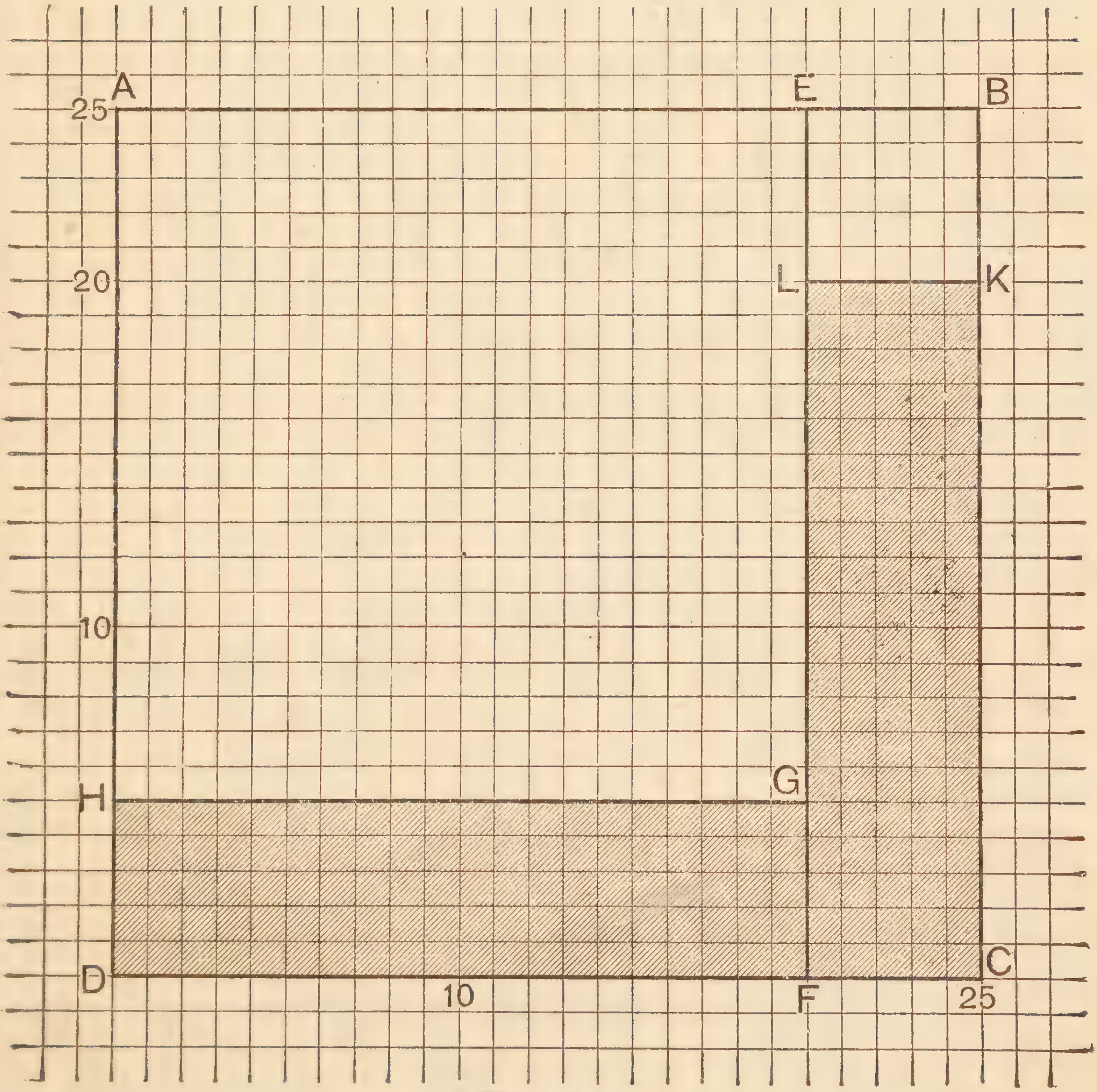

FIG. 7

Draw the square ABCD to represent 25^2; and since $25^2 = 25 \times 25 = 20 \times 25 + 5 \times 25$, divide the square into two rectangles, AEFD representing 20×25 and EBCF representing 5×25. Divide again the rectangle AEFD into two rectangles AEGH representing 20^2 and HGFD representing 20×5; also divide EBCF into square EBKL representing 5^2 and rectangle LKCF representing 5×20.

Now HGFD and LKCF are equal to one another since each contains the same number of squares, viz. 20×5.

∴ HGFD and LKCF = twice HGFD.

∴ Square ABCD = square AEGH + twice HGFD + square EBKL.

∴ $25^2 = 20^2 +$ twice $20 \times 5 + 5^2$.

If a and b denote any two numbers $(a+b)^2 = a^2 + 2ab + b^2$.

The square of the sum of two numbers is equal to the square of the first number plus *the square of the second number* plus *twice the product of the two numbers.*

EXAMPLES.—ORAL.

1. Find the product of—

1. 19×17. 2. 21×18. 3. 33×23. 4. 26×25.
5. 42×28. 6. 29×32. 7. 37×27. 8. 39×29.

2. Find the squares of any of the numbers from 17 to 99.

Practical Exercise.

Draw diagrams on squared paper to shew that—

1. $15^2 = 10^2 +$ twice $10 \times 5 + 5^2$. 2. $19^2 = 10^2 +$ twice $10 \times 9 + 9^2$.
3. $23^2 = 20^2 + 2 \times 20 \times 3 + 3^2$. 4. $35^2 = 30^2 + 2 \times 30 \times 5 + 5^2$.

Draw diagrams similarly for the squares of any numbers given out by the teacher.

---

45. Multiplication and Subtraction in one operation.

EXAMPLE. Subtract 16 times 49584 from 870123.

Multiplying by 16 and subtracting by the method of complementary addition, we have

870123	$16 \times 4 = 64$ and 9, 73; write 9 and carry 7.
49584 × 16	$16 \times 8 + 7 = 135$ and 7, 142; ,, 7 ,, ,, 14.
————	$16 \times 5 + 14 = 94$ and 7, 101; ,, 7 ,, ,, 10.
76779 Ans.	$16 \times 9 + 10 = 154$ and 6, 160; ,, 6 ,, ,, 16.
	$16 \times 4 + 16 = 80$ and 7, 87; ,, 7 ,, ,, 8.
	8 and 0, 8.

EXAMPLES. II.

To be done in a single step—

1. From a purse containing Rs. 750, a sum of Rs. 132 is taken thrice. What is left in the purse?

2. If I make 5 payments each of Rs. 245, what will remain out of Rs. 16,250?

3. A boy has to travel a distance of 5,280 yards. He has walked 4 hours at the rate of 1,236 yards an hour. How far is he from his journey's end?

4. The population of a city is 4 lakhs. If the number of Hindus, Mahomedans and Christians be each 86,543, what is the strength of the others?

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46. To shew graphically that $16^2 - 13^2 = 29 \times 3$.

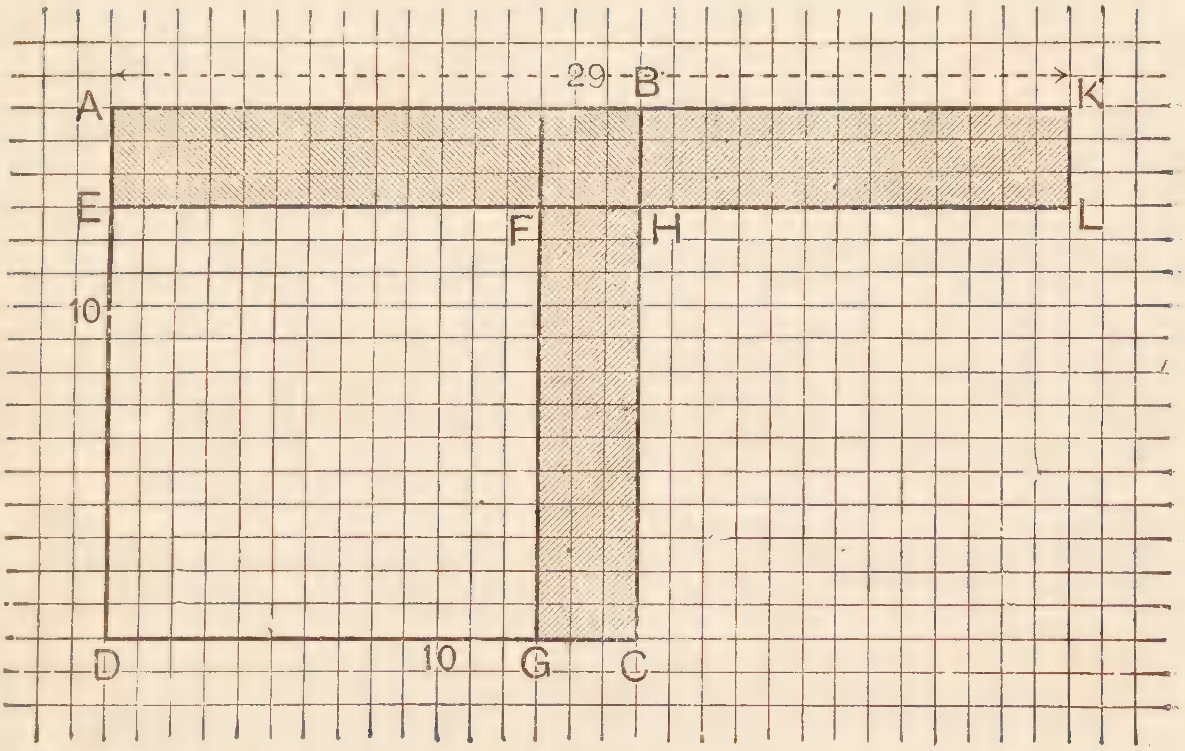

FIG. 8

Let each division represent 1 unit.

Draw the figure ABCD to represent the square of 16; and the figure EFGD representing the square of 13. Produce EF to H. From ABCD take away EFGD; and the remainder = ABHE + FGCH.

From the point B measure a length BK equal to CH, i.e. 13 units; and complete the rectangle BKLH.

The rectangle BKLH is equal to the rectangle FGCH, since each represents 13×3 and contains 39 squares.

$\therefore$ The remainder $=$ ABHE $+$ BKLH $=$ AKLE.

And the rectangle AKLE represents the product of 29×3.

$\therefore 16^2 - 13^2 = 29 \times 3$.

29 is the sum of 16 and 13 ; and 3 is the difference between 16 and 3. Therefore $16^2 - 13^2 = (16 + 13) \times (16 - 13)$.

Hence if a and b stand for any two numbers, $a^2 - b^2 = (a + b) \times (a - b)$.

The difference between the squares of two numbers is equal to the product of the sum and difference of the two numbers.

Practical Exercise.

Draw figures on squared paper to shew that—

1. $12^2 - 4^2 = 16 \times 8$.
2. $15^2 - 12^2 = 27 \times 3$.
3. $18^2 - 13^2 = 31 \times 5$.
4. $19^2 - 12^2 = 31 \times 7$.
5. $21^2 - 15^2 = 36 \times 6$.
6. $24^2 - 16^2 = 40 \times 8$.
7. $23^2 - 18^2 = 41 \times 5$.
8. $27^2 - 20^2 = 47 \times 7$.

EXAMPLE. Find by the easiest method, the product of—

(1) 81×79. (2) 63×57.

(1) $81 \times 79 = (80 + 1)(80 - 1) = 80^2 - 1^2 = 6400 - 1 = 6399$.

(2) $63 \times 57 = (60 + 3)(60 - 3) = 60^2 - 3^2 = 3600 - 9 = 3591$.

EXAMPLES.—ORAL.

Required the value of—

1. $19^2 - 11^2$.
2. $21^2 - 18^2$.
3. $18^2 - 13^2$.
4. $25^2 - 21^2$.
5. $30^2 - 21^2$.
6. $32^2 - 27^2$.
7. $35^2 - 25^2$.
8. $40^2 - 35^2$.
9. 31×29.
10. 53×47.
11. 62×58.
12. 74×68.

---

47. To multiply by 9, 99, 999, etc.

(1) To multiply by 9, 99, 999, add *one* cypher, *two* cyphers, *three* cyphers respectively, at the end of the multiplicand and subtract the multiplicand since $9 = 10 - 1$, $99 = 100 - 1$, and $999 = 1000 - 1$.

EXAMPLES.—ORAL.

Find the product of—

1. 19×9.
2. 25×9.
3. 36×9.
4. 16×12.
5. 18×19.
6. 25×29.
7. 27×29.
8. 18×21.
9. 27×31.
10. 32×31.
11. 35×41.
12. 45×52.

48. In some cases, arranging the digits of the multiplier in a different order helps to lessen labour in multiplication.

EXAMPLE 1. Multiply 87694 by 357.

$$\begin{array}{r} 87694 \\ \underline{357} \\ 613858 \\ \underline{3069290} \\ 31306758 \end{array}$$

357 = 350 + 7 = 7 + 350 = 7 + 7 × 50 = 7 + 7 × 5 tens. Multiply by 7 and write the product. Multiply this product by 5 tens, i.e. multiply by 5 and write the first resulting figure in the tens' place.

EXAMPLE 2. Find the value of 35678 × 18272.

$$\begin{array}{r} 35678 \\ \underline{18272} \\ 71356 \\ 642204 \\ \underline{2568816} \\ 651908416 \end{array}$$

18272 = 18000 + 200 + 72 = 200 + 18000 + 72
= 2 hundred + 9 tens × 200 + 18 × 4 units.

The first partial product is the product by 200; the next partial product is the product of this product by 9 tens; the third partial product is the product of the second partial product by 4.

EXAMPLES. III.

1. Multiply in two lines—

(1) 6298 by 639. (2) 8764 by 366.
(3) 1899 by 355. (4) 7812 by 497.

2. Multiply using only three partial products—

(1) 879045 by 84217. (2) 967508 by 840426.
(3) 1234567 by 256648. (4) 2345678 by 486819.

3. Find the product in two lines of—

(1) 4985 and 864. (2) 45678 and 735.
(3) 56789 and 963. (4) 67890 and 872.

4. Multiply using the shortest method possible—

(1) 97536 by 6118. (2) 80605 by 16232.
(3) 73965 by 27354. (4) 7891011 by 24872.

CHAPTER III.

SIMPLE DIVISION.

49. Division is the process of finding how often one quantity is contained in another or of separating a given quantity into a certain number of equal parts.

The former process is known as **Quotition** and the latter is known as **Partition.**

Finding how often one quantity is contained in another is the same as finding how many times one quantity can be *subtracted* from another.

Thus to find how many times Rs. 10 is contained in Rs. 40 is to find how many times Rs. 10 can be taken away from Rs. 40. In this sense, division may be regarded as repeated subtraction where the subtrahend is constant.

50. The quantity to be divided is called the **Dividend**, the quantity by which it is divided is called the **Divisor**; and the result obtained is known as the **Quotient**; and the quantity left at the end of the process is known as the remainder.

The sign of division is ÷, which indicates that the quantity preceding it should be divided by the quantity following it.

51. In Quotition, both dividend and divisor are of the same name.

EXAMPLE. (1) 112 ÷ 16, also written $\frac{112}{16} = 7$, where dividend, divisor and quotient are abstract numbers.

(2) Rs. 112 ÷ Rs. 16 = 7, where dividend and divisor are concrete, while the quotient is abstract.

In partition, the dividend is concrete, the divisor is abstract and the quotient is concrete.

EXAMPLE. Rs. 112 ÷ 16 = Rs. 7.

52. Graphic representation of Simple Division.

EXAMPLE. Divide 192 by 16.

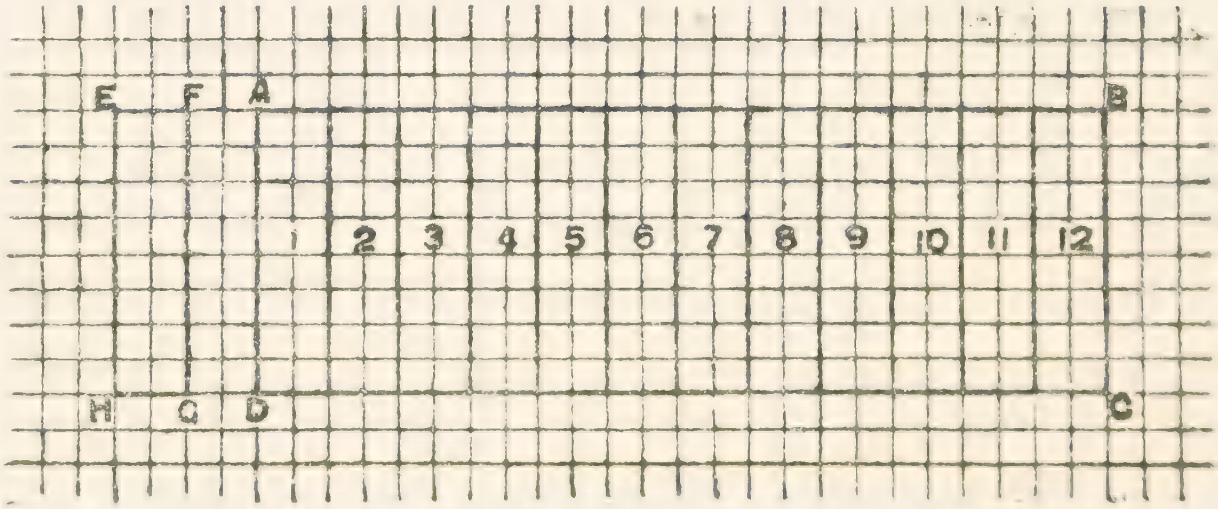

FIG. 9

Let each small square represent 1 unit.

The rectangle ABCD which contains 192 squares represents 192 units; and the rectangle EFGH, containing 16 squares represents 16 units.

The larger rectangle ABCD can be divided as shewn in the figure into 12 smaller rectangles each containing 16 squares, and therefore each equal to EFGH; and there is no remainder.

∴ $192 \div 16 = 12$.

Or, measure a length of 192 divisions on squared paper and mark off distances representing 16 divisions and find how many times this could be done.

EXAMPLE. Divide Rs. 9,256 equally among 8 persons.

$$\begin{array}{r|cccc} & \text{Th} & \text{H} & \text{T} & \text{U} \\ 8 & 9 & 2 & 5 & 6 \\ \hline \text{Rs.} & 1 & 1 & 5 & 7 \end{array}$$

9 thousand by 8, 1 thousand and remainder 1 thousand, 1 thousand and 2 hundreds = 12 hundreds; 12 hundreds by 8, 1 hundred and remainder 4 hundreds; 4 hundreds and 5 tens = 45 tens; 45 tens by 8, 5 tens and remainder 5 tens; 5 tens and 6 units = 56 units. 56 units by 8 = 7 units; and no remainder.

Verification.– Multiply the quotient by the divisor and so get the dividend. Thus $1157 \times 8 = 9256$.

53. To multiply a number by 5, multiply it by 10, that is, add *one* cypher to the number, and divide the number thus formed by 2; for $5 = 10 \div 2$.

EXAMPLE. $78 \times 5 = 780 \div 2 = 390$.

To multiply a number by 25, multiply it by 100, that is, add *two* cyphers to the number, and divide the number thus formed by 4; for $25 = 100 \div 4$.

EXAMPLE. $87 \times 25 = 8700 \div 4 = 2175$.

To multiply a number by 125, multiply it by 1000, that is, add *three* cyphers to the number and divide the number thus formed by 8; for $125 = 1000 \div 8$.

EXAMPLE. $27 \times 125 = 27000 \div 8 = 3375$.

54. In exact division, Dividend = Divisor × Quotient.

Dividend ÷ Divisor = Quotient.

Dividend ÷ Quotient = Divisor.

∴ Any two of these three, Dividend, Divisor and Quotient being given, the third may be found.

Practical Exercise.

Divide graphically by taking rectangles or straight lines on squared paper—

1. 80 by 5, 8, 10 and 16 respectively.
2. 96 by 8, 12, 16 and 24 ,,
3. 112 by 7, 8, 16 and 28 ,,

EXAMPLES.—ORAL.

1. How often can Rs. 9 be taken from Rs. 72 ?
2. How many lengths of 11 inches can be cut off from 121 inches ?
3. How often is Rs. 7 contained in 56, 91, 105, 119 rupees ?
4. How often can £12 be taken from 96, 132, 180, 156 pounds ?
5. Multiply each of the following by 5 :—23, 29, 37, 46, 58, 69, 84, 98.
6. Multiply each of the following by 25 :—13, 17, 19, 22, 26, 29, 32, 48, 51.
7. Multiply each of the following by 125 :—24, 32, 48, 64, 15, 25, 35, 45.
8. Give out the number omitted in the following :—

	Dividend	Divisor	Quotient
(1)	—	17	12
(2)	112	—	8
(3)	210	—	15
(4)	—	18	9

EXAMPLES. Divide 10774 by 12.

12|10774
897–10 units
Quotient 897 and
Remainder 10.

107 hundreds by 12, 8 hundreds and remainder 11 hundreds. 11 hundreds and 7 tens = 117 tens ; 117 tens by 12 ; 9 tens and 9 tens remainder ; 9 tens and 4 units = 94 units ; 94 units by 12, 7 units and remainder 10 units.

55. Verification. Multiply the divisor by the quotient and add the remainder to the product and the result should be equal to the dividend.

In this Example, $897 \times 12 + 10 = 10764 + 10 = 10774$.

56. In inexact division, that is, when there is a remainder, Dividend = Divisor × Quotient + remainder.

Graphic Illustration. Divide 245 by 12.

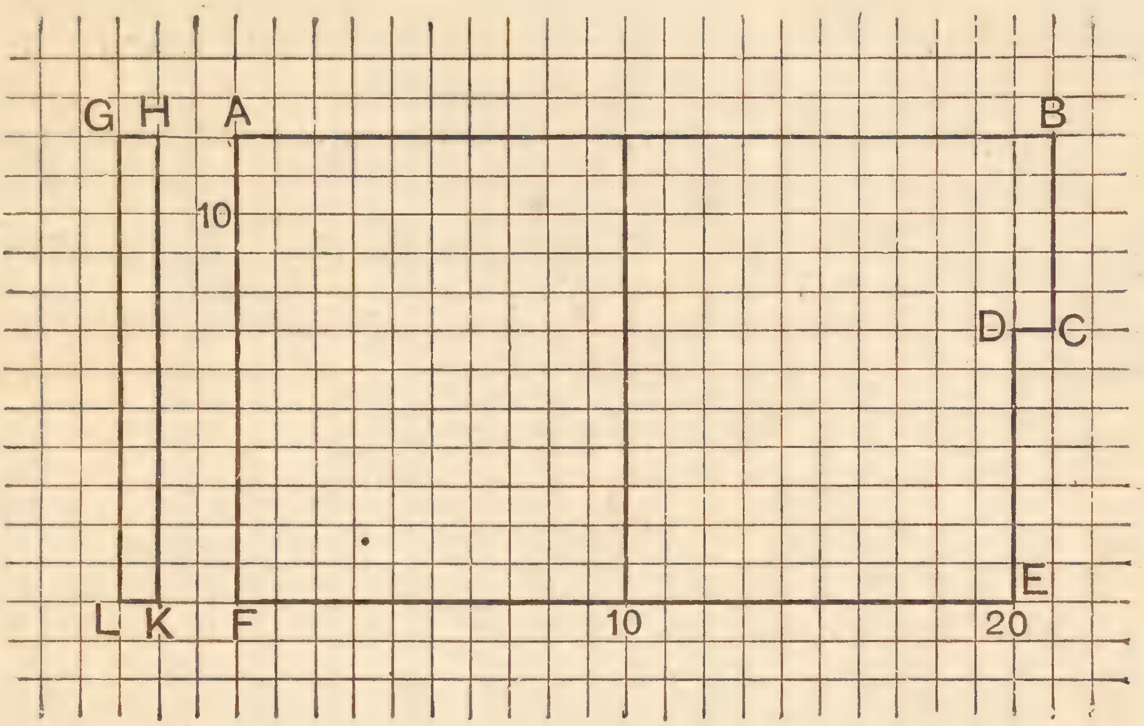

FIG. 10

Let each small square represent 1 unit. Then the figure ABCDEF which contains 245 squares represents the dividend 245; and the rectangle GHKL containing 12 squares represents the divisor 12. The figure ABCDEF is divided into 20 such rectangles as GHKL and there is a remainder of 5 squares.

$\therefore$ ABCDEF = 20 times GHKL + 5 squares.

= 20 × 12 squares + 5 squares.

$\therefore\quad 245 = 20 \times 12 + 5.$

Practical Exercise.

1. Divide graphically 91 by 5, 9, 13, 15, separately and find the remainder in each case. Use your diagram to shew Divisor × Quotient + Remainder = Dividend.

2. Prove graphically the same truth in the division of 174 by 8, 11, 13, 15, 16 separately.

---

57. Inexact division, that is, when there is a remainder, since Dividend = Divisor × Quotient + Remainder, we get

Divisor = (*Dividend* — *Remainder*) ÷ *Quotient* ;

Quotient = (*Dividend* — *Remainder*) ÷ *Divisor* ; *and*

Remainder = *Dividend* — (*Divisor* × *Quotient*).

Any three of these four being given, the fourth may be found.

EXAMPLE. (1) If the divisor is 15, the quotient 14, and the remainder 7, the dividend = 15 × 14 + 7 = 217.

(2) If the dividend is 217, the quotient 14, and the remainder 7, the divisor = (217 − 7) ÷ 14 = 15.

(3) If the dividend is 217, the divisor 15, and the remainder 7, the quotient = (217 − 7) ÷ 15 = 14.

(4) If the dividend is 217, the divisor 15, and the quotient 14, the remainder = 217 − (15 × 14) = 7.

EXAMPLES.—ORAL.

1. Divisor 16, quotient 12, remainder 11 ; dividend ?
2. Divisor 15, quotient 14, remainder 4 ; dividend ?
3. Dividend 287, quotient 20, remainder 7 ; divisor ?
4. Dividend 259, quotient 15, remainder 4 ; divisor ?
5. Dividend 263, divisor 25, remainder 13 ; quotient ?
6. Dividend 260, remainder 4, divisor 16 ; quotient ?
7. Dividend 226, divisor 17, quotient 13 ; remainder ?
8. Dividend 242, divisor 16, quotient 15 ; remainder ?

EXAMPLE. Find the least number of Rs. that should be taken from Rs. 31 so that it may be exactly divisible among 9 persons.

Let each division in the squared paper denote 1 rupee.

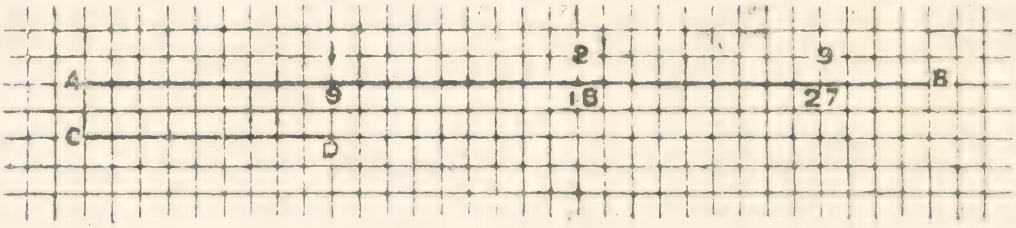

FIG. 11

Take A B equal to 31 rupees and C D equal to 9 rupees. Mark off from A B as many lengths equal to C D, as you can. This can be done 3 times ; and the remainder is Rs. 4. ∴ The least number of Rs. that should be taken is Rs. 4.

If the least number of Rs. that should be added is required, it is evident that it is 5, i.e. 9 − 4.

EXAMPLES.—ORAL.

1. A purse contains Rs. 100 ; what is left in it after giving 12 persons Rs. 8 each ?

2. I measure 5 lengths of 16 inches each from a tape 84 inches long. What length remains over ?

3. Find the remainders when Rs. 180 is divided equally among 13, 14, 16 and 17 persons respectively.

4. What is the least number that must be taken from 120 that it may be divisible exactly by 7, 9, 11, 13, 14 and 16 respectively ?

5. Find the least number that should be added to 172 so that the sum may be exactly divisible by 8, 9, 11, 12, 13, 15 and 16 respectively.

Division by factors.

58. When one number consists of the product of two or more numbers, each of the latter is called a **factor** of the product.

$6 \times 7 = 42$;. 6 and 7 are factors of 42.

$3 \times 4 \times 5 = 60$; 3, 4 and 5 are factors of 60.

59. **A prime factor** is a factor which is divisible only by unity or by itself.

6 and 7 are factors of 42 ; here 6 is not a prime factor, for it is divisible by 2 and 3 ; but 7 is a prime factor for no other number except 1 and 7 can divide 7 exactly.

60. When the division is exact, division by factors presents no difficulty ; but when the division is inexact, the following examples will shew how the remainders are obtained.

EXAMPLE 1. Divide 1874 by 42.

$$42 = 6 \times 7.$$

Explanation

$$\begin{array}{r|l} 6 & 1874 \text{ units} \\ \hline 7 & 312 \text{ groups of 6 and 2 units over} \\ \hline & \;\; 44 \text{ groups of 42 and 4 groups of 6 over} \end{array}$$

Working

$$\begin{array}{r|llll} 6 & 1874 & & & \\ \hline 7 & 312 & . & . & . \; 2 \\ \hline & \;\; 44 & . & . & . \; 4 \end{array}$$

∴ Complete remainder = 4 groups of 6 + 2 units = $4 \times 6 + 2 = 26$.

∴ Quotient = 44 ; and remainder = 26.

EXAMPLE 2. Divide 4569 by 112 by means of 3 factors.

$$112 = 4 \times 4 \times 7.$$

Eplanation

$$\begin{array}{r|l} 4 & 4569 \text{ units} \\ \hline 4 & 1142 \text{ groups of 4 and 1 unit over} \\ \hline 7 & 285 \text{ groups of 16 and 2 groups of 4 over} \\ \hline & \;\; 40 \text{ groups of 112 and 5 groups of 16 over} \end{array}$$

Working

$$\begin{array}{r|llll} 4 & 4569 & & & \\ \hline 4 & 1142 & . & . & . \; 1 \\ \hline 7 & 285 & . & . & . \; 2 \\ \hline & \;\; 40 & . & . & . \; 5 \end{array}$$

Complete remainder $= 5 \times 16 + 2 \times 4 + 1 = 80 + 8 + 1 = 89$.

61. To find the complete remainder in short division. **Graphic representation.**

37 divided by 3 and 5 successively.

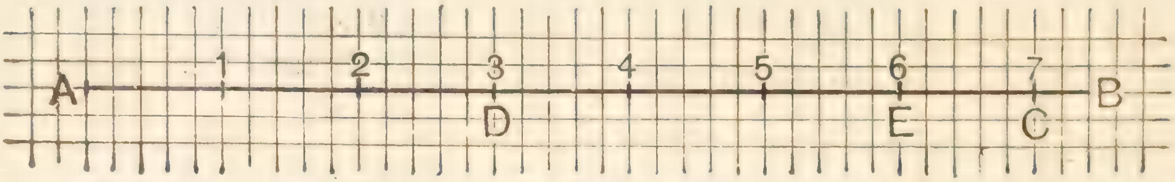

FIG. 12

Let one division represent 1 unit.

Take a line AB, 37 units in length, and measure off from it lengths of 5 units each. This can be done 7 times and there is a remainder CB of 2 units.

Next mark off these 5 units-lengths 3 at a time and get the points D and E; and the remainder is EB which contains 7 units.

$$\begin{aligned} EB &= EC + CB \\ &= 1 \text{ length of } 5 \text{ units} + 2 \text{ units} \\ &= 5 \text{ units} + 2 \text{ units} = 7 \text{ units.} \end{aligned}$$

62. By Rectangles. To divide 123 by 35, i.e. by 7 × 5 and to find the complete remainder.

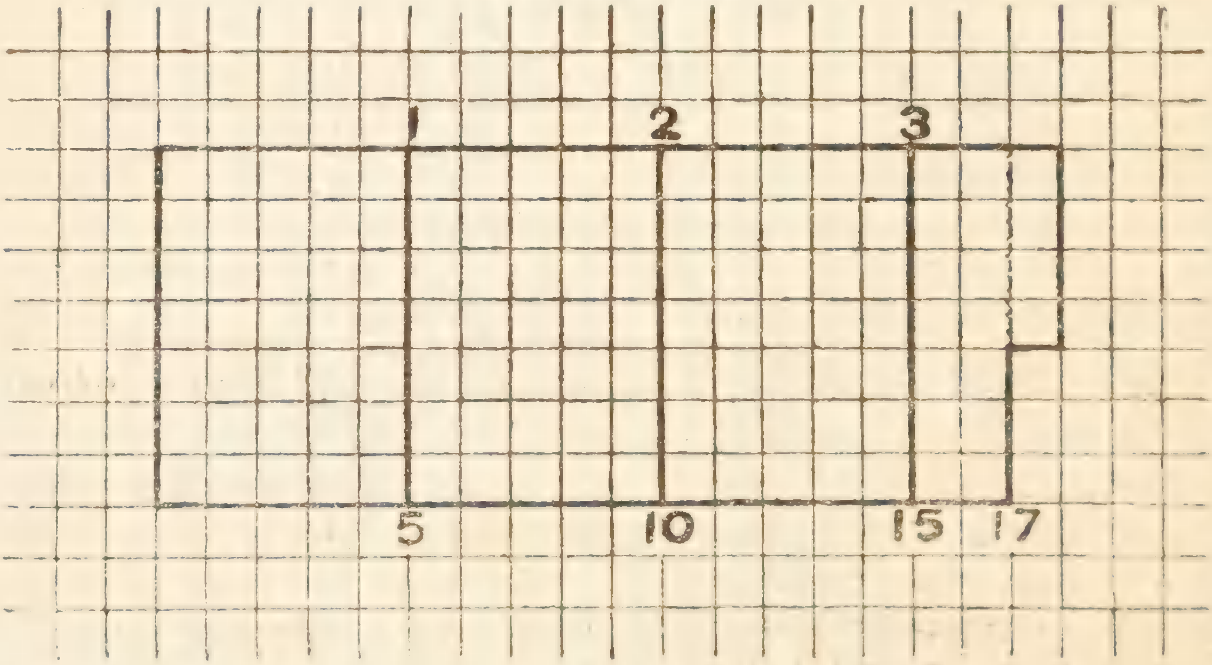

FIG. 13

Let each square denote 1 unit.

The figure represents 123 units, because it contains 123 squares.

Dividing the figure into columns of 7 squares each, we get 17 columns and 4 squares over.

Now take 5 columns of 7 squares at a time, and we see this can be done 3 times; and the remainder is 2 columns of 7 squares each and 4 squares.

∴ The complete remainder = 2 columns of 7 squares + 4 squares.
= 14 squares + 4 squares.
= 18 squares = 18 units.

∴ The remainder is of the form $2 \times 7 + 4$; and this is the form of the complete remainder in the division of 123 by 7×5.

EXAMPLES.—ORAL.

Required the factors of—

(1) 24. (2) 25. (3) 35. (4) 49. (5) 56. (6) 64.
(7) 72. (8) 84. (9) 96. (10) 108. (11) 144. (12) 192.

Give the prime factors of—

(1) 24. (2) 42. (3) 56. (4) 72. (5) 84. (6) 108.

EXAMPLES. I.

Prove graphically the correctness of your result in examples 5 and 6.

Divide using factors—

1. 8625 by 25.
2. 479168 by 64.
3. 170016 by 84.
4. 420255 by 165.

Divide by factors and find the remainder—

5. 125 by 24, 32, 36 and 42.
6. 232 by 25, 42, 63 and 84.

Divide using three factors in each case—

7. 8765 by 112, 132, 144 and 168.
8. 10697 by 105, 125, 128 and 192.
9. Divide by means of prime factors—
   235948 by 75, 99, 154 and 165.

63. Hitherto the divisor or the factors of a divisor did not exceed 16. We shall now deal with division where the divisor is a larger number.

EXAMPLE 1. Divide 625582 by 167.

```
167)625582(3000
    501000  700
    ------   40
    124582    6
    116900
    ------
      7682
      6680
      ----
      1002
      1002
      ----
```

As in multiplication, the final cyphers in the successive products are omitted in division and the process is shewn thus:

```
       3746
       ----
167)625582
    501
    ----
    1245
    1169
    ----
     768
     668
    ----
    1002
    1002
    ----
```

It is advantageous to write the first digit of the quotient which, in this example, is *thousands* directly *above the thousands' place* in the dividend; and then the other digits follow in their proper places.

65. The Italian Method of Division. In this method, the partial products are omitted; and the processes of finding these products and subtracting them from the successive dividends are done in *one* operation, the remainders only being written, as shewn below.

```
       3746
       ----
167)625582
    ----
    1245
    ----
     768
    ----
    1002
    ----
```

The mental work is as follows:—

3 × 7 = 21 and 4, 25; write 4 and carry 2
3 × 6 + 2 = 20 and *2*, 22 ,, 2 ,, 2
3 × 1 + 2 = 5 and *1*, 6 ,, 1 ,,
Bring down the 5 from the dividend.
7 × 7 = 49 and *6*, 55; write 6 and carry 5
7 × 6 + 5 = 47 and 7, 54; ,, 7 ,, 5
7 × 1 + 5 = 12 and 0, 12; Bring down the 8
4 × 7 = 28 and 0, 28; write 0 and carry 2
4 × 6 + 2 = 26 and 0, 26; ,, 0 ,, 2
4 × 1 + 2 = 6 and *1*, 7; ,, 1. Bring down the 2
6 × 7 = 42 and 0, 42; ,, 0 and carry 4
6 × 6 + 4 = 40 and 0, 40; ,, 0 ,, 4
6 × 1 + 4 = 10 and 0, 10. No remainder.

65. Verification of the result of division by casting out nines. Cast out the 9's from the quotient and from the divisor, and multiply these two remainders. Cast out the 9's again, if necessary, and add the result to that obtained by casting out the 9's from the remainder. Cast out the 9's again if necessary; and the result must be that obtained by casting out the 9's from the dividend.

EXAMPLE. Apply this process to Example 2 below.

(1) Casting out the 9's from the quotient, the remainder is 2.
(2) Casting out the 9's from the divisor, the remainder is 0.
(3) The product of these two remainders is 0.
(4) Casting out the 9's from the remainder, the remainder is 8.
(5) Adding (3) and (4) we get $0 + 8 = 8$.
(6) Casting out 9's from the dividend, the remainder is 8.

Since (5) and (6) are equal, the answer is *probably* correct; for if any of the integers be wrongly placed, the test will be of no value in determining the correctness of the result.

EXAMPLE 2. Divide true to the nearest unit 142550 by 297.

Working by the Italian method, we have.

$$\begin{array}{r} 479 \\ 297)\overline{142550} \\ \overline{2375} \\ \overline{2960} \\ \overline{287} \end{array}$$

Here the final remainder 287 is more than half of the divisor 297; and the quotient true to the nearest unit is 480; because 480 × 297 or 142560 is nearer 142550 than 479 × 297 which is 142263.

If the remainder is less than half of the divisor, the quotient itself is the number required to the nearest unit. Thus in the division of 678509 by 987, the quotient is 687 and the remainder is 440; and since 440 is less than half of the divisor 987, the answer is 687.

EXAMPLES. II.

Divide by the Italian method—

1. Rs. 13249 among 128 boys. 2. £16727 among 392 men.
3. £14255 among 297 girls. 4. Rs. 653585 among 252 women.
5. 3968901 by 735. 6. 59996156 by 1848.
7. 35300897 by 2268. 8. 63439456 by 1575.

Test the correctness of the answers by casting out nines.

Find the number nearest to—

9. 789101 which is exactly divisible by 75.
10. 307090 ,, ,, ,, 209.
11. 786543 ,, ,, ,, 625.
12. 7089906 ,, ,, ,, 599.

Divide true to the nearest unit—

13. 24763 by 672
14. 143255 by 432.
15. 345678 by 587.
16. 6810249 by 979.

---

66. Some special methods for shortening work in Division will now be given.

To divide by 5, 25 and 125.

(1) To divide a number by 5, multiply it by 2 and divide the product by 10.

EXAMPLE 1. $785 \div 5 = 785 \times 2 \div 10 = 1570 \div 10 = 157$.

2. $697 \div 5 = 697 \times 2 \div 10 = 1394 \div 10 = 139$ quotient; and remainder is not 4 but $4 \div 2$ or 2, since the dividend has been multiplied by 2.

(2) To divide a number by 25, multiply it by 4 and divide the product by 100.

EXAMPLE 1. $1800 \div 25 = 1800 \times 4 \div 100 = 7200 \div 100 = 72$.

2. $1879 \div 25 = 1879 \times 4 \div 100 = 7516 \div 100 = 75$ quotient; and the remainder is not 16, but $16 \div 4$, that is 4.

(3) To divide a number by 125, multiply it by 8 and divide the product by 1000.

EXAMPLE 1. $8625 \div 125 = 8625 \times 8 \div 1000 = 69000 \div 1000 = 69$.

2. $9876 \div 125 = 9876 \times 8 \div 1000 = 79008 \div 1000 = 79$ quotient; and the remainder is $8 \div 8$ or 1.

Tests of Divisibility.

67. A number that is exactly divisible by another number is said to be its **multiple.**

EXAMPLE. 10 is divisible by 2; and so 10 is a multiple of 2.

10 is also divisible by 5; and so 10 is a multiple of 5.

5×10 or 50 is a multiple of 2, or 5, or 10; for each of these divides 50 exactly.

68. The Test for 2 and 5.

Since 2 divides 10, it divides any multiple of 10.

Since 5 divides 10, it divides any multiple of 10; and any multiple of 10 ends in 0.

Therefore 2 and 5 each divides a number that ends in 0.

Any number greater than 10 which does not end in *a cypher* is composed of a multiple of 10 + the units' digit; and such a number can only be divisibe by 2 or 5, if the last digit, viz. the units' digit is divisible by 2 or 5.

Example 1. 128 = 120 + 8 = a multiple of 10 + 8.
= a multiple of 10 + a multiple of 2

∴ 128 is a multiple of 2 and is divisible by 2.

Example 2. 725 = 720 + 5 = a multiple of 10 + 5
= a multiple of 5 + a multiple of 5

∴ 725 is divisible by 5.

69. The Test for 4 and 25.

100 is divisible both by 4 and by 25. Any number ending in *two cyphers* is a multiple of 100 and is therefore divisible both by 4 and by 25.

Any number greater than 100 which does not end in 2 cyphers is composed of a multiple of 100 + the number expressed by the last two digits; and such a number can only be divisible by 4 or 25, if the number expressed by the last two digits is divisible by 4 or 25.

Example 1. 3148 = 3100 + 48 = 31 × 100 + 4 × 12
= a multiple of 100 + a multiple of 4
= a multiple of 4 + a multiple of 4

∴ 3148 is divisible by 4.

Example 2. 7875 = 7800 + 75 = 78 × 100 + 3 × 25
= a multiple of 25 + a multiple of 25

∴ 7875 is divisible by 25.

70. The Test for 8.

1000 is divisible by 8; and any number ending in *three cyphers* is a multiple of 8 and is therefore divisible by 8. Any number greater than 1000 which does not end in *3 cyphers* is a multiple of 1000 + the number expressed by the three last digits. If this number is divisible by 8, the whole number is divisible by 8.

Example. 18824 = 18000 + 824 = 18 × 1000 + 8 × 103
= a multiple of 1000 + a multiple of 8

Since the number 824 is divisible by 8, the number 18824 also is divisible by 8.

71. The Test for 9 or 3.

Since $10 = 9 + 1$, any number greater than 10 can be expressed as a multiple of $9 +$ the sum of its digits.

Take, for example, the number 8937

$$\begin{aligned}
8937 &= 8000 + 900 + 30 + 7.\\
&= 8 \times 1000 + 9 \times 100 + 3 \times 10 + 7.\\
&= 8 \times (999 + 1) + 9 \times (99 + 1) + 3 \times (9 + 1) + 7.\\
&= 8 \times 999 + 8 + 9 \times 99 + 9 + 3 \times 9 + 3 + 7.\\
&= \text{a multiple of } 9 + 8 + \text{a multiple of } 9 + 9 + \text{a multiple of } 9 + 3 + 7.\\
&= \text{a multiple of } 9 + 8 + 9 + 3 + 7.\\
&= \text{a multiple of } 9 + \text{the sum of the digits of the number } 8937.
\end{aligned}$$

Now this number can only be divisible by 9, if the sum of its digits is divisible by 9. The sum 27 is divisible by 9; and therefore 8937 is divisible by 9.

Since 9 is a multiple of 3, the number can be expressed as a multiple of $3 +$ the sum of its digits.

A number is therefore divisible by 3, if the sum of its digits is divisible by 3.

72. The Test for 11.

$$\begin{aligned}
10 &= 11 - 1\\
100 = 99 + 1 &= 11 \times 9 + 1 = \text{a multiple of } 11 + 1\\
1000 = 1001 - 1 &= 11 \times 91 - 1 = \text{a multiple of } 11 - 1\\
10000 = 9999 + 1 &= 11 \times 909 + 1 = \text{a multiple of } 11 + 1 \text{ and so on.}
\end{aligned}$$

In the same way, the number 7348 may be thus written:

$$\begin{aligned}
7348 &= 7000 + 300 + 40 + 8.\\
&= 7 \times 1000 + 3 \times 100 + 4 \times 10 + 8\\
7000 &= 7 \times 1000 = \text{a multiple of } 11 - 7\\
300 &= 3 \times 100 = \text{a multiple of } 11 + 3\\
40 &= 4 \times 10 = \text{a multiple of } 11 - 4\\
8 &= + 8\\
\\
7348 &= \text{a multiple of } 11 + 8 + 3 - 4 - 7\\
&= \text{a multiple of } 11 + 11 - 11\\
&= \text{a multiple of } 11 + 0.
\end{aligned}$$

The sum of 8 and 3 are $11 =$ the sum of the digits in the odd places; and the sum of -4 and -7 or $-11 =$ the sum of the digits in the even places.

The number 7348 is divisible by 11.

Hence a number is divisible by 11, if the difference between the sum of its digits in the odd places and the sum of its digits in the even places is 0, or is divisible by 11.

73. Hence, we have the following tests of divisibility :—

A number is exactly divisible by—

2, if it ends in 0 or if its last digit is divisible by 2.

EXAMPLE. 50, 56.

3, if the sum of its digits is divisible by 3.

EXAMPLE. 456 is divisible by 3 ; since 4 + 5 + 6 or 15 is divisible by 3.

4, if the number expressed by its two last digits is divisible by 4.

EXAMPLE. 3148 is divisible by 4, since 48 is divisible by 4. For the same reason, 5108 is divisible by 4.

5, if its last digits is 0 or 5. EXAMPLES. 230, 725.

6, if it is divisible by 2 and 3. EXAMPLES. 1842, 2340.

8, if the number expressed by its three last digits is divisible by 8.

EXAMPLE. 18,824 is divisible by 8, since 824 is divisible by 8. For the same reason 15,008 and 29,016 are each divisible by 8.

9, if the sum of its digits is divisible by 9.

EXAMPLE. 89,037 is divisible by 9, since 8 + 9 + 0 + 3 + 7 or 27 is divisible by 9.

10, if it ends in cypher.

11, if the difference between the sum of its digits in the odd places and the sum of its digits in the even places be 0 or be divisible by 11.

EXAMPLE 1. 17,248 is divisible by 11 ; some of the odd digits = 1 + 2 + 8 = 11 ; sum of the even digits = 7 + 4 = 11 : 11 — 11 = 0.

EXAMPLE 2. 917,092 is divisible by 11 : 9 + 7 + 9 = 25 : 1 + 0 + 2 = 3 ; 25 — 3 = 22, which is divisible by 11.

12, if it is divisible by 3 and 4. EXAMPLES. 1032, 4044.

15, if it is divisible by 3 and 5. EXAMPLES. 1125, 3045.

25, if it ends in two cyphers or if the last two digits are divisible by 25. EXAMPLES. 1700, 1875.

EXAMPLES.—ORAL.

State which of the following are divisible by 2, or by 3, or by 4, or by 5, or by 8, or by 9, or by 11, or by 6, or by 12, or by 15, or by 25.

492, 235, 9,316, 1,011, 1,233, 8,074, 9,810, 87,125, 87,131, 20,016, 79,008, 12,738, 93,228, 285,714, 30,216, 32,016, 80,640, 285,625.

74. The tests of divisibility enable us to resolve a composite number into its factors.

Example. Resolve 2772 into factors.

9|2772.........satisfies the test for 9
4|308.........satisfies the test for 4
7|77
11

$\therefore 2772 = 9 \times 4 \times 7 \times 11.$

If prime factors are required.

$$2772 = 3 \times 3 \times 2 \times 2 \times 7 \times 11 = 3^2 \times 2^2 \times 7 \times 11.$$

75. When the divisors cannot be easily determined, prime numbers should be tried.

A prime number is that which is divisible only by itself and by unity. Thus, 17 is a prime number.

The following is a list of the prime numbers from 1 to 160—

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157.

Example. Resolve 17017 into factors.

11|17017......satisfies the test for 11
7|1547......next try the prime No. 7
13|221 ,, ,, , ..13
17

$\therefore 17017 = 11 \times 7 \times 13 \times 17.$

76. To find out whether a number is prime or not.

Try the prime numbers as divisors beginning with 2 successively till you get a quotient less than the divisor. If no factor has been obtained by this time, the number is prime.

Example. Find out if 163 is a prime.

Divide 163 successively by 2, 3, 5, 7, 11, 13; and the quotient of 163 ÷ 13 is 12 which is less than the divisor. If 163 contained a factor less than 13, it should have already appeared as a divisor; and if it contained a factor greater than 13, it should have appeared as a quotient. Leaving out itself and unity, 163 has no factor less than 13 or greater than 13. Therefore 163 is a prime.

77. Odd and Even numbers.

Numbers which are exactly divisible by 2, such as 2, 4, 6, 8, 10, etc., are called *Even* numbers. Hence no even number except 2 can be a prime.

Numbers which are not exactly divisible by 2, such as 1, 3, 5, 7, 9, 11, etc., are called *Odd* numbers. Hence when we divide an odd number by 2, the remainder is always 1.

Every even number therefore is of the form $2n$ and every odd number is of the form $2n + 1$ or $2n - 1$.

78. Consecutive numbers are a series of numbers in which each is greater than its preceding number by 1.

EXAMPLE. 9, 10, 11, 12, 13. Consecutive odd numbers beginning from 9 are 9, 11, 13, 15, 17, etc. Consecutive even numbers beginning from 10 are 10, 12, 14, 16, 18, etc.

EXAMPLES.—ORAL.

1. Name all the prime numbers from 1 to 20, from 25 to 60, from 70 to 100, and from 101 to 160.
2. Name all the composite numbers from 21 to 50; 61 to 90; and 100 to 160, and give their prime factors.
3. Name all the even numbers from 500 to 1,000.
4. Name all the odd numbers from 1,001 to 2,000.

EXAMPLES. III.

I. Resolve into prime factors—

1. 208. 2. 704. 3. 676. 4. 2,890.
5. 1,705. 6. 7,920. 7. 6,534. 8. 10,125.

II. Determine which of the following are prime numbers and find the prime factors of the composite numbers:—

469, 607, 867, 1,365, 2,127, 2,197, 3,019 and 8,712.

79. Resolution of numbers into factors may be used to determine whether a given number is a square or not and hence to find its square root or to ascertain the least factor by which the number should be multiplied to make it a square.

EXAMPLE 1. Resolve into factors 9801; and find if it is a square.

9 | 9801 ...... satisfies the test for 9
9 | 1089 ...... satisfies the test for 9
11 | 121
11

$\therefore 9801 = 9 \times 9 \times 11 \times 11 = 9^2 \times 11^2$.

9801 is a square, since it is the product of two squares; and its square root is 9×11 or 99

EXAMPLE 2. Find the least factor necessary to make 21780 a perfect square.

10 | 21780 $21780 = 10 \times 2 \times 9 \times 11 \times 11$
2 | 2178 $= 5 \times 2 \times 2 \times 9 \times 11 \times 11$
9 | 1089 $= 5 \times 2^2 \times 3^2 \times 11^2$
11 | 121
11

Since we have not the square of 5, the number 21780 should be multiplied by 5 to make it a perfect square.

EXAMPLES. IV.

Resolve into factors and find the square root of the numbers that are squares—

1. 441. 2. 484. 3. 576. 4. 720.
5. 1,225. 6. 1,280. 7. 1,936. 8. 3,072.
9. 5,929. 10. 8,748. 11. 13,689. 12. 10,368.

Find the least factor required to complete the squares in the above; also in the following:—

13 3,072. 14. 8,450. 15. 8,748. 16. 13,475.
17. 13,122. 18. 25,920. 19. 23,232. 20. 12,096.

Problems.

EXAMPLE. Rama and Govind have together Rs. 58,072, and Rama has Rs. 14,180 more than Govind. What has each?

Rs. 58,072 – Rs. 14,180 or Rs. 43,892 is the sum which Rama and Govind have equally between them.

∴ Rs. 43,892 ÷ 2 or Rs. 21,946 is the sum which Govind has.

∴ Rs. 21,946 + Rs. 14,180 or Rs. 36,126 is the sum which Rama has.

EXAMPLES. V.

1. Two topes A and B produced 87,870 cocoanuts between them. A produced 1,248 more than B. How many did each produce?

2. The contents of two bags are together one lakh of rupees. One contains Rs. 5,678 more than the other. Find what each contains.

3. Divide Rs. 68,984 into two parts whose difference shall be Rs. 486.

4. The sum of two numbers is one million, and their difference 5,248; find the numbers.

EXAMPLE. How many cows at Rs. 33 each should be given in exchange for 108 oxen each worth Rs. 55?

The value of 108 oxen at Rs. 55 each = Rs. 55 × 108 = Rs. 5,940.

∴ The number of cows at Rs. 33 each = Rs. 5,940 ÷ Rs. 33 = 180.

EXAMPLES. VI.

1. How many sheep worth Rs. 3 each should be given in exchange for 21 cows worth Rs. 18 each?

2. How many chairs at Rs. 4 each can be bought for the value of 28 tables each worth Rs. 37?

3. How many horses each worth Rs. 210 could be purchased by selling 280 sheep at Rs. 12 each?

4. What number multiplied by 150 will give the product of 375,000 by 125?

EXAMPLES. VII.

MISCELLANEOUS.

1. Add together without re-arranging in columns, 3759261, 8431914, 21086, 7678439, 346525.

2. Find the sum of 9 consecutive even numbers, of which the middle number is 38.

3. Find the sum of 8 consecutive odd numbers, whose 4th number is 41.

4. In the 1st class there are 8 girls and 6 boys; in the 2nd as many girls as there are boys in the 1st and 4 more boys than girls; in the 3rd as many boys as there are girls in the 1st two classes, and as many girls as there are boys in the 2nd. How many girls are there in the school?

5. The population of 5 towns is respectively 19,870, 23,729, 15,678, 21,987, and 17,896. By how many does the total fall short of 1 lakh (in one step)?

6. A man bought 3 houses; the 1st for Rs. 3,250, the 2nd for Rs. 5,725 and the 3rd for Rs. 500 less than the 1st two. If he sold the houses for Rs. 18,000, what did he gain?

7. A man leaves Rs. 2,50,000; his eldest son gets Rs. 35,000, his second son Rs. 27,750 and his wife Rs. 20,250. He owes Rs. 17,500 to one and Rs. 12,575 to another. The rest of his property goes to charitable institutions; what sum do these get?

8. Two baskets contain plantains; the 1st 400 and the 2nd 550; if 60 be sold out of the 1st, and 48 be put out of the 2nd into the 1st, and then 150 be sold out of the 2nd and 84 be put out of the 1st into the 2nd, how many plantains will there be then in each basket?

Find by the shortest method possible the product of—

9. 87,609 by 728.

10. 8,765,439 by 74,963.

11 109,089 and 997.

12. 100000002 and 9876543.

13. In a page of the *Madras Mail* there are 7 columns; in each column, on an average, there are 170 lines; and in each line 35 letters. How many letters are there in a daily issue of 12 pages?

14. A boy's age is 16; and his elder brother 6 years ago was twice as old as he then was. How old is the elder brother now?

15. Four years hence, a father will be four times as old as his son, now six years old. How old was the father when the son was born?

16. What must be taken from the sum of 7 times the square of 59 and 5 times the cube of 26 to make one lakh?

17. A sum of money is divided among 111 persons; each man gets Rs. 84 and Rs. 86 is left over. Find the sum.

18. Find the number next above 234567 which can be exactly divided by 897.

19. The distance between two stations is 38 miles; how many times must a motor-car run to and fro to make up a distance of 3,000 miles?

20. 5,280 feet make one mile; how many miles are there in 12,345,678 ft. What length remains over?

21. Find the smallest number of rupees that must be added to Rs. 49,876, so that the sum may be equally distributed among 325 persons?

22. What is the nearest sum to one lakh of rupees that can be invested in shares of Rs. 93 each?

23. How many times must 356 be added to 1754 so that the sum may be 6026?

24. How many times must Rs. 575 be taken from Rs. 25,850, so that Rs. 4,000 may be left?

25. 15 men are partners each having an equal number of shares in a business in which the receipts in a certain year amounted to £18,750 and the expenses to £15,525. What is each man's share of the profits?

26. Divide Rs. 1,200 into two parts, whose difference shall be Rs. 120.

27. During a harvest, a husband and his wife earned 84 measures of paddy in a week. The husband earned 26 measures more than his wife; find how much each earned.

28. A road 520 yards long has 27 lantern posts placed at equal intervals. What is the interval between two posts?

29. In 1760 yards of railroad, telegraph posts are placed at equal intervals of 88 yards. How many are there?

30. How many cows at Rs. 25 each could be bought from the money realized by selling 20 sheep at Rs. 15 each?

31. In dividing 89076 by 578, a boy copied the dividend wrongly and got 140 quotient and 56 remainder. What was his mistake?

32. Another boy in working the above sum copied the divisor incorrectly and got 148 quotient and 572 remainder. What mistake did he make?

33. In the division of 345678 by a certain number, the three successive remainders are respectively 819, 286 and 231. Find the divisor and the quotient.

34. Fill in the missing numbers in the following :—

3 * *
523) * * * * * *
1105
594
71 remainder.

35. *|62341
8|* * * * .. remainder 4.
* * * .. remainder 3. Complete remainder 37.

36. 7| * * * * *
9| * * * * .. remainder 5.
5| * * * .. remainder 3.

Quotient 81 .. remainder 4.

37. In a cricket match, of 3 boys A, B and C, A and B scored together 100 runs ; A and C together 95 ; and B and C together 105. Find the score of each.

38. Of three persons A, B and C, A and B are together worth one lakh of rupees, A and C are together worth Rs. 75,000 ; and B and C are worth Rs. 91,000. Find what each is worth.

39. A party of 100 schoolboys gather 2,783 flowers in one garden and 891 flowers in another ; but there are so few flowers in a third garden that while some are able to find only one each, the others can find none. The total number of flowers collected being equally divisible among all the boys, how many are unable to find flowers in the third garden ?

40. A fruit seller ordered 1,000 mangoes at the rate of 20 a rupee and 450 at 15 for a rupee ; and on arrival found 40 of the former and 30 of the latter were damaged and could not be sold. What profit did he make by selling the rest at the uniform rate of 12 a rupee ?

CHAPTER IV.

COMPOUND QUANTITIES : INDIAN AND ENGLISH TABLES OF MONEY.

80. We have in previous chapters made use of certain simple quantities such as 5 rupees, 3 feet, 15 years, 10 pounds, etc. These are quantities in naming which only one unit is used. But there are quantities in naming which more than one unit of the same kind is used ; that is terms of different denominations, but all of the same kind are used. Such quantities are called **Compound Quantities.**

Thus 5 rupees 10 annas 8 pies is a compound quantity; for there are terms of three different denominations used in it; viz. rupees, annas and pies.

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81. INDIAN TABLE OF MONEY.

12 *Pies* (*p.*) *make* 1 *Anna* (*a.*)
16 *Annas* ,, 1 *Rupee* (*R. or Re.*)

A gold *mohur* of the value of Rs. 16 and a *pagoda* worth Rs. 3-8 as. which were formerly current are not now in use.

82. In Ceylon the rupee is divided into 100 equal parts called **Cents.**

83. Reduction is the process of expressing (*a*) numbers of a higher denomination in units of a lower denomination; (*b*) numbers of a lower denomination in units of a higher denomination; as in—

(*a*) reducing Rs. 162-7-3 to pies; 162 Rs. 73 cents to cents.
(*b*) reducing 18489 pies to Rs. as. p.; 18488 cents to Rs.

(*a*) Rs. A. P.
162 7 3
16
2599 as.
12
Ans. 31191 p.

First multiply Rs by 16 and add 7 as. Then multiply the annas by 12 and add the 3 p.

(*b*) 12 | 18489 p.
16 | 1540 as. 9 ps.
96 Rs. 4 as.
Ans. Rs. 96-4-9.

Rs. Cents.
162 73
100
16273 cents.

Rs 162 is multiplied by 100 and 73 cents are added.

100 | 184,88 cents.
184 Rs. 88 cents.

84. The Ceylon coinage is a decimal coinage since the change from one denomination to the other is effected by multiplying or dividing by 100, i.e. by 10^2. In this case there is no labour involved in reduction.

Thus 8725 cents = 87 Rs. 25 cents;
and 123 Rs. 40 cents = 12340 cents.

EXAMPLE. How many quarter-annas are there in Rs. 369 12 as. 6 p.?

Reducing Rs. 869-12 as. to annas, we get 13916 as. Since there are 4 quarter-annas in an anna and 2 quarter-annas in 6 p., we multiply 13916 by 4 and add 2; we thus get 55, 666 quarter-annas.

EXAMPLES.—ORAL.

Reduce to pies—

1. 13 as. 10 p. 2. 14 as. 11 p. 3. 15 as. 6 p. 4. 15 as. 11 p.
5. Rs. 1-12-6. 6. Rs. 2-4-9. 7. Rs. 3-0-6. 8. Rs. 4-5-4.

How many half-anna stamps can be bought for—

1. 10 as. 2. 13 as. 3. 15 as. 4. 14 as. 6 p.
5. Rs. 1-1-6. 6. Rs. 1-13-6. 7. Rs. 2-4 as. 8. Rs. 3-8 as.
9. Rs. 3-3-6. 10. Rs. 3-11-6. 11. Rs. 4-7-6. 12. Rs. 4-10-6.

How many quarter-anna stamps can be had for—

1. 13 as. 3 p. 2. 15 as. 9 p. 3. Rs. 1-2 as. 4. Rs. 1-7 as.
5. Rs. 2-4 as. 6. Rs. 3-6 as. 7. Rs. 1-3-6. 8. Rs. 1-6-9.
9. Rs. 1-14-6. 10. Rs. 2-0-9. 11. Rs. 3-1-6. 12. Rs. 3-4-9.

How many quarter-rupees should be given in change for—

1. Rs. 16. 2. Rs. 24. 3. Rs. 29. 4. Rs. 32.
5. Rs. 15-12 as. 6. Rs. 16-8 as. 7. Rs. 18-4 as. 8. Rs. 21-8 as.

How many two-anna pieces are there in—

1. Rs. 9. 2. Rs. 12. 3. Rs. 14. 4. Rs. 16.
5. Rs. 7-14 as. 6. Rs. 8-12 as. 7. Rs. 11-4 as. 8. Rs. 12-10 as.
9. Rs. 13-6 as. 10. Rs. 14-2 as. 11. Rs. 15-8 as. 12. Rs. 16-6 as.

Express as annas and pies or as Rs. a. p.

1. 100 p. 2. 124 p. 3. 183 p. 4. 190 p.
5. 208 p. 6. 300 p. 7. 400 p. 8. 500 p.
9. 620 p. 10. 700 p. 11. 800 p. 12. 966 p.

Reduce to quarter-rupees and give the remainder—

1. 140 p. 2. 180 p. 3. 200 p. 4. 300 p.
5. 460 p. 6. 580 p. 7. 620 p. 8. 700 p.

EXAMPLES. I.

Reduce—

1. Rs. 2,578-14-8; Rs. 8,765-4 as.; Rs. 4,567-8-10; Rs. 9,976-5-1 to pies.

2. Rs. 1,125-6 as.; Rs. 2,345-12 as.; Rs. 5,678-8 as.; Rs. 5,087-4 as. to two-anna pieces.

3. Rs. 7,803-9 as.; Rs. 869-7-6; Rs. 9,876-11-9; Rs. 2,345-12-3 to quarter-annas.

4. 964,789; 789,010; 1,011,123; 1,915,453 pies to Rs. a. p.

5. 891,035; 999,999; 12,045½; 25,683½ quarter-annas to Rs. a. p.

6. 6,94,578; 9,10,112; 10,23,675; 21,22,234 pies to quarter-rupees.

85. ENGLISH TABLE OF MONEY.

4 *Farthings* (*f.*) make 1 *Penny* (*d.*)
12 *Pence* ,, 1 *Shilling* (*s.*)
20 *Shillings* ,, 1 *Pound* (*£*)

2 farthings = a half-penny.
1 farthing is written ¼*d.*
2 farthings are written ½*d.*
3 farthings are written ¾*d.*

The following coins are current in England:—

Bronze Coins.
A farthing
A half-penny
A penny

Gold 'Coins'
A Half-sovereign
A Sovereign

'Silver' Coins.
Three-penny piece
Six-penny piece
Shilling
Florin (2*s.*)
Half-crown (2*s.* 6*d.*)
Double florin.

The guinea of the value of 21*s.* is not now issued; but the word *guinea* is still used to denote 21*s.*; and half a guinea = 10*s.* 6*d.*

EXAMPLE 1. (*a*) Find the number of farthings in £28-17-6½.

(*b*) Express 97081 farthings as £ *s. d.*

(*a*)
£ *s.* *d.*
28 17 6½
20
577 *s.*
12
6930 *d.*
4
27722 *f.*

(*b*)
4 | 97081 *f.*
12 | 24270 ¼*d.*
20 | 2022 *s.* 6¼*d.*
101 £ 2*s.*
Ans. £101-2-6¼.

(*a*) We multiply the £ by 20 and add 17*s.*; next multiply the shillings by 12 and add 6*d.*; and then multiply the pence by 4 and add the 2*f*. in ½*d.*

EXAMPLE 2. Express 97080 farthings as half-crowns.

4 | 97080 *f.*
6 | 24270 *d.*
5 | 4045 six-penny pieces.
809 half-crowns.

We divide pence by 6 and the quotient by 5; because in half a crown there are 5 six-penny pieces.

EXAMPLES.—ORAL.

Express as pence—

1. 12*s.* 8*d.* 2. 13*s.* 9*d.* 3. 15*s.* 10*d.* 4. 18*s.* 6*d.*
5. £1-0-3. 6. £1-2-6. 7. £1-10-4. 8. £1-12-6.

Find the number of half-pennies in—

1. 11*s.* 10*d.* 2. 14*s.* 6*d.* 3. 16*s.* 5*d.* 4. 19*s.* 8*d.*
5 £1-1-6. 6. £1-3-4. 7. £1-5-2. 8. £1-10-6.

Express as farthings—

1. 8*s.* 4¼*d.* 2. 11*s.* 8½*d.* 3. 12*s.* 6¾*d.* 4. 13*s.* 4¾*d.*
5. 15*s.* 0½*d.* 6. 17*s.* 6½*d.* 7. 18*s.* 4¼*d.* 8. 19*s.* 2¼*d.*

How many six-penny pieces are there in—

1. 14*s.* 6*d.* 2. 19*s.* 3. £1-2-6. 4. £1-12-6.
5. £1 15*s.* 6. £2 1*s.* 7. £2 18*s.* 8. £3 10*s.*

How many three-penny pieces are there in—

1. 8*s.* 9*d.* 2. 12*s.* 6*d.* 3. 13*s.* 3*d.* 4 15*s.* 9*d.*
5. £1-2-3. 6. £1-8-6. 7. £10-1-9. 8. £1-16-6.

Find the number of half-crowns in—

1. 12*s.* 6*d.* 2. 17*s.* 6*d.* 3. £1. 4. £1 5*s.*
5. £1 15*s.* 6. £2-2-6. 7. £2-17-6. 8. £3 10*s.*

Express as shillings and pence or as £. *s.* *d.*—

1. 150*d.* 2. 189*d.* 3. 200*d.* 4. 232*d.*
5. 162*f.* 6. 190*f.* 7. 230*f.* 8. 254*d.*
9. 242*d.* 10. 260*d.* 11. 284*d.* 12. 300*d.*
13. 420*d.* 14. 560*d.* 15. 768*d.* 16. 842*d.*

EXAMPLES. II.

1. Express as pence—
   (1) £219-17-11. (2) £382-13-9.
   (3) £1,491-19-7. (4) £2,867-9-8.
2. Reduce to half-pennies—
   (1) £234-15-7½. (2) £538-13-6½.
   (3) £789-19-9½. (4) £1,579-11-8½.

3. Reduce to farthings—
(1) £29-11-7¾. (2) £107-16-5½.
(3) £298-9-6¼. (4) £876-18-8¾.

4. Reduce to six-penny pieces—
(1) £350-19-6. (2) £453-13*s.*
(3) 35 guin. 9*s.* 6*d.* (4) 79 guin. 17*s.* 6*d.*

5. How many half-crowns are there in—
(1) £54-2-6. (2) 20 guin. 15*s.* (3) £26-7-6. (4) £32-2-6.

6. Reduce to £ *s. d.*—
(1) 99,999*d.* (2) 202,077*d.* (3) 77,777*f.* (4) 506,530*f.*

7. How many half-crowns are there in—
(1) 101,880*f.* (2) 130,680*d.* (3) 262,977*d.* (4) 485,264*d.*

THE FOUR COMPOUND RULES.

86. Compound Addition is the process by which compound quantities are collected into one sum.

EXAMPLE. Add together—

(1)

Rs.	cents.
87	75
123	80
49	29
378	44
19	36
526	57
1,185	21 Ans.

In (1), we proceed exactly as in simple addition, 'carrying' 3 i.e. 300 from the cents' column to the rupees.

(2)

£	*s.*	*d.*
35	8	9½
76	10	4¾
25	8	9
4	0	7½
36	12	5¼
42	5	9¾
220	6	9¾

In (2), adding the farthings the sum 11*f.* = 2¾*d.* we write ¾ and carry 2*d.* Adding the pence we get 45*d.* which is 3*s.* 9*d.* Writing 9 under pence, we carry 3*s.* and so on.

EXAMPLES. III.

Add the following vertically and horizontally :—

	1			2			3			4		
I.	RS.	A.	P.	RS.	A.	P.	RS.	A.	P.	RS.	A.	P.
5.	3,654	10	9	789	13	11	6,010	8	7	7,609	13	9
6.	4,378	9	8	97	12	8	590	15	11	70	14	7
7.	1,312	11	10	3,456	7	9	83	14	8	678	11	5
8.	456	7	6	6,789	10	11	3	0	9	2,354	9	10
9.	79	8	11	345	6	7	1,326	7	3	300	10	3
10.	9	13	7	16	9	5	987	5	6	3,067	8	9

II.	1 £	s.	d.	2 £	s.	d.	3 £	s.	d.	4 £	s.	d.
5.	65	9	5¾	78	7	6	3,456	7	8¼	6,789	10	11½
6.	87	13	3¼	395	10	9¾	234	5	6½	654	3	8¾
7.	295	15	9¼	857	9	8½	789	10	11¾	987	6	5½
8.	436	19	8½	3,976	12	11¾	906	19	7	19	16	3¼
9.	89	11	11¼	693	8	6¼	97	18	10¾	3,096	19	7½
10.	14	8	7¾	89	17	7½	4,579	13	9½	674	7	0¼

87. Compound Subtraction is the method of finding the difference between two compound quantities.

EXAMPLE.

(1)	RS.	cents
From	374	25
Take	189	75
Rs.	184	50

Here we proceed as in simple subtraction.

(2)	£	s.	d.
From	25	15	9½
Take	16	17	10¾
Ans.	8	17	10¾

Subtracting ¾*d.* from 1½*d.* i.e. 3*f.* from 6*f.* we get 3*f.* or ¾*d.*

EXAMPLES. IV.

1. Find the difference between 28,067 Rs. 24 cents and 21,278 Rs. 42 cents.

2. From a purse containing Rs. 2,000, the following sums are taken :—195 Rs. 95 cents, 280 Rs. 80 cents, 395 Rs. 70 cents, 525 Rs. 25 cents. Find how much is left.

Perform the following subtractions—

3. RS.	A.	P.	4. RS.	A.	P.	5. £	s.	d.	6. £	s.	d.
10,000	0	0	3,04,050	13	4	87	12	10	275	10	7½
785	8	9	78,965	15	8	39	15	8¾	129	14	9¼

Find the missing quantities in—

7. RS.	A.	P.	8. RS.	A.	P.	9. £	s.	d.	10 £	s.	d.
8,98,765	13	7	9,68,070	12	3	23,456	10	9½	30,506	14	8¼
......			......			......			......		
3,98,897	14	11	1,68,613	12	5	14,686	14	10¾	7,049	18	9¾

11. What must I take away from Rs. 11,111-11-1 to leave Rs. 9,999-9-9 ?

12. If I gain £36,234-9-6½ and lose £50,306-1-9, how much shall I have gained or lost altogether ?

Example. A horse and its saddle are worth £120-17-6; the saddle alone is worth £12-7-6. By how much does the value of the horse exceed that of the saddle?

The horse and the saddle are worth.......... £120-17-6
But the saddle is worth.............................£12-7-6
Subtracting, the horse is worth...............£108-10-0

The value of the horse exceeds that of the saddle by £108-10-0 – £12-7-6 or by £96-2-6.

EXAMPLES. V.

1. A picture with its frame costs Rs. 25; the picture costs Rs. 12-10-8. How much does the one cost more than the other?

2. A house with its furniture is worth £4,759-10-9¼; the house alone is worth £1,494-11-9¾. By how much does the value of the furniture exceed that of the house?

3. A has 19*s.* 8½*d.*, and B has 17*s.* 11*d.*; A gives B 7*s.* 10¾*d.*, and B gives A 13*s.* 9½*d.* Which of the two has more, and how much?

4. A person has £17-8-6; how much has he left, after paying away £1-6-10½, £3-2-8, 4½ guineas, 14*s.* 7½, and £1-0-9¾?

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88. Compound Multiplication is the process of finding the amount of any compound quantity, when it is repeated a number of times.

When the multiplier exceeds 16, the easiest method will be to separate the multiplier into factors or into factors and parts.

Example. Multiply (1) 524 Rs. 24 cents by 54. (2) Rs. 285-10-7 by 87. (3) £79-14-6¾ by 76.

	(1) Rs.	cents.	(2) Rs.	A.	P.	(3) £	*s.*	*d.*	*f.*
54 = 6 × 9	524	24	285	10	7	79	14	6	3
		6			87				76
	3,145	44	1999	10	1	478	7	4	
		9	2285	4	8	558	1	11	
Answer	28,308	96	24852	8	9	£6059	6	9	

(2) The first line is the product by 7. The next line is the product by 80 or 8 tens and so the pies, annas and rupees are written beginning at the tens' place. 81 p. = 6 as. 9 p. 56 as. = Rs. 3-8 as.

MENTAL ARITHMETIC.

89. Multiplying a number of pies or pence by 12 gives the same number of annas or shillings.

Thus, 7 p. × 12 = 7 as. ; 5*d*. × 12 = 5*s*.
$4\frac{3}{4}d. \times 12 = 4\frac{3}{4}s. = 4s.\ 9d.$

Hence, to find the value of a dozen articles, reckon each pie in the given value as an anna, and each penny as a shilling.

Thus, the value of a dozen articles at 3 as. 4 p. each is 40 as., i.e. Rs. 2-8 as. ; and at 4*s*. 8*d*. each is 56*s*., i.e. £2 16*s*.

EXAMPLE. Find the value of 1 dozen articles at Rs. 2-5-8 each.

The value of 12 articles at Rs. 2-5-8 each = the value of the 12 articles at Rs. 2 each + the value of the 12 articles at 5 as. 8 p. each.

The value of the 12 articles at Rs. 2 each = Rs. 24.
and..............................5 as. 8 p. = 68 as. = Rs. 4-4 as.
∴ The......................Rs. 2-5-8 = Rs. 28 4 as.

EXAMPLE. Find the value of 84 articles at £3-3-6 each.
84 = 7 dozen.
The value of 1 dozen at £3-3-6 = £36 + 42*s*. = £38-2*s*.
∴ The ....7 .... = £38-2*s*. × 7 = £266-14*s*.

90. To find the value in Indian money of 16 articles, reckon each anna in the given value as a rupee, since 16 as. make one rupee.

Thus, the value of 16 articles at 5 as. each is Rs. 5.

EXAMPLE. Find the value of 16 articles at 12 as. 6 p. each.

12 as. 6 p. = $12\frac{1}{2}$ as. Required value = Rs. $12\frac{1}{2}$, i.e. Rs. 12-8 as.

91. To find the value in English money of 20 articles reckon each shilling in the given value as £1, since 20*s*. make £1.

Thus, the value of 20 articles at
(1) 3*s*. 3*d*. = value at $3\frac{1}{4}s.$ = £$3\frac{1}{4}$ = £3-5*s*.
(2) 13*s*. 9*d*. = £$13\frac{3}{4}$ = £13-15*s*.

EXAMPLES.—ORAL.

1. Find the value of a dozen articles at (1) $5\frac{1}{2}$*d.* (2) $4\frac{1}{4}$*d.* (3) $6\frac{3}{4}$*d.* (4) $7\frac{1}{2}$*d.* (5) $9\frac{1}{4}$*d.* (6) $10\frac{3}{4}$*d.* (7) $8\frac{1}{4}$*d.* (8) $11\frac{3}{4}$*d.*

2. Find the value of a dozen articles at (1) 7 as. 2 p. (2) 9 as. 1 p. (3) 10 as. 6 p. (4) 11 as. 10 p. (5) 12 as. 5 p. (6) 13 as. 11 p. (7) 14 as. 9 p. (8) 15 as. 10 p. each.

3. Find the value of a dozen things at (1) 8*s.* 10*d.* (2) 9*s.* 11*d.* (3) 10*s.* 4*d.* (4) 11*s.* 3*d.* (5) 12*s.* 5*d.* (6) 13*s.* 6*d.* (7) 14*s.* 7*d.* (8) 15*s.* 8*d.* (9) 16*s.* 9*d.* (10) 17*s.* 4*d.* (11) 18*s.* 6*d.* (12) 19*s.* 3*d.* each.

4 Find the value of a dozen articles at (1) Rs. 1-10-3. (2) Rs. 1-12-4. (3) Rs. 2-5-4. (4) Rs. 2-10-8. (5) Rs. 2-12-6. (6) Rs. 2-14-2. (7) Rs. 3-1-6. (8) Rs. 3-4-8. (9) Rs. 3-6-4. (10) Rs. 3-10-6 each.

5. Find the value of a dozen articles at (1) £1-1-4. (2) £1-2-8 (3) £1-3-4. (4) £1-5-9. (5) £1-6-8. (6) £1-8-6. (7) £1-12-9. (8) £2-2-4 (9) £2-6-8. (10) £2-8-4 each.

6. Find the value at (*a*) 6*s.* 4*d.* each. (*b*) 7 as. 8 p. each of (1) 36, (2) 48, (3) 60, (4) 84, (5) 120, (6) 144, (7) 180, (8) 192 articles.

7. Find the value of 16 articles at (1) 6 as. 5 p. (2) 7 as. 9 p. (3) 8 as. 8 p. (4) 9 as. 6 p (5) 10 as. 3 p. (6) 12 as. 9p. (7) 14 as. 6 p. (8) 15 as. 8 p. (9) Rs. 1-2-6. (10) Rs. 2-3-9. (11) Rs. 3-1-3. (12) Rs. 3-4-9 each.

8. Find the value of 20 articles at (1) 13*s.* 6*d.* (2) 15*s.* 3*d.* (3) 16*s.* 9*d.* (4) 18*s.* 3*d.* (5) 19*s.* 6*d.* (6) £1-2-3. (7) £1-10-9. (8) £1-12-6 each.

9. Find the value of (1) 60, (2) 80, (3) 120, (4) 180, (5) 220, (6) 260 articles at £1-4-3 each.

EXAMPLES. VI.

1. If the cost of making a ring comes to Rs. 30-10-8, find the cost of making 27 such rings.

2. What amount is given to 29 persons at 25 guineas each?

3. How much is spent in a year of 365 days at the daily rate of Rs. 17-14 as.?

4. If an acre of land is sold for Rs. 802-9-3, what is realized by selling 789 acres of similar land?

5. A person had 8 as. 8 p. after giving each of 19 persons Rs. 33-12 as. What had he at first?

6. How much should I have to give to each of 17 people 3*s*. 8¼*d*. and to each of 23 others 15*s*. 0¾*d*.

7. A person has 27 half-rupees, 31 quarter-rupees and 49 two-anna pieces. Find the total sum he has.

8. What sum is necessary to give each boy in a school of 53, a sovereign, a half-crown, a shilling, a six-penny piece and a three-penny piece ?

9. 24 coolies are engaged for 6 days a week, working 8 hours a day at three-quarters of an anna per hour. What has to be paid to them at the end of 4 weeks ?

10. The straw from three paddy fields each yielding 62 bundles, each weighing 62 pounds is sold at the rate of 10 pies per pound. What is the amount realized ?

11. What is the total sum received from 13 first class railway passengers paying Rs. 27-12 as. each, from 27 second class passengers paying Rs. 13-14 as. each and from 234 third class passengers at Rs. 4-10 as. each ?

12. A man bought 2,000 sheep-skins at 12 as. each and 1,200 goat-skins at Rs. 1-8 as. each ; and paid carriage hire to the drying shed at the rate of 4 as. per 100. Find the total cost of the skins.

---

92. Compound Division is the method (1) of dividing a compound number into as many equal parts as the divisor contains units ; and (2) of finding how often one compound number is contained in another of the same kind ; as in—

(1) Dividing Rs. 2,893-8 as. by 144.
(2) Dividing £6,052-14-6 by £123-10-6.

In the division of a concrete number by an abstract number, we separate the former into as many parts as are represented by the latter ; the quotient is one of such parts and is therefore concrete.

$$\frac{\text{Concrete number}}{\text{Abstract number}} = \text{Concrete number.}$$

EXAMPLE. Divide Rs. 2,893-8 as. by 144.

It is easy to divide by means of factors, whenever the divisor can be split up into factors; 144 = 12 × 12.

	Rs.	A.	P.
12	2,893	8	0
12	241	2	0
Rs.	20	1	6

EXAMPLES.—ORAL.

What is the value of? 1. 4*s*. 9*d*. ÷ 2, 3, 6. 2. 12*s*. 6*d*. ÷ 3, 4, 6, 8, 12.
3. 6 as. 8 p. ÷ 4, 8. 4. 15. as. ÷ 4, 6, 8, 12. 5. 9*s*. $6\frac{3}{4}$*d*. ÷ 3.
6. 8*s*. 4*d*. ÷ 8. 7. 5*s*. $10\frac{1}{2}$*d*. ÷ 2. 8. 5*s*. $4\frac{1}{2}$*d*. ÷ 3.
9. 18*s*. ÷ 4, 8, 12, 16. 10. Rs. 1-10-8 ÷ 4, 8, 10, 16.
11. £1-16-8 ÷ 4, 5, 8, 16. 12. £2-6-8 ÷ 4, 5, 8, 10, 16.

EXAMPLES. VII.

1. Divide equally among 127 persons the following sums of money (1) 87,654 Rs. 80 cents. (2) 1,00,987 Rs. 52 cents. (3) 2,34,567 Rs. 75 cents. (4) 5,60,789 Rs. 48 cents.

2. Divide £16,501-8-$4\frac{1}{4}$ equally among 197 persons.

3. If 204 articles cost £658-15*s*., find the cost of one.

4. If a man earned Rs. 114-6-4 in a year of 366 days what was his daily earning?

5. If the total cost of 3,465 horses amounted to £119,232-1*s*.-$10\frac{1}{2}$*d*., what was the average cost of each?

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93. By the division of one concrete quantity by another of the same kind is meant the process of finding how often the second is contained in the first; the quotient therefore is an abstract number.

$$\frac{\text{Concrete number}}{\text{Concrete number}} = \text{abstract number.}$$

Reduce both quantities to the same denomination and then divide the one result by the other; but it is advantageous to choose that denomination, in which the given quantities are expressed by the smallest numbers.

Example. Among how many persons can £6,052-14-6 be divided so that the share of each may be £123-10-6 ?

No. of six-penny pieces in £123-10-6 = 4,941.
No. of six-penny pieces in £6,052-14-6 = 242,109.
242,109 six-pennies ÷ 4,941 six-pennies = 49. Ans. 49.

EXAMPLES.—ORAL.

How often is the first of these contained in the others ?

1. 8 p. in 12 as. 8 p. ; 13 as. 4 p. ; 10 as. 8 p. ; Re. 1.
2. 9 p. in 10 as. 6 p. ; 13 as. 3 p. ; 15 as. ; Re. 1-5 as.
3. 1 an. 4 p. in 12 as. ; 14 as. 8 p. ; Rs. 1-2-8 ; Rs. 2.
4. 1*s.* 4*d.* in £1 ; £1-8*s.* ; £2-2-8 ; £2-10-8.
5. 1*s.* 3*d.* in £1 ; £1-10*s.* ; £2-5*s.* ; £2-15*s.*

EXAMPLES. VIII.

1. How many payments of 9 as. 6 p. can be made from Rs. 285 ?

2. The rupee being worth 1*s.* 4*d.* find the number of rupees that can be exchanged for £30-4*s.*

3. How many sovereigns can be had for Rs. 506-5-4, if the sovereign be worth Rs. 16-5-4 ?

4. How many times can £1-10-6¼ be taken from £160-4-8¼ ?

5. Among how many persons must Rs. 776-2-10½ be divided so that the share of each may be Rs. 36-15-4½ ?

6. If an article cost 12*s.* 6*d.*, how many dozen can be had for £22-10*s.* ?

7. How many more half sovereigns than guineas are there in £157-10*s.* ?

8. How many more dollars than pagodas are there in Rs. 1,984-8 as. ? One dollar = Rs. 2-4 as. and one pagoda = Rs. 3-8 as.

Example. How many cows at Rs 21-12 as. each should be given in exchange for 120 sheep at Rs. 7-4 as. each ?

120 sheep are worth Rs. 7-4 as. × 120 or Rs. 870.
∴ No. of cows required = Rs. 870 ÷ Rs. 21-12.
= Rs. 870 × 4 qr. rs. ÷ 87 qr. rs.
= 40.

EXAMPLES. IX.

1. A man sold 30 sheep at Rs. 3-8 as. each; and with the money, bought ducks at 10 as 6 p. each. How many did he buy?

2. How many sheep worth Rs. 7-8 as. should be given in exchange for 20 bulls worth Rs. 25-8 as. each?

3. How many ducats of 4*s.* $11\frac{3}{4}$*d.* each are worth 55,926 dollars of 4*s.* $10\frac{1}{2}$*d.* each?

4. What quantity of jaggery at Rs. 2-10-8 a maund should be given in exchange for $4\frac{1}{2}$ mds. of sugar at Rs. 10 a maund?

CHAPTER V.

COMPOUND QUANTITIES.—*Continued.*

INDIAN AND BRITISH TABLES OF WEIGHTS AND MEASURES

TABLES OF WEIGHT.

94. MADRAS COMMERCIAL WEIGHT.

3 *Tolas* (*Tol.*) *make* 1 *Palam* (*Pal.*)
40 *Pals.* ,, 1 *Viss.*
8 *Viss.* ,, 1 *Maund* (*Md.*)
20 *Mds.* ,, 1 *Candy* (*Can.*)

8 Palams = 1 Seer.
5 Seers = 1 Viss.

One rupee weighs 1 Tola or 180 grs. Troy.

The South Indian or Madras Maund is nearly 25 lbs. Avoir.

95. INDIAN IMPERIAL WEIGHT.

This table is used for weighing salt and goods sent by Railway.

80 *Imperial Tolas make* 1 *Imperial Seer.*
40 *Imperial Seers* ,, 1 *Imperial Maund.*

An Indian Imperial Maund = $82\frac{2}{7}$ *lb. Avoirdupois.*

EXAMPLES. I.

Reduce—

1. 12 can. 15 mds. 4 viss 4 seers 5 pals. to palams.
2. 45 can. 18 mds. 7 viss 3 seers 1 pal. to palams.

3. 300 can. 10 mds. 2 viss 20 pals. to tolas.
4. 35 can. 13 mds. 4 viss 18 pals. to tolas.

Reduce to candies, etc.—5. 95,403 seers. 6. 700,500 pals.
7. 67,56,789 tolas. 8. 87,65,430 half-tolas.

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96. TABLE OF AVOIRDUPOIS WEIGHT.

This table is used in weighing all articles and all objects of commerce.

16 *Drams (drs.) make* 1 *Ounce (oz.)*
16 *oz.* ,, 1 *Pound (lb.)*
28 *lbs.* ,, 1 *Quarter (qr.)*
4 *qrs.* ,, 1 *Hundredweight (cwt.)*
20 *cwt.* ,, 1 *Ton.*

1 lb. Avoir. weighs 7,000 grs. Troy.
1 stone (st.) = 14 lbs. Avoir.

EXAMPLE. Reduce 20,806 lbs. to tons, etc.

28 { 4|20806
7|5201 four lbs. 2 lbs. Instead of dividing by 28, we
4|743 quarters. divide by its factors 4 and 7.
20|185 cwt. 3 qrs. Ans. 9 tons 5 cwt. 3 qrs. 2 lbs.
9 tons 5 cwt.

EXAMPLES. II.

Reduce—

1. 2 cwt. 21 lbs. to drams.
2. 19 cwt. 2 qrs. 10 lbs. to lbs.
3. 8 tons 3 cwt. 21 lbs. to lbs.
4. 81 tons 8 cwt. 3 qrs. to lbs.
5. 9,800 drams to qrs.
6. 46,789 lbs. to tons.
7. 736,990 oz. to tons.
8. 908,706 oz. to tons.

TABLES OF TIME.

97. INDIAN TABLE OF TIME.

60 *Vinadies make* 1 *Naliga.*
7½ *Naligas* ,, 1 *Jamam.*
8 *Jamams* ,, 1 *Day.*

1 Naliga = 24 minutes.
2½ Naligas = 1 hour.

98. English Table of Time.

60 *Seconds* (*sec.*) *make* 1 *Minute* (*min.*)
60 *Mins.* ,, 1 *Hour* (*hr.*)
24 *Hrs.* ,, 1 *Day.*
7 *Days* ,, 1 *Week* (*wk.*)
4 *Wks.* ,, 1 *Lunar Month.*

13 Lunar months or 12 Calendar months or 365 days make 1 year.
366 days make 1 leap year.
52 wks. are considered as one year.

99. The number of days in the months is remembered by means of the following lines :—

Thirty days hath September,
April, June and November ;
February has twenty-eight alone,
And all the rest have thirty-one ;
But leap year coming once in four,
February then has one day more.

The following is a very easy method :—Take the knuckles, omitting the thumb on either hand with the depressions between them. Call the first beginning at either end January, and the depression February, the next knuckle March, the next depression April and so on ; the last knuckle is July. As there is no depression following, begin again at the beginning with August and so on to December. Then every month which is represented by a knuckle has 31 days, the others have 30 days (except February which has 28 or 29 days).

100. If the number that denotes a year is divisible by 4, that year is a leap year. Ex. 1888, 1892, 1896.

But years which complete centuries are not to be taken as leap years, unless the numbers which express the centuries are exactly divisible by 4 : thus 1600 and 2000 are leap years, but 1700, 1800, 1900 are not leap years.

EXAMPLES. III.

1. How many days were there from January 1st to September 13, in the year 1900 ?

2. Find the number of days from the 1st of August to the 15th of December, both days included.

3. By how many days did the second half year of 1904 exceed the first half?

4. A person was born on the 15th of May, 1877. How many days old was he on the 8th February, 1892?

Reduce—

5. 20 yrs. 50 days 20 hrs. to minutes.
6. 4 wks. 6 days 50 sec. to seconds.
7. 4,567,893 min. to yrs., days, etc.
8. 567,890,000 sec. to yrs., days, etc.

101. TABLE OF ENGLISH LINEAL MEASURE.

This table is used in measuring distances, lengths, breadths, heights, depths and the like, of places or things.

12	*Inches* (*in.*)	*make*	1 *Foot* (*ft.*)
3	*Ft.*	,,	1 *Yard* (*yd.*)
220	*Yds.*	,,	1 *Furlong* (*fur.*)
8	*Fur.* (1760 *yds.*)	,,	1 *Mile* (*mi.*)

5½ Yds.	make	1 Pole (po.) rod or perch.
40 Poles	,,	1 Furlong.

Used in surveying lands.

100 Links	make	1 Chain (22 yds.)
10 Chains	,,	1 Furlong.

3 Miles = 1 league.
6 Feet = 1 fathom (used for measuring the depth of the sea.)
4 Inches = 1 hand (used for measuring horses).

EXAMPLE. Reduce 3 fur. 5 chains 85 links to links.

3 fur. 5 ch. 85 links = 35 ch. 85 links = 3,585 links.
7,375 links = 73 ch. 75 links = 7 fur. 3 ch. 75 links.

Compare this with the reduction of inches to furlongs in the following example.

Reduce 1,000,000 inches to miles, furlongs, yds., etc.

12 | 1000000 inches.
3 | 83333 feet 4 in.
220 { 20 | 27777 yards 2 ft.
11 | 1388 twenty-yds. + 17 yds. 2 twenty-yds. + 17 yds.
8 | 126 furlongs 2 twenty-yds. = 40 yds. + 17 yds. = 57 yds.
15 miles 6 fur. Ans. 15 mi. 6 fur. 57 yds. 2 ft. 4 in.

EXAMPLES. IV.

The first 8 examples are to be done orally.

Reduce to links—

1. 7 fur. 8 ch. 64 links. 2. 5 fur. 6 ch. 90 links.
3. 6 fur. 7 ch. 85 links. 4. 4 fur. 5 ch. 46 links.

Reduce to furlongs, chains, links—

5. 7,986 links. 6. 8,765 links.
7. 5,987 links. 8. 8,648 links.

Reduce—

9. 3 miles to inches. 10. 2 mi. 6 fur. 20 yds. to feet.
11. 7 fur. 110 yds. 2 ft. to in. 12. 5 mi. 208 yds. to feet.

Reduce to miles, furlongs, yards, etc.—

13. 5,000,000 in. 14. 891,011 ft.
15. 2,345,678 in. 16. 45,678,910 ft.

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102. MADRAS MEASURE OF CAPACITY.

8 *Ollocks* (*oll.*) *make* 1 *Measure* (*Meas.*)
8 *Measures* ,, 1 *Marakkal* (*Mar.*)
5 *Marakkals* ,, 1 *Parah* (*Par.*)
80 *Parahs* ,, 1 *Garce* (*Gar.*)
12 Marakkals = 1 Kalam.
One Measure = 100 Cubic inches.

EXAMPLES. V.

Reduce—

1. 36 gar. 50 par. 3 mar. 4 meas. to measures.
2. 385 gar. 75 par. 3 mar. 7 meas. to measures.
3. 18 gar. 60 par. 4 mar. 6 oll. to ollocks.
4. 120 gar. 35 par. 2 mar. 5 meas. 3 oll. to ollocks.

Reduce—

5. 753,103 oll. to kalams. 6. 56,789 meas. to garce, etc.
7. 876,543 oll. to garce, etc. 8. 1,234,567 oll. to garce, etc.

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103. English Measures of Capacity.

For measuring liquids such as water, wine, beer, etc.

4 *Gills* (*g.*) *make* 1 *Pint* (*pt.*)
2 *Pints* ,, 1 *Quart* (*qt.*)
4 *Quarts* ,, 1 *Gallon* (*gal.*)
136 *Gallons* ,, 1 *Barrel.*

A pint of pure water weighs a pound and a quarter.
∴ A gallon weighs 10 lbs.

For measuring dry goods.

2 *Gals.* *make* 1 *Peck* (*pk.*)
4 *Pks.* ,, 1 *Bushel* (*bus.*)
8 *Bus.* ,, 1 *Quarter* (*qr.*)
5 *Qrs.* ,, 1 *Load.*

EXAMPLES. VI.

Reduce—

1. 9 gals. 3 qts. to pints. 2. 3 bar. 10 gals. 2 qts. to pints.
3. 4 qrs. 7 bus. to gals. 4. 3 loads 3 qrs. 3 bus. to pecks.
5. 1,600 pints to barrels. 6. 8,756 pints to barrels.
7. 126,252 gals. to qrs. 8. 7,340 gals. to qrs.

---

104. Table of Number.

12 *Units* *make* 1 *Dozen.*
12 *Dozen* ,, 1 *Gross.*
20 *Units* ,, 1 *Score.*
24 *Sheets of Paper* ,, 1 *Quire.*
20 *Quires* ,, 1 *Ream.*
10 *Reams* ,, 1 *Bale.*

EXAMPLES. VII.

Reduce—

1. 14 reams 17 quires; 2. 5 bales 6 reams 9 quires to sheets.
3. 804,468 sheets; 4. 567,890 sheets to bales, reams, etc.

EXAMPLES. VIII.

Add the following vertically and horizontally:—

	1				2				3				4			
I.	Md.	vs.	pl.	tol.	Md.	vs.	pl.	tol.	Md.	vs.	pl.	tol.	Md.	v.	pl.	tol.
5.	15	7	25	2	17	6	15	1	19	5	28	2	17	6	17	1
6.	17	6	33	1	19	3	19	0	66	3	17	1	9	5	28	2
7.	19	4	19	2	15	4	27	2	15	6	29	2	10	3	32	2
8.	18	2	16	1	18	7	39	2	18	4	32	2	15	6	19	1
9.	16	5	9	2	6	2	8	1	14	7	9	0	18	4	8	2
10.	8	3	27	2	14	5	16	2	17	2	36	1	15	7	6	1

	1				2			3			4		
II.	Cwt.	qrs.	lb.	oz.	Cwt.	qrs.	lb.	Cwt.	qrs.	lb.	Ton.	cwt.	qrs.
5.	7	3	24	12	6	2	16	17	1	18	9	0	3
6.	6	2	9	7	17	3	19	9	3	20	17	17	2
7.	9	3	15	9	9	2	20	6	2	12	8	9	1
8.	8	1	19	10	13	3	8	5	3	16	6	4	2
9.	5	2	18	8	7	2	9	8	1	24	15	18	2
10.	4	3	14	10	16	3	24	2	2	22	14	14	3

	1				2				3				4			
III.	Fur.	yds.	ft.	in.	Mi.	yds.	ft.	in.	Mi.	yds.	ft.	in.	Mi.	yds.	ft.	in.
5.	6	120	2	8	3	100	1	6	4	219	2	11	8	207	2	9
6.	7	208	1	9	4	208	2	8	3	128	1	10	6	74	1	10
7.	4	62	2	6	7	192	1	9	5	200	2	9	4	128	2	11
8.	8	74	1	7	5	82	2	11	6	61	1	8	2	92	2	8

EXAMPLES. IX.

Perform the following subtractions:—

1.	Can.	mds.	vis.	sr.	pal.
	2345	9	4	3	4
	925	12	6	3	5

2.	Can.	mds.	vis.	sr.	pal.
	7446	14	5	4	7
	2789	18	6	3	5

3.	Tons.	cwt.	qrs.	lb.
	720	17	2	14
	358	19	3	27

4.	Tons	cwt.	qrs.	lb.	oz.
	1000	6	3	12	8
	...	...	...	...	...
	567	18	3	16	12

5.	Cwt.	qrs.	lb.	oz.	dr.
	15	2	18	10	6
	...	...	...	...	...
	7	3	24	13	9

6	Mi.	fur.	ch.	links.
	21	4	7	45
	15	6	8	72

Perform the following subtractions:—

7.

Mi.	fur.	ch.	links.
100	6	8	60
71	7	9	85

8.

Mi.	fur.	yds.	ft.
368	4	104	1
...	...	...	..
99	6	215	2

9.

Gar.	par.	mar	meas.	oll.
709	69	4	6	5
382	71	4	7	6

10.

Gar.	par.	mar.	meas.	oll.
7080	63	2	4	3
237	49	3	6	5

11.

Bales	reams	quires	sheets
50	4	12	18
23	6	13	20

12.

Bales	reams	quires	sheets
123	5	3	10
...	...	...	...
67	7	10	16

EXAMPLES. X.

Multiply—

1. 35 can. 3 mds. 4 viss 5 seers 3 pals. by 235.
2. 475 can. 12 mds. 2 viss 23 pals. by 459.
3. 12 cwt. 3 qrs. 20 lb. 12 oz. by (1) 87. (2) 149.
4. 99 tons 10 cwt. 2 qrs. 24 lbs. by (1) 456. (2) 397.
5. 405 tons 15 cwt. 1 qr. 16 lbs. by (1) 789. (2) 654.
6. 320 yds. 1 ft. 8 in. by (1) 71. (2) 83.
7. 4 mi. 3 fur. 110 yds. 2 ft. 4 in. by (1) 35. (2) 88.
8. 3 days 9 hrs. 8 min. 31 sec. by 410.
9. 13 wks. 3 days 20 hrs. 30 min. 20 sec. by 463.
10. 72 gar. 45 par. 3 mar. 6 meas. by 123.
11. 45 kals. 10 mar. 3 meas. 4 oll. by 271.
12. 500 kals. 8 mar. 6 meas. 6 oll. by 365.

EXAMPLE. The circumference of a carriage wheel is 12 ft. Find the distance travelled when it has made 880 revolutions.

In 1 revolution it travels a distance of 12 ft.

∴ In 880 revolutions it travels a distance of 880 × 12 ft. or 2 miles.

EXAMPLES. XI.

1. The circumference of a carriage wheel is 12 ft. Find the distance travelled when it has made 1,100 revolutions.

2. The circumference of a wheel is 11 ft. Find the distance travelled when it has made 800 revolutions.

3. A carriage wheel is 13 ft. round; in what distance will it make 440 revolutions?

4. Find the distance travelled by a carriage wheel 14 ft round, when it has made 990 revolutions.

EXAMPLES. XII.

Divide—

1. 487 tons 13 cwt. 1 qr. 25 lbs. 10 oz. by 9.
2. 409 tons 3 cwt. 26 lbs. 5 drs. by 11.
3. 5 tons 8 cwt. 2 qrs. 21 lbs. by 37.
4. 409 tons 3 cwt. 25 lbs. 12 oz. by 96.
5. 217 tons 14 cwt. 3 qrs. 14 lbs. by 98.
6. 402 can. 3 mds. 1 sr. 7 pals. by 405.
7. 9,999 can. 19 mds. 7 viss 39 pals. by 889.
8. 16 gar. 7 par. 2 mar. 1 meas. by 184.
9. 40 gar. 50 par. 4 mar. 4 meas. 4 oll. by 333.
10. 350 days 15 hrs. 24 min. by 120.
11. 882 yrs. 253 days 9 min. 54 sec. by 243.
12. 27 mi. 274 yds. 9 in. by 881.

EXAMPLE. Find the circumference of a carriage wheel which makes 660 revolutions in going over a distance of 1 mi. 4 fur.

In 660 revolutions, the wheel goes over 1 mi. 4 fur. or 12 × 220 yds.

∴ In 1 revolution, the wheel goes over 12 × 220 yds. ÷ 660 or 4 yds
∴ The circumference of the wheel is 4 yds.

EXAMPLES. XIII.

1. Find the circumference of a carriage wheel which makes 440 revolutions in a distance of 1 mi. 4 fur.

2. A carriage wheel makes 1,210 revolutions in going over a distance of 2 mi. 6 fur. Find its circumference.

3. What is the circumference of a wheel which turns round 800 times in going over 1 mi. 2 fur.?

4. A wheel makes 514 complete revolutions in passing over 1 mile 467 yds. 1 ft. Find its circumference.

105. Division of one compound quantity by another.

EXAMPLE. Divide 2 tons 4 cwt. 2 qrs. 11 lbs. into parcel of 1 cwt. 2 qrs. 17 lbs. each.

Both the quantities ought to be reduced to the same denomination, and then the division done.

2 tons 4 cwt. 2 qrs. 11 lbs. = 4995 lbs.
1 cwt. 2 qrs. 17 lbs. = 185 lbs.
∴ No. of parcels required = 4995 ÷ 185 = 27.

EXAMPLES. XIV.

1. How many periods of 3 days 11 hrs. 33 min. are there in 41 wks. 1 day 22 hrs. 39 min. ?

2. How many packages of 15 oz. can be made out of 8 tons 8 cwt. 1 qr. 1 lb. 10 oz. ?

3. Show that there are seven times as many palams in 2 can. 13 mds. 7 viss 8 pals. as there are ollocks in 7 par. 3 mar. 4 meas.

4. Show that there are 13 times as many inches in 14 mi. 4 fur. 61 yds. as there are ounces in 1 ton 19 cwt. 2 qrs. 3 lbs. 8 oz.

5. How many revolutions does a carriage wheel 13 ft. 6 in. in circumference make in going over a distance of 18 miles ?

6. A man in walking takes steps each 32 in. long. How many steps does he take in going over 3 miles ?

7. A post man walks 14 miles a day. How many steps does he take in a week (6 days), each step being 2 ft. in length ?

8. The fore-wheel of a carriage is 10 ft. in circumference, and the hind-wheel is 16 ft. How many revolutions will the one make more than the other in 10 miles ?

CHAPTER VI.

MISCELLANEOUS PROBLEMS ON COMPOUND RULES.

EXAMPLE 1. A house and its furniture are worth £900-11-3 ; but the house is worth 8 times as much as the furniture. What is the house worth ?

The value of the house is 8 times the value of the furniture.
∴ The value of the furniture + the value of the house
= 9 times the value of the furniture = £900-11-3.
∴ The value of the furniture £900-11-3 ÷ 9 = £100-1-3.
∴ The value of the house = £100-1-3 × 8 = £800-10*s*.

EXAMPLES. I.

1. A house and furniture are worth £6,734-5-9. The house is worth 8 times as much as the furniture. What is the house worth?

2. A coat and trousers cost Rs. 18-6-6. The former costs twice as much as the latter. What does the coat cost?

3. A carriage and a horse cost Rs. 1,800. The carriage costs 5 times as much as the horse. Find the cost of each.

4. A man's luggage weighs 6 times as much as himself. The weight of both is 1,211 lbs. How many lbs. does the luggage weigh more than the man?

EXAMPLE 2. An equal number of rupees, half-rupees and quarter-rupees amounts to Rs. 56. How many of each coin are there?

1 rupee + 1 half-rupee + 1 quarter-rupee = 7 quarter-rupees.
And Rs. 56 contains 56 × 4 or 224 qr. rs.
∴ No. of coins reqd. = 224 ÷ 7 or 32 coins of each kind.

EXAMPLES. II.

1. The sum of an equal number of rupees, half-rupees, quarter-rupees and two-anna pieces amounts to Rs. 20-10 as. How many of each sort are there?

2. Divide 270 guineas into an equal number of guineas, half-guineas, crowns, half-crowns, shillings and six-pennies.

3. An equal number of guineas, pounds, half-guineas, crowns and half-crowns amounts to £398-5*s*. How many of each coin are there?

4. The same number of men and women are employed in a piece of work. Each man gets 4 as. 6 p. a day and each woman 3 as. a day. Their daily earnings amount to Rs. 49-11 as. Find the number of men and women.

EXAMPLE 3. The fore-wheel of a carriage, whose circumference is 6 ft. 8 in. makes 176 revolutions more than the hind-wheel in 2 miles. Find the circumference of the hind-wheel.

2 mi. = 2 × 5280 × 12 in. ; 6 ft. 8 in. = 80 in.
The fore-wheel makes 2 × 5280 × 12 ÷ 80 or 1584 revolutions
∴ The hind-wheel makes (1584 − 176) or 1408 revolutions.
∴ The circumference of the hind-wheel
= 2 miles ÷ 1408 = 2 × 5280 × 12 in. ÷ 1408 = 90 in. = 7 ft. 6 in

EXAMPLES. III.

1. The fore-wheel of a carriage, 11 ft. in circumference, makes 1,680 turns more than the hind-wheel in a distance of 10 mi. 4 fur. Find the circumference of the hind-wheel.

2. The fore-wheel of a carriage which is 7 ft. in circumference makes 792 turns more than the hind-wheel in a journey of 3 mi. 4 fur. Find the circumference of the hind-wheel.

3. The fore-wheel of a carriage 7 ft. 6 in. in circumference makes 280 revolutions more than the hind-wheel in a distance of 1 mi. 440 yds. Find the circumference of the hind-wheel.

4. The hind-wheel of a carriage is 7 ft. 6 in. in circumference; and it makes 429 revolutions less than the fore-wheel in 1 mi. 4 fur. Find the circumference of the fore-wheel.

EXAMPLE 4. A person buys 220 yds. of cloth at 4 as. 6 p. a yard and sells it at 5 as. 4 p. a yard. What does he gain?

His gain on 1 yd. = 5 as. 4 p. − 4 as. 6 p. = 10 p.
∴ His gain on 220 yds. = 220 × 10 p. = 2,200 p. = Rs. 11-7-4.

EXAMPLES. IV.

1. A paper merchant bought 25 reams of paper at Rs. 4 12 as. per ream and sold it at Rs. 5-2 as. a ream. What did he gain?

2. A person bought 1,650 yds. of cloth at 4*s.* 6½*d.* a yd. and retailed it at 6*s.* a yard. What was his profit?

3. The cost price of 243 candles is Rs. 11-6-3 and the selling price is Rs. 13-14-9. What is the gain on each?

4. 172 tons of goods costing £430 were sold for £440 15*s.* What was the gain on one ton?

EXAMPLE 5. A person buys 13 mds. 6 viss of sugar at Rs. 160 per candy, and retails it at Re. 1-0-6 per viss; find his gain.

1 candy or 160 viss is bought for Rs. 160.
∴ 1 viss is bought for Re. 1.
But 1 viss is sold for Re. 1-0-6.
∴ The gain on 1 viss = 6 pies.
The gain on 13 mds. 6 viss or 110 viss = 110 × 6 p.
= 660 p. = Rs. 3-7 as.

EXAMPLES. V.

1. Jaggery bought at 13 as. 4 p. per viss is sold at 5 pies per palam. What is the gain on 1 candy?

2. If sugar be bought at £1-19-6 a cwt. and sold at $6\frac{1}{2}d$. a lb., what is the profit on a cask of 7 cwt. 1 qr. 14 lbs.?

3. Goods are bought at $6\frac{1}{4}d$. per lb., and the cost of carriage is $1\frac{1}{2}d$. per lb. They are sold at £4-10*s*. per cwt. What is the loss or gain per ton?

4. 320 steel nibs bought at 11 as. a gross are sold at 1 an. 2 p. a dozen. Find the gain.

EXAMPLE 6. 267 shawls were sold for Rs. 4,005; and the gain on each shawl was Rs. 4. Find the cost price of a shawl.

267 shawls were sold for Rs. 4,005.
The gain on 267 shawls is Rs. 267 × 4 or Rs. 1,068.
∴ 267 shawls were bought for Rs. 4,005—Rs. 1,068 or Rs. 2,937.
∴ The cost price of 1 shawl = Rs. 2,937 ÷ 267 = Rs. 11.

EXAMPLES. VI.

1. 250 books were sold for Rs. 562-8 as.; and the gain on each was 12 as. Find the cost price of a book.

2. A flock of 375 sheep was sold for Rs. 1,218-12 as.; and the gain on each was 3 as. Find the cost price of a sheep.

3. 3 750 horses were sold for £153,000; and the gain on each was £3-5*s*. How much did each horse originally cost?

4. A person sold 367 sheep for £759-19-11 gaining thereby 16*s*. on each sheep. How much did he buy a sheep for?

EXAMPLE 7. Six tables and a dozen chairs were bought for Rs. 91-8 as. If the chairs cost Rs. 3-8 as. each, find the price of each table.

The cost of 1 chair = Rs. 3-8 as.
∴ The cost of 12 chairs = Rs. 3-8 as. × 12 = Rs. 42
But the cost of 6 tables and 12 chairs = Rs. 91-8 as.
∴ The cost of 6 tables = Rs. 91-8 as. — Rs. 42
= Rs. 49-8 as.
∴ The cost of 1 table = Rs. 49-8 as. ÷ 6 = Rs. 8-4 as.

EXAMPLES. VII.

1. 17 chairs and 13 tables together cost Rs. 135-12 as. If each chair cost Rs. 2-4 as., find the price of each table.

2. 20 sheep and 12 cows together cost Rs. 316. If a cow cost Rs. 15-8 as., find the price of a sheep.

3. A has 50 sheep worth Rs. 6-12 as. each ; B gives A 15 cows and Rs. 37-8 as. cash and takes the sheep. What is the value of a cow ?

4. A gives to B 98 sheep worth Rs. 12-12 as. each, and gets in return Rs. 409-8 as. and 576 yds. of cloth. What is the value of the cloth per yard ?

EXAMPLE 8. A number of chairs was bought for Rs. 420 and sold for Rs. 560 ; and the gain on each chair was Re. 1-2-8. How many were bought ?

The gain on the chairs is Rs. 560 — Rs. 420 or Rs. 140.
But the gain on 1 chair = Rs. 1-2-8.

$$\therefore \text{The number of chairs} = \frac{\text{Rs. } 140}{\text{Rs. } 1\text{-}2\text{-}8} = \frac{140 \times 192}{224} = 120.$$

EXAMPLES. VIII.

1. A certain number of articles was bought for Rs. 800 and sold for Rs. 1,000 ; and the gain on each was Re. 1-4 as. How many were bought ?

2. A person bought a certain number of cows for Rs. 625 and sold them for Rs. 703-2 as. The gain on each being Rs. 3-2 as., find how many were bought.

3. The cost price of a certain number of watches was £3,479, the selling price £3,763 and the gain on each £1-6-8. How many were sold ?

4. The cost price of a number of books is Rs. 335-15-5 and the selling price Rs. 374-5-9 ; and the gain on each is 6 as. 4 p. Find the number sold.

EXAMPLE 9. A farmer bought a number of sheep for £80 at 16*s*. a sheep, and after reserving a certain number for himself, sold the rest at 18*s*. a sheep for £81. How many did he reserve for himself ?

Since 16*s*. is the cost price of 1 sheep.
£80 or 1600*s*. is the cost price of 1600 ÷ 16 or 100 sheep.
Since 18*s*. is the selling price of 1 sheep.
£81 or 1620*s*. is the selling price of 1620 ÷ 18 or 90 sheep.
∴ He reserved for himself (100 — 90) sheep or 10 sheep.

EXAMPLES. IX.

1. A person buys a number of sheep at Rs. 7-8 as. each for Rs. 600 ; after some time a few of them died, and he sold the rest at Rs. 8 each for Rs. 560. How many died ?

2. A person lays out £43-9-4 in spirits at 5*s*. 4*d*. a gallon, some of which leaked out in the carriage. He sold the remainder however for £54 at the rate of 7*s*. 6*d*. a gallon. How many gallons leaked out ?

3. Bought soda-water bottles at 4 as. a bottle for Rs. 12-8 as. Some of them were broken ; and the remaining ones were sold for Rs. 15 at 5 as. 4 p. each. How many were broken ?

4. A person bought lemonade at 1 an. 6 p. a bottle for Rs. 27-4-6, some of which he reserved for himself. The remainder was sold for Rs. 35 at 2 as. a bottle. How many bottles did he reserve for himself ?

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CHAPTER VII.

METRIC TABLES OF LENGTH AND WEIGHT.

106. In England, all lengths are commonly measured by the **yard,** which is the standard or unit of length. But in France and some other countries of Europe, another unit called the **metre** is used for measuring lengths. Scientific men throughout the world use the metre as the unit of length.

The **metre** is divided into 10 equal parts called **decimetres,** each decimetre is sub-divided into 10 equal parts called **centimetres** and the centimetre is further sub-divided into 10 equal parts called **millimetres.**

A length of 10 metres is called a **decametre**; a length of 10 decametres is called a **hectometre**; and a length of 10 hectometres is called a **kilometre.**

In these names the prefix—

Deci	means	one-tenth.
Centi	,,	one-hundredth.
Milli	,,	one-thousandth.
Deca	,,	ten times.
Hecto	,,	hundred times.
Kilo	,,	thousand times.

107. METRIC TABLE OF LENGTH.

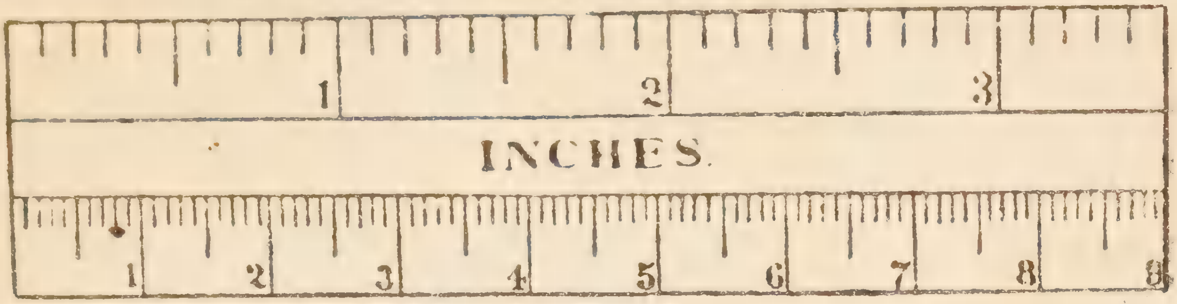
10	*Millimetres* (*mm.*)	*make*	1 *Centimetre* (*cm.*)
10	*Centimetres*	,,	1 *Decimetre* (*dm.*)
10	*Decimetres*	,,	1 **Metre** (*m.*)
10	*Metres*	,,	1 *Decametre* (*Dm.*)
10	*Decametres*	,,	1 *Hectometre* (*Hm.*)
10	*Hectometres*	,,	1 *Kilometre* (*Km.*)

This table may also be written thus :

1,000 mm. = 100 cm. = 10 dm. = 1 m.

1,000 m. = 100 Dm = 10 Hm. = 1 Km.

Of these, decametres and hectometres are little used. Long distances are expressed in kilometres and metres ; short distances in metres, centimetres and millimetres; but very short lengths are expressed in centimetres and millimetres.



CENTIMETRES

FIG. 14.

The figure shows the comparison between inches and centimetres.

Practical Work

Take a scale reading to half a centimetre, tenths of a centimetre and tenths of an inch; examine and satisfy yourself that half a centimetre = 5 mm.; 1 cm. = 10 mm.; 2 cm. and 5 mm. = 25 mm.

3 cm. 7 mm. = 37 mm.; also 45 mm. = 4 cm. 5 mm.

1. Mark the following lengths in your notebook :—

(1) 1 in. 8 tenths, 2 in. 3 tenths, 3 in. 7 tenths, 4 in. 6 tenths, 5 in. 9 tenths. (2) 47 mm., 56 mm., 73 mm., 94 mm., 120 mm. (3) 2 cm. 5 mm., 6 cm. 4 mm., 1 dm. 2 cm. 3 mm., 2 dm. 0 cm. 7 mm.

2. Measure any one or some of your text-books and express the length in (1) inches and tenths of an inch, (2) millimetres, (3) centimetres and millimetres, (4) centimetres and tenths of a centimetre.

3. Express the length and breadth of your desk, the length, width and height of the table as in the preceding exercise.

4. Take your dividers, adjust to read on the inch scale from 0 to 1 inch; and then transfer to the millimetre scale so as to have one of the points on 0 mm.; and find the number of millimetres corresponding to one inch; and do the same for 2, 3, 4,...........12 in. = Record your results thus :

1 inch = ... mm. = ... cm.
2 in. = ... mm. = ... cm.
3 in. = ... mm. = ... cm.
... ... = ... = ...

108. Reduction in British and Metric Lengths compared.

EXAMPLE 1. Reduce (*a*) 9 yds. 2 ft. 8 in. to inches.
(*b*) 9 m. 2 dm. 8 cm. to cm.

British Length.	*Metric Length.*
9 yds. 2 ft. 8 in.	9 m. 2 dm. 8 cm.
3	= 92 dm. 8 cm., since 9 m. = 90 dm.
29 ft.	= 928 cm., since 92 dm. = 920 cm.
12	The multiplication is by 10 only.
356 in.	
We first multiply by 3 and then by 12.	

EXAMPLE 2. Reduce (1) 5678 inches to yards.
(2) 5678 mm. to metres, etc.

British Length.	*Metric Length.*
12)5678 in.	5678 mm. = 567 cm. 8 mm., dividing by 10.
3)473 ft. 2 in.	= 56 dm. 7 cm. 8 mm., dividing cm. by 10.
157 yds. 2 ft.	
Divisions first by 12 and then by 3.	= 5 m. 6 dm. 7 cm. 8 mm, dividing dm. by 10.
	Division throughout by 10.

109. In the Metric System, labour in the process of Reduction is done away with and Reduction can be done mentally.

Thus, 6 m. 0 dm. 5 cm. = 605 cm.
and 8706 mm. = 8 m. 7 dm. 0 cm. 6 mm.

EXAMPLES.—ORAL.

Express in millimetres—

1. 7 m.
2. 5 dm.
3. 6 cm.
4. 9 cm.
5. 7 cm. 5 mm.
6. 9 dm. 8 cm.
7. 3 m. 4 cm.
8. 5 m. 5 mm.
9. 7 m. 6 mm.
10. 6 m. 4 cm. 5 mm.
11. 5 m. 6 dm. 7 mm.
12. 8 m. 8 cm. 5 mm.

How many centimetres are there in—

1. 5 m. 7 dm. 3 cm.
2. 8 m. 6 dm. 5 cm.
3. 7 m. 5 dm. 8 cm.
4. 5 m. 9 cm.
5. 10 m. 8 cm.
6. 15 m. 7 cm.
7. 5 Dm. 7 m.
8. 7 Hm. 6 dm.
9. 6 Km. 4 Hm.

Reduce to metres—

1. 6 Km. 7 Hm.
2. 5 Km. 6 Dm.
3. 7 Km. 3 m.
4. 5 Km. 8 m.
5. 10 Km. 7 Dm. 5 m.
6. 12 Km. 7 Dm. 8 m.

Express in Km. Hm., etc.

1. 128 m. 2. 634 m. 3. 3208 m. 4. 5034 m.
5. 56789 dm. 6. 78090 cm. 7. 234056 dm. 8. 305078 cm.

Express in m. dm., etc.

374 cm., 504 cm., 1035 cm., 2309 cm.
356 mm., 857 mm., 2304 mm., 30406 mm.

110. METRIC TABLE OF WEIGHT.

10 *Milligrams* (*mg.*) *make* 1 *Centigram* (*cg.*)
10 *Centigrams* ,, 1 *Decigram* (*dg.*)
10 *Decigrams* ,, 1 **Gram (g.)**
10 *Grams* ,, 1 *Decagram* (*Dg.*)
10 *Decagrams* ,, 1 *Hectogram* (*Hg.*)
10 *Hectograms* ,, 1 *Kilogram* (*Kg.*)

This table may also be written thus :

1,000 mg. = 100 cg. = 10 dg. = 1 g.
1,000 g. = 100 Dg. = 10 Hg. = 1 Kg.

The most commonly used weights are the milligram, the centigram, the gram and the kilogram. For very heavy weights, a **quintal** equal to 100 kilograms and a **tonne** equal to 1,000 kilograms are used.

EXAMPLE 3. Reduce (1) 4 cwt. 3 qrs. 5 lb. 6 oz. to ounces.
(2) 4 g. 3 dg. 5 cg .6 mg. to milligrams.

(1) *British Weight.*

4 cwt. 3 qrs. 5 lb. 6 oz.
4
19 qrs.
4
76
7
537 lb.
16
8598 oz.

(2) *Metric Weight.*

4 g. 3 dg. 5 cg. 6 mg.
= 43 dg. 5 cg. 6 mg.
= 435 cg. 6 mg.
= 4356 mg.

Each higher denomination is multiplied by 10 to bring it to the lower.

EXAMPLE 4. Reduce (1) 17285 oz. to cwt., etc.
(2) 17285 mg. to grams, etc.
(3) 17285 g. to Kg., etc.

	British Weight.	*Metric Weight.*
28	16|17285 oz.	17285 mg. = 1728 cg. 5 mg.
	4|1080 lb. 5 oz.	= 172 dg. 8 cg. 5 mg.
	7|270 four lb.	= 17 g. 2 dg. 8 cg. 5 mg.
	4|38 qrs. 4 four lb.	17285 g. = 1728 Dg. 5 g.
	9 cwt 2 qrs.	= 172 Hg. 8 Dg. 5 g.
	Ans. 9 cwt. 2 qrs. 16 lbs. 5 oz.	= 17 Kg. 2 Hg. 8 Dg. 5 g.

Expressing 7085 mg. in grams, we may at once say 7 g. 0 dg. 8 cg. 5 mg.; or 5 g. 6 cg. 7 mg. may be expressed as 5067 mg.

EXAMPLES.—ORAL.

Express in milligrams—

1. 8 g. 2. 6 dg. 3. 5 cg. 4. 9 cg.
5. 5 cg. 7 mg. 6. 8 dg. 9 cg. 7. 7 cg. 6 mg. 8. 8 dg. 7 cg.
9. 3 g. 9 mg. 10. 5 g. 7 cg. 11. 7 g. 7 mg. 12. 5g. 4cg. 6mg.

Express in centigrams—

1. 7 g. 2. 5 g. 5 cg. 3. 6 g. 9 cg. 4. 4 g. 5 dg. 6 cg.
5. 6 g. 7 dg. 8 cg. 6. 5 Dg. 7g. 7. 1 Dg. 7g. 3dg. 8. 2 Dg. 4g. 6cg.

Express in grams—

1. 6 Kg. 7 g. 2. 7 Kg. 5 g. 3. 4 Kg. 4 Hg. 4 Dg.
4. 6 Kg. 7 Dg. 5 g. 5. 7 Kg. 4 Dg. 6 g. 6. 8 Kg. 5 Dg. 7 g.

Express in Kg., etc.—

1. 3405 g. 2. 5430 g. 3. 6078 g. 4. 8009 g.
5. 24058 cg. 6. 30456 cg. 7. 45607 cg. 8. 123454 cg.

Express in g. cg. mg.—

1. 485 cg. 2. 736 cg. 3. 840 cg. 4. 1002 cg.
5. 2078 mg. 6. 2403 mg. 7. 3456 mg. 8. 5070 mg.

---

111. Decimal Notation.

5555 = 5 thousands + 5 hundreds + 5 tens + 5 units: and
5555 metres = 5 Km. + 5 Hm. + 5 Dm. + 5 m.; also
5555 mg. = 5 g. + 5 dg. + 5 cg. + 5 mg.

We thus see that just as in the ordinary system of notation, so in the metric tables of length and weight, each figure in a number has *ten* times the value of the figure next on its right and *one-tenth* the value of the figure next on the left.

In other words, each figure denoting a certain denomination has ten times the value of the figure next on the right which indicates a lower denomination, and one-tenth the value of the figure next on the left which indicates a higher denomination. The length or weight denoted by any one of the digits depends upon the *place* value of the particular digit.

Thus 5555 mm. = 5 m. + 5 dm. + 5 cm. + 5 mm.

The right-hand digit 5 represents one-tenth of the length which it would represent, if it were moved one place to the left.

Since a millimetre is one-tenth of a centimetre 5555 mm. may be written 555 cm. 5 mm. ; or in terms of the centimetre, it may be written 555 cm. and 5 tenths of a centimetre.

112. To distinguish the tenths of a centimetre from the units' figure in the centimetres, a point is placed to the right of the units' digit and a little above it ; and this point is called the **decimal** point.

Thus 555 cm. 5 mm. is written 555·5 cm., which means five hundred and fifty-five cm. and 5 tenths of a centimetre.

Extending this notation still further to the right of the decimal point, 55·55 dm. means 55 dm. 5 tenths dm. and 5 hundredths dm. that is 5 m. 5 dm. 5 cm. 5 mm.

Also 5·555 g. = 5 g. 5 tenths g. 5 hundredths g. 5 thousandths g.
= 5 g. 5 dg. 5 cg. 5 mg.

In the same way, 5·65 inches = 5 inches 6 tenths of an inch and 5 hundredths of an inch ; £1·28 means £1, two-tenths of £1 and 8 hundredths of £1 ; and 3·456 tons = 3 tons, 4 tenths of a ton, 5 hundredths of a ton and 6 thousands of a ton. We have thus a means of expressing figures to the right of the units' figure, by naming them tenths, hundredths, thousandths, etc., of the given unit.

Extending this notation to abstract numbers;

55·555 units = 5 tens + 5 units + 5 tenths of a unit + 5 hundredths of a unit + 5 thousandths of a unit.
= 5 tens + 5 units + 5 tenths + 5 hundredths + 5 thousandths.

Here the 5 in the units' place has one-tenth of the value of the 5 next on the left, but ten times the value of the 5 next on the right, that is the 5 immediately after the decimal point.

113. Take any one digit that is on the right of the decimal point, that digit has always one-tenth of the value of the next digit on its left and ten times the value of the next digit on its right. Thus, moving a digit one place to the left makes its value ten-fold and moving it one place to the right makes its value one-tenth.

In the ordinary system of notation which deals with whole numbers, we have seen that the moving of a digit one place either to the left or to the right has exactly the same effect.

Therefore figures to the right of the decimal point have the same place values as the figures to the left of the decimal point.

Hence this system of notation called the decimal notation is simply an extension of the ordinary notation.

114. This notation may be shewn as follows :—

	Millions	H. Thousands	Ten ,,	Thousands	Hundreds	Tens	Units	Decl. point	Tenths	Hundredths	Thousandths	Ten ,,	H. ,,	Millionths	
. . . . .	7	6	5	4	3	2	1	.	1	2	3	4	5	6	. . . . .

Numeration.—

7·6 is read seven and 6 tenths ; 0·25 is read 2 tenths, 5 hundredths ; 0·1023 is read 1 tenth, 2 thousandths, 3 ten thousandths ; the cypher after 1 indicates the absence of hundredths.

7·6 is also read seven, decimal (or point) 6 ;
0·25 is ,, ,, decimal (or point) two, five.
0·1023 is ,, ,, ,, one, cypher, two, three.

Notation.—

Three hundreds and five-tenths is written 300·5.

Seven and nine-hundredths is written 7·09, the cypher indicating the absence of tenths.

Twenty + two hundredths + two ten thousandths is written 20·0202.

EXAMPLES.—ORAL.

Assign to each figure its local value ; and express in words—

(1) 20·8. (2) ·15. (3) ·275. (4) 55·37. (5) 305·305. (6) 23·06. (7) 19·035. (8) 5·0027. (9) 20·00835. (10) 2073·0273. (11) 1502·8076. (12) 8706·1502.

Read off as decimals—

1. Eight and nine-tenths.
2. Fifteen ann five-tenths.
3. Thirteen and three-hundredths.
4. One millionth.
5. Three hundred and thirty-three, and three ten thousandths.
6. Thirty-one, and nine-thousandths.
7. Seven hundred, and seven hundred-thousandths.
8. One-thousandth and one ten-millionth.

---

115. We have seen that 5·79 m. = 5 m. 7 dm. 9 cm. Conversely, 5 m. 7 dm. 9 cm. may be expressed as 5·79 m. or as 57·9 dm. or as 579 mm.

Also 3 m. 5 dm. 6 cm. 8 mm. = 3·568 m. = 35·68 dm.
= 356·8 cm. = 3568 mm.
And 5 g. 6 dg. 7 cg. 8 mg. = 5·678 g. = 56·78 dg.
= 567·8 cg. = 5678 mg.
In the same way, 5 m. 4 cm. = 5 m. 0 dm. 4 cm. = 5·04 m.
= 50·4 dm. = 504 cm.
And 0·234 gram. = 2 dg. 3 cg. 4 mg. = 2·34 dg.
= 23·4 cg. = 234 mg.
Also 7 Rs. 75 cents = 7·75 Rs.
2 chains 14 links = 2·14 chains.
6 fur. 2 chains = 6·2 furlongs.
6 fur. 2 ch. 14 links = 62·14 chains = 6·214 furlongs.

EXAMPLES.—ORAL.

Express as metres and decimals of a metre or as grams and decimals of a gram, as the case may be—

1. 9 m. 8 dm.
2. 7 g. 6 dg.
3. 12 m. 7 dm. 8 cm.
4. 5 g. 6 dg. 7 cg.
5. 9 m. 8 dm. 7 cm.
6. 7 g. 6 dg. 5 cg.
7. 9 m. 4 cm.
8. 8 g. 9 cg.
9. 18 g. 8 cg.
10. 6 m. 7 cm. 8 mm.
11. 4 g. 8 cg. 6 mg.
12. 14 m. 9 cm. 6 mm

Express as (1) dm. (2) cm. (3) mm. or as (1) dg. (2) cg. (3) mg. as the case may be—

1. 3 m. 2 dm. 4 mm.
2. 4 g. 5 cg. 6 mg.
3. 6 m. 7 cm. 8 mm.
4. 7 g. 6 dg. 5 mg.
5. 7 m. 8 cm.
6. 5 g. 9 cg.
7. 0·75 m.
8. 0·234 g.
9. 0·456 m.
10. 0·065 g.
11. 4·056 m.
12. 12·678 g.

Express in metres or grams—

1. 376 dm.
2. 486 dg.
3. 869 dm.
4. 1006 dg.
5. 698 cm.
6. 987 cg.
7. 4070 cm.
8. 5386 cg.

Express in Rupees—

1. 8 Rs. 15 c. 2. 25 Rs. 25 c. 3. 92 Rs. 80 c. 4. 78 Rs. 84 c.
5. 985 cents. 6. 729 c. 7. 1089 c. 8. 2075 c.

Express as (1) chains, (2) furlongs—

1. 7 fur. 4 ch. 2. 6 fur. 8 ch. 3. 5 fur. 6 ch.
4. 8 ch. 75 links. 5. 14 ch. 15 l. 6. 25 ch. 80 l.
7. 678 links. 8. 985 l. 9. 789 l.
10. 2 fur. 3 ch. 50 l. 11. 7 fur. 5 ch. 48 l. 12. 10 fur. 7 ch. 36 l.

CHAPTER VIII.

METRIC TABLES AND DECIMALS.

ADDITION AND SUBTRACTION.

EXAMPLE 1. Add together—

g.	dg.	cg.	mg.
5	6	7	8
8	7	6	5
7	6	5	9
9	8	8	4
31	9	8	6

26 mg. = 2 cg. 6 mg.
28 cg. = 2 dg. 8 cg.
29 dg. = 2 g. 9 dg.

(1) Expressing each in centigrams.	(2) Or expressing each in decigrams.	(3) Verifying the correctness of the answer by expressing each in grams.
567·8	56·78	5·678
876·5	87·65	8·765
765·9	76·59	7·659
988·4	98·84	9·884
3198·6 cg.	319·86 dg.	31·986 grams.

ADDITION AND SUBTRACTION OF DECIMALS.

116. Extending this method of addition to abstract decimals, we have the following rule for the addition of decimals :—*Place the numbers under each other, units under units, tens under tens, etc., one-tenths under one-tenths, one-hundredths under one-hundredths, etc., so that the decimal points may be all under each other ; then proceed as in simple addition and* *place the decimal point in the sum under the decimal point above.*

EXAMPLE 2. Add together 43·903, 301·5, ·4612, 7 and 5·69.

```
 43·903
301·5
   ·4612
  7·
  5·69
--------
358·5542.
```

EXAMPLES. I.

Add together examples 1 to 8 and verify the correctness of your answer by expressing each in units of any the same denomination as shewn above in example 1.

1

g.	dg.	cg.	mg.
7	8	9	6
5	6	7	8
8	9	0	4
6	7	8	9
9	4	6	5

2

m.	dm.	cm.	mm.
9	4	6	7
4	8	9	3
5	7	4	9
0	8	7	6
8	3	5	8

3

fur.	ch.	links.
6	7	18
7	9	72
10	8	45
8	9	6
9	6	39

4. 17 g. 9 dg. 7 cg. 6 mg., 8 dg. 9 cg. 7 mg., 9 g. 7 cg. 8 mg., 29 g. 6 dg. 5 mg.

5. 23 m. 5 dm. 6 cm. 8 mm., 4 m. 6 dm. 9 mm., 18 m. 7 dm., 15 m. 6 mm., 19 m. 7 cm. 5 mm.

6. 9 fur. 7 ch. 8 l., 8 fur. 72 l., 7 fur. 9 ch. 7 l., 8 ch. 99 l., 6 fur. 6 ch., 4 fur. 40 l.

7. 23 Kg. 8 g. 7 dg., 77 Kg. 7 g., 8 dg. 7 cg., 54 Kg. 9 g. 7 mg., 49 Kg. 6 dg. 8 cg., 95 Kg. 4 g. 9 mg.

8. 78 Km. 9 m. 4 cm., 45 Km. 6 m. 7 cm., 29 Km. 8 dm., 17 Km. 4 cm. 8 mm., 96 Km. 8 m. 9 mm.

Add together as decimals—

9. Rs. 8·75, Rs. 19·25, Rs. 13·5, Rs. 25·125, Rs. 57·375.

10. Rs. 101·625, Rs. 99·375, Rs. 80·25, Rs. 72·5, Rs. 0·875.

11. 172 Rs. 72 c., 294 Rs. 48 c., 576 Rs. 95 c., 678 Rs. 64 c., 1,009 Rs. 9 c., 2,479 Rs. 8 c.
12. 58 Rs. 9 c., 29 Rs. 75 c., 37 Rs. 5 c., 96 Rs. 96 c., 73 Rs. 48 c.
13. £8·25, £0·175, £812·875, £23·5, £19·625.
14. 3430·01 cwt., 99·9875 cwt., 5·95 cwt., 20·725 cwt.
15. 13·04 ft., 27·56 ft., 32·45 ft., 87·09 ft., 93·72 ft.
16. 0·12 ton, 7·035 ton, 13·456 ton, 27·875 ton, 15·4 ton.
17. Add together—
    (1) 12·3, 120·6, 54·235, 2·79.
    (2) ·003, 36·4, 4·567, 54·00025.
    (3) 500·3, 45·3785, 3·0234, ·012, 17·2369.
    (4) 765·12, 2·345, 50·1357, ·9,44·55.
18. Find the sum of—
    (1) 0·056 + 282·1 + ·75 + 2 + 36·001.
    (2) 874·09 + 9·997 + ·0003 + 25.
    (3) 1000 + ·0001 + 993·993 + ·0023.
    (4) 109·1 + ·0059 + 25·998 + ·876.
    (5) 1234·05 + ·6789 + 78·91012 + 85.
    (6) 100·0001 + 20·0203 + 899·988 + ·732.
    (7) 8·26 + ·0175 + 812·538 + 23·259.
    (8) 0·01 + 3430 + 99·9875 + 5·95.

---

117. 9 m. 3 dm. 5 cm. = 9 m. 3 dm. 5 cm. 0 mm.

Expressing each as metres.

9·35 m. = 9·350 m. : or 9·350 m. = 9·35 mm.

The cypher to the right of the last figure in the decimal has no value. Therefore any cyphers may be added to or omitted from the right of the last decimal figure without altering the value of the decimal.

∴ 9·35 = 9·350 ; and 8·7200 = 8·72.

EXAMPLE 1. Subtract 7 m. 5 dm. 6 cm. 3 mm.
from 9 m. 3 dm. 5 cm.

	m.	dm.	cm.	mm.
From	9	3	5	0
Take	7	5	6	3
	1	7	8	7

Or expressing in metres.

9·350 metres.
7·563 ,,
1·787 metres.

118. Subtraction of Decimals—

Put the less under the greater placing the numbers as in addition and suppose cyphers to be supplied, if necessary, in the upper line to the right of the last decimal figure; then proceed as in simple subtraction and place the decimal point under the decimal point above.

EXAMPLE 2. Subtract 7·563 from 9·35.

9·350
7·563
Ans. 1·787

EXAMPLES. II.

In Examples 1 to 12, perform the subtraction in two ways, (1) as compound quantities; (2) as concrete decimals.

	1.				2.				3.		
	m.	dm.	cm.	mm.	g.	dg.	cg.	mg.	fur.	ch.	l.
From	10	7	3	5	8	6	7	6	6	3	45
Take	5	8	5	6	4	6	8	7	4	5	60

	4.				5.				6.		
From	7	5	0	4	10	0	6	8	13	6	72
Take	3	6	7	8	6	6	7	9	7	8	94

	7.				8.			
From	13	0	6	5	20	5	0	6
Take	7	8	6	8	13	6	4	8

9. Subtract 3 m. 7 dm. 6 cm. 5 mm. from 7 m. 6 dm. 4 cm. 3 mm.

10. Take 4 g. 6 dg. 5 cg. 4 mg. from 8 g. 4 dg. 6 cg. 2 mg.

11. From 10 fur. 6 ch. 72 l. take 8 fur. 8 ch. 80 l.

12. Subtract 23 g. 6 cg. 7 mg. from 40 g. 5 dg. 6 cg. 4 mg.

13. Subtract—

(1) 88·479 from 90·898. (2) 469·735 from 694·398.
(3) 625·6789 from 700·89. (4) 523·450678 from 650.

14. Find the difference between—

(1) 205·987 and 189·098765. (2) 1001·2345 and 999·98765.
(3) 989·989 and 765·7654321. (4) 879·978 and 789·89765.

15. Subtract twice in succession—

(1) 12·3456 from 25·9. (2) 19·8706 from 40·02.
(3) 103·579 from 300·05. (4) 47·89101 from 100.

Example 3. (*a*) Find the value of 8·03 — 1·265 + 15·6045 — 19·3879.

8·03	1·265	23·6345
15·6045	19·36879	20·63379
23·6345	20·63379	3·00071 Ans.

(*b*) What decimal added to the sum of 0·0007, 3·6, ·06, 3·0546 and ·048 will make the result an integer?

·0007
3·6
·06
3·0546
·048
6·7633

The next higher number is 7.
∴ Subtracting 6·7633 from 7.
7·0000
6·7633
0·2367 must be added.

EXAMPLES. III.

Find the value of—

1. 100·1 — 75·432 + 13·456 — 10·1357.
2. 755·01 — 50·963 + ·0705 — 600·8359.
3. 18·703 — 11·9 + 21·7534 — 2·3579.
4. 3·01 + 17·234 + ·6789 — ·05 — 9·867 — 8·76543.
5. Find the difference between the sum and difference of 1875·01 and 974·789.

6. Find the difference between the sum and difference of 15·45678 and 29·0203.

7. What number added to the sum of 7·05, ·567, 4·6 and 17·80604 will make the result an integer?

8. What number added to the sum of 0·0007 + 2·4 + ·05 + 3·0436 + ·047 will make an integer?

CHAPTER IX.

METRIC TABLES AND DECIMALS.

MULTIPLICATION AND DIVISION.

119. By Powers of ten.

We know that 5 dm. 5 cm. × 10 = 5 m. 5 dm.; and 4 cg. 3 mg. × 10 = 4 dg. 3 cg.

Again 5 dm. 5 cm. ÷ 10 = 5 cm. 5 mm.; 4 dg. 3 cg. ÷ 10 = 4 cg. 3 mg.

We thus see that in the metric tables of length and weight multiplying by 10 *has the effect of raising the different denominations of the quantity by one denomination and dividing by* 10 *has the effect of lowering each of the different denominations of the quantity by one denomination.*

EXAMPLE. 0 m. 5 dm. 3 cm. 6 mm. × 10 = 5 m. 3 dm. 6 cm.
0 m. 6 dm. 7 cm. 0 mm. × 100 = 67 m.
3 g. 4 dg. 5 cg. 0 mg. ÷ 10 = 0 g. 3 dg. 4 cg. 5 mg.
1 m. 2 dm. 0 cm. 0 mm. ÷ 100 = 0 m. 0 dm. 1 cm. 2 mm.

EXAMPLES.—ORAL.

Give the value of the following:—

	g.	dg.	cg.	mg.			g.	dg.	cg.	mg.	
1	0	4	5	6	× 10	6.	0	0	7	8	× 1000
2.	0	6	7	8	× 10	7.	5	6	7	0	÷ 10
3.	0	0	8	9	× 100	8.	0	7	6	0	÷ 10
4.	0	5	6	0	× 100	9.	104	2	0	0	÷ 100
5.	10	4	5	6	× 10	10.	5 Kg. ÷ 100.				

Give the value of the following :—

	m.	dm.	cm.	mm.			m.	dm.	cm.	mm.	
11.	0	7	0	8	× 10	16.	0	0	6	7	× 1000
12.	0	0	8	9	× 10	17.	6	9	7	0	÷ 10
13.	0	0	9	6	× 100	18.	0	8	4	0	÷ 10
14.	0	9	5	0	× 100	19.	4	6	0	0	÷ 100
15.	17	4	8	6	× 10	20.	8 m. ÷ 100.				

EXAMPLE. 23·579 m. = 235·79 dm., same as multiplying m. by 10, as 10 dm. = 1 m.
= 2357·9 cm. ,, ,, dm. by 10 or m. by 100.
= 23579 mm. ,, ,, cm. by 10 or m. by 1000.

120. We thus see that to multiply by **10,** the decimal point is moved **one** place to the **right,** and multiplying by **100** is the same as moving the point **two** places to the **right;** and multiplying by **1000** is equivalent to moving the decimal point **three** places to the **right.**

Conversely, 3568 mm. = 356·8 cm., same as dividing mm. by 10.
= 35·68 dm. ,, ,, cm. by 10 or mm. by 100.
= 3·568 m. ,, ,, dm. by 10 or mm. by 1000.

121. We see that to divide by **10,** the decimal place is moved **one** place to the **left** and to divide by **100,** it is moved **two** places to the **left** and to divide by **1000,** the point is moved **three** places to the **left.**

122. The number of places moved either to the right or to the left is the same as the index denoting the power of ten in the multiplier or the divisor, i.e. the number of cyphers in the multiplier or the divisor.

Thus $2·4 \times 10 = 2·4 \times 10^1 = 24$; point moved 1 place to the right.
$0·0024 \times 10000 = 0·0024 = 10^4 - 24$,, ,, 4 places ,, ,,
$2·4 \div 100 = 2·4 \div 10^2 = 0·24$,, ,, 2 ,, ,, left.
$2·4 \div 1000 = 2·4 \div 10^3 = ·0024$,, ,, 3 ,, ,, ,,

EXAMPLES.—ORAL.

1. Multiply each of the following by (1) 10, (2) 100, (3) 1000.
   1. 4·523 m. 2. 15·678 g. 3. 12·075 m. 4. 10·789 g.
   5. 0·256 m. 6. 0·753 g. 7. 0·046 m. 8. 0·032 g.

2. Give the numbers which are *ten times* the following :—
   (1) ·5 ; (2) ·05 ; (3) 2·3 ; (4) ·0006 ; (5) ·00005 ; (6) 15·36 ; (7) ·3045.
3. Multiply each decimal in the above by (1) 100, (2) 1000, (3) 10000.
4. Multiply (1) ·716 separately by 100, 10000, 10.
   (2) 8·0134 by 10000, 1000000 and by 1000.
5. Divide each of the following by (1) 10, (2) 100, (3) 1000.
   1. 4567 m. 2. 8576 m. 3. 4029 g. 4. 8705 g.
   5. 213·45 g. 6. 154·678 m. 7. 34·75 g. 8. 52·56 m.
6. Give the numbers which are a *tenth* of the following :—
   (1) 2 ; (2) 30 ; (3) ·4 ; (4) ·03 ; (5) 1·2 ; (6) 10·5 ; (7) 120·64 ; (8) 75·025.
7. Divide each decimal in the above by (1) 100, 1000, 10000.
8. Divide : (1) ·004 separately by 100, 10000, 1000000.
   (2) 17·123 ,, ,, 1000, 100, 100000.

123. MULTIPLICATION BY A SINGLE FIGURE (INTEGRAL.)

EXAMPLE. Multiply 7 m. 6 dm. 5 cm. 4 mm. by 8.

m.	dm.	cm.	mm.	Expressing in metres and multiplying
7	6	5	4	7·654 metres
			8	8
61	2	3	2	61·232 metres.

EXAMPLES. I.

Multiply (1) as compound quantities, (2) as decimals of grams or metres.

	g.	dg.	cg.	mg.		m.	dm.	cm.	mm.
1.	2	3	4	5 by 5.	4.	7	6	5	4 by 3.
2.	8	9	0	7 by 8.	5.	6	0	7	8 by 8.
3.	9	7	8	9 by 9.	6.	0	8	9	7 by 9.
7.	7 fur. 6 ch. 72 l. by 8.				8.	9 fur. 7 ch. 86 l. by 9.			

EXAMPLE 1. Multiply 12·345 by (*a*) 7, (*b*) 70.

12·345
7
86·415

The process is exactly the same as in simple multiplication ; only the decimal point is placed in the product directly under the point in the multiplicand before passing on to the multiplication of the units.

```
12·345
    70
──────
864·15
```

70 = 7 × 10; we first multiply by 7 and then by 10, i.e. the decimal point is moved one place to the right. The first figure in the product is written under the multiplier that produces it.

EXAMPLE 2. Multiply 39·475 by 8.

```
39·475
     8
──────
315·800 = 315·8 (Art. 116.)
```

EXAMPLES.—ORAL.

Multiply—

1. 3·3 by 2. 2. 2·3 by 3. 3. 3·2 by 4. 4. 4·1 by 5.
5. 0·5 by 7. 6. 0·6 by 8. 7. 0·7 by 6. 8. 0·8 by 9.
9. 2·7 by 4. 10. 3·5 by 5. 11. 4·6 by 6. 12. 5·7 by 8.
13. 6·4 by 5. 14. 7·5 by 8. 15. 12·5 by 4. 16. 11·5 by 8.

17. The length of each of 4 strings is 4·3 m. What is the total length?

18. A side of a square measures 0·25 m. What is the distance all round it.

19. A metre is 39·3 in. long, find the length of 3 metres.

20. A kilogram weighs 2·2 lb. nearly. What is the approximate weight of 9 Kg.?

21. Multiply 2·06 by 5, 50, 500.

22. Multiply 3·15 by 6, 60, 100.

23. Multiply 0·13 by 9, 90, 900.

24. Multiply 0·05 by 8, 80, 800.

25. 2·185 × 5 = 10·925; read off the products by 50, 500 and 5000.

26. 25·87 × 7 = 181·09; read off the products by 70, 700 and 7000.

EXAMPLES. II.

Multiply—

1. 13·79 yards by 5, 7, 40, 800.
2. Rs. 25·75 by 6, 9, 30, 400.
3. £72·625 by 6, 8, 50, 700.
4. 20·0125 tons by 5, 8, 400, 600.
5. 230·56 by 6, 8, 70, 900.
6. 109·049 by 7, 9, 60, 800.
7. 56·0789 by 6, 9, 50, 700.
8. 99·0965 by 6, 8, 70, 900.

---

124. MULTIPLICATION BY A SINGLE FIGURE (DECIMAL)

EXAMPLE 1. Multiply 3·457 by 0·9.

```
3·457
  0·9
──────
3·1113
```

0·9 = 9 tenths; multiply by 9 and divide by 10; this step is done by moving the decimal point one place to the left.

Placing the units' figure of the multiplier directly under the last figure of the multiplicand brings the first figure in the product exactly below the figure that produces it.

Example 2. Multiply 3·457 by 0·09.

3·457
0·09
———
·31113

0·09 = 9 hundredths; we multiply by 9 and divide by 100, both being done in one step.

Example 3. 0·003 × ·07 = ·00021.

In Example 1, the No. of decimal figures in the product is 4, i.e. 3 + 1; in Examples 2 and 3 it is 5, i.e., 3 + 2 respectively.

From this we see when two decimals are multiplied, the number of decimal figures in the product is equal to the sum of the decimal figures in the multiplicand and the multiplier.

EXAMPLES.—ORAL.

Multiply—

1. 9·6 by 0·7.
2. 0·07 by 0·9.
3. 0·25 by 0·7.
4. 12·8 by 0·02.
5. 13·5 by 0·03.
6. 14·7 by 0·04.
7. 8·75 by 0·8.
8. 9·35 by 0·6.
9. 7·94 by 0·05.
10. Given 18·09 × 0·8 = 14·472, read off the products by 0·08 and 0·008.
11. Given 29·24 × ·9 = 26·316, read off the products by 9, 0·09, 0·009.
12. Given 32·05 × ·7 = 22·435, read off the products by 70, 0·07, 0·0007.

125. Multiplication by Integers (with two or more digits).

Example. Multiply (*a*) 0·06532 by 16; (*b*) 8·037 by 2468.

```
(a) 0·06532      (b) 8·037
         16            2468
     ———————         ———————
    1·04512        16074·
                    3214·8
                     482·22
                      64·296
                    ————————
                   19835·316
                    ————————
```

2468 = 2000 + 400 + 60 + 8.

Begin with the digit at the extreme left and write the first figure in the product under the figure that produces it as in ordinary multiplication: and continue thus to the end of the process. This has the effect of placing the decimal points one under the other.

EXAMPLES. III.

Multiply—

1. 0·049 by (1) 56, (2) 103, (3) 178, (4) 3007.
2. (1) 135·125 by 16. (2) 19·625 by 24.
   (3) 4·72 by 25. (4) 304·24 by 125.
3. 0·000348 by (1) 85, (2) 125, (3) 2345, (4) 2004.
4. (1) 0·0000012 by 150. (2) 15·0257 by 1700.
   (3) 30·710204 by 734000. (4) 0·003579 by 30030000.

---

126. Multiplication by Decimals.

Example 1. Multiply 5·125 by 3·85.

$$\begin{array}{r} 5{\cdot}125 \\ 3{\cdot}85 \\ \hline 15{\cdot}375 \\ 4{\cdot}1000 \\ {\cdot}25625 \\ \hline 19{\cdot}73125 \end{array}$$

Write the units' figure of the multiplier under the last figure of the multiplicand and each partial product as explained in the example in Art. 125.

A rough approximation of the product may be found as follows :— The multiplicand is a little over 5 and the multiplier is a little under 4. Therefore the product should be somewhat under 20.

127. A decimal is said to be of the **Standard Form** when it has only one integral digit. Examples. 9·74, 3·0648.

In Example 1, the multiplier is of the standard form. When the multiplier is not of the standard form, it may by suitable multiplication or division by some power of 10 be brought to that form. In that case the multiplicand also must be divided or multiplied by the same power of 10, lest the product should be affected.

Example 2. Find the square of 29·237.

Bring the multiplier 29·237 to the standard form 2·9237 by dividing it by 10. The multiplicand has now to be multiplied by 10 ; so that, we have to find the product of 292·37 by 2·9237.

$$\begin{array}{l}
292{\cdot}37 \\
\quad\;\; 2{\cdot}9237 \\
\hline
584{\cdot}74 \\
263{\cdot}133 \\
\;\;5{\cdot}8474 \\
\quad{\cdot}87711 \\
\quad{\cdot}204659 \\
\hline
854{\cdot}802169 \\
\hline
\end{array}$$

Write the partial products as in the previous example.

The product should be greater than the square of 29, since each of the factors is a little over 29; and $29^2 = 841$.

Example 3. Find the product of 278·3 by 0·000072.

$$\begin{array}{r}
0{\cdot}002783 \\
7{\cdot}2 \\
\hline
0{\cdot}019481 \\
5566 \\
\hline
0{\cdot}0200376
\end{array}$$

Here the multiplier has to be multiplied by 10^5 to bring it to the standard form 7·2. The multiplicand has to be divided by 10^5 and it becomes 0·002783.

EXAMPLES. IV.

1. Multiply (1) 25·73 by 9·25. (2) 0·012 by 2·01.
(3) 18·75 by 3·125. (4) 23·236 by 2·052.
(5) 0·207 by ·035. (6) 23·456 by ·075.
(7) 0·0235 by ·458. (8) 8·076 by 10·201.

2. Find the product of (1) 0·476 and 120·65; (2) 1·0093 and 0·826; (3) 20·7025 × 9·092; (4) 0·0000125 × 1250.

3. Find the value of (*a*) (12·305 + 8·75) × 20·1892.
(*b*) (8 − 6·5764) × ·01234.
(*c*) (87·6543 + 29·1) × 8·567.
(*d*) 1·37 × ·00057 + 1·23.

4. Multiply (*a*) 35·28 by ·000032 and hence write down the product of ·3528 by 32.
(*b*) 3254·76 separately by ·900901 and ·099099 and add the results.
(*c*) 12·5 × ·009 × 21·05.
(*d*) ·005 × ·005 × 25·05.

5. Find the value of (1) $(37{\cdot}368)^2$; (2) $(73{\cdot}875)^2$; (3) $(18{\cdot}95)^3$; (4) $(25{\cdot}67)^3$.

CHAPTER X.

METRIC TABLES AND DECIMALS.

DIVISION.

123. By Integers (Single figure).

EXAMPLE. Divide (*a*) 17 g. 6 dg. 7 cg. 2 mg. by 8.
(*b*) 25 m. 7 dm. 6 cm. 7 mm. by 9.

both as compound quantities and as decimals of gram and metre respectively.

(*a*)	8|17 g. 6 dg. 7 cg. 2 mg. 2 g. 2 dg. 0 cg. 9 mg.	8|17·672 g. 2·209 g.
(*b*)	9|25 m. 7 dm. 6 cm. 7 mm. 2 m. 8 dm. 6 cm. 3 mm.	9|25·767 m. 2·863 m.

The division in each case is exactly the same as in simple division.

EXAMPLES.—ORAL

Divide (1) as compound quantities and (2) as decimals of metres or grams :—

	m.	dm.	cm.	mm.		g.	dg.	cg.	mg.
1.	15	6	7	3 ÷ 7.	7.	23	4	5	4 ÷ 6.
2.	9	9	6	8 ÷ 8.	8.	12	9	8	5 ÷ 7.
3.	5	6	8	8 ÷ 9.	9.	4	5	7	6 ÷ 8.
4.	9	8	8	5 ÷ 5.	10.	11	7	6	3 ÷ 9.
5.	18 fur.	7 ch.	28 l.	÷ 8.	11.	10 fur.	8 ch	72 l.	÷ 9.
6.	24 ,,	6 ,,	47 l.	÷ 7.	12.	23 ,,	7 ,,	87 l.	÷ 9.

EXAMPLE 1. Divide 64·75 chains by 7.

7|64·75 ch.
9·25 ch.

EXAMPLE 2. Divide (*a*) 4·84 by 4 ; (*b*) 18·272 by 8.

(*a*)	4|4·84 1·21	(*b*)	8|18·272 2·284

In these examples, we proceed as in ordinary division.

EXAMPLE 3. Divide (*a*) 4·84 by 8 ; (*b*) 457·71 by 6.

8|4·84,0 0·605	6|457·71,0 76·285

In these two examples, a cypher is added to the right of the last decimal figure to complete the process of division ; such cyphers do not alter the value of the decimal. (Art. 117.)

EXAMPLE 4. Divide 0·01428 by 70.

0·01428 ÷ 70 = 0·001428 ÷ 7; dividing by 10 is the same as moving the decimal point one place to the left. (Art. 121.)

7|0·001428
0·000204.

EXAMPLES.—ORAL.

Divide—

1. 4·8 by 2, 4 and 8.
2. 9·6 by 2, 3, 4, 6 and 8
3. 6·3 by 3, 7 and 9.
4. 8·4 by 4 and 7.
5. 2·4 by 6 and 8.
6. 5·6 by 4 and 8.
7. 0·4 by 8.
8. 0·4 by 5.
9. 0·18 by 6 and 9.
10. 0·72 by 8.
11. 1·5 by 6.
12. 1·2 by 8.
13. 2·7 by 5.
14. 0·81 by 9.
15. 7·2 by 5.
16. 3·2 by 80.
17. 4·5 by 90.
18. 0·002 by 20.
19. ·014 by 70.

EXAMPLES. I.

1 *to* 4 *to be done orally*—

1. What is the ninth part of £13·437, 29·097 m.?
2. What is the eighth part of Rs. 81·256, 36·808 g.?
3. Divide 42·848 by 4 and 8.
4. 0·07209 by 3 and 9.
5. 5·004 by 8.
6. 1·1 by 8.
7. 1·5257 by 8.
8. 212·4 by 8.
9. 130·8 by 60.
10. 0·266 by 700.
11. 0·477 by 90.
12. 25·375 by 50, 700 and 200.

---

129. Division by Integers with two or more digits.

EXAMPLE 1. Divide (*a*) 486·794 by 29. (*b*) 0·004625 by 185.

(*a*)
16·786 = quotient.
29)486·794
196
227
249
174

(*b*)
·000025 = quotient.
185)0·004625
925

We divide as in simple division by the Italian method and write the quotient-digits above the dividend, as each digit is obtained.

EXAMPLES. II.

Divide—

1. 819·28665 by 273.
2. 796·6766 by 199.
3. 2009·1148 by 287.
4. 796·109 by 3331.

Divide—

5. 4760·697 by 4913.
6. ·00237711 by 79.
7. 34·503 by 371.
8. 19·81849 by 283.

---

130. When the divisor can be split up into factors, it is best to carry on the process of division by factors; otherwise, use long division.

Example 2. Divide (*a*) ·022015 by 350. (*b*) 8·547 by 132.

350 = 10 × 5 × 7

10 | ·022015
5 | ·0022015
7 | ·0004403
·0000629

132 = 11 × 12.

11 | 8·547
12 | 0·77700
0·06475

EXAMPLES. III.

Divide—

1. 0·44352 by 8400.
2. 0·34104 by 5600.
3. 219·641 by 919000.
4. 124·227 by 387000.
5. 4·009 by 152.
6. ·07171 by 284.
7. 0·8547 by 132.
8. 9·197 by 136000.

Divide, expressing the quotient in each case as a decimal—

9. 713 by 32.
10. 25 by 160.
11. 100653 by 1680.
12. 2345698 by 6250.

Division by Decimals.

131. If the **divisor** be a **decimal,** we can in every case change it into an integer by multiplying it by 10, 100, 1000... according as the number of decimal places in the divisor is one, two, three... ; and this is the same as moving the decimal point, one, two, three...places to the right. (Art. 120.)

For instance, if the divisor be ·004, we change it into the integer 4 by multiplying it by 1000.

We may multiply a divisor by any number, if we multiply the dividend also by the same number.

For example if the divisor be 5 and the dividend 25, we may multiply each by 50; then the divisor becomes 250 and the dividend 1250; and whether we divide 25 by 5, or 1250 by 250, the quotient is the same number 5.

EXAMPLE 1. Divide 0·00184 by (*a*) 0·8, (*b*) 0·08

Now, $0{\cdot}00184 \div 0{\cdot}8 = \dfrac{{\cdot}00184}{{\cdot}8} = \dfrac{{\cdot}00184 \times 10}{{\cdot}8 \times 10} = \dfrac{{\cdot}0184}{8} = {\cdot}0023.$

Here we have multiplied both the dividend and the divisor by 10 and thereby made the divisor a whole number.

In (*b*) to make the divisor an integer, it has to be multiplied by 100; we therefore multiply the dividend also by 100, and get $\dfrac{{\cdot}184}{8} = {\cdot}023.$

EXAMPLES.—ORAL.

Divide—

1. 17·5 by ·5, ·7, ·05, ·07.
2. 20·4 by ·4, ·6, ·04, ·06.
3. 0·016 by ·8, ·5.
4. 0·36 by ·6, ·9, ·5, ·8.
5. 9·63 by ·09, ·6.
6. 10·26 by ·9, ·03, ·5, ·06.
7. How many £0·8 in £5·6?
8. How many 0·4 m. in 2·8 m.?
9. How many 0·6 g. in 7·2 g.?
10. How many three-tenths in 3·6?
11. How many nine-tenths in 6·3?
12. How many seven-tenths in 2·1? 4·9?

---

132. When the divisor contains more decimal figures than one, the same method of converting it into an integer may be adopted, and the division done as in the case of division by integers.

It is not however necessary that the divisor should in every case be converted into an integer. It is sufficient if by multiplying or dividing the divisor and the dividend by a suitable power of ten, the divisor is brought to the STANDARD FORM.

For example, in dividing 37·128 by 0·476, to bring the divisor to the standard form it should be multiplied by 10; and then the dividend also has to be multiplied by 10. In other words, the decimal point has to be moved one place to the right both in the divisor and in the dividend.

Thus, $\dfrac{37{\cdot}128}{0{\cdot}476} = \dfrac{371{\cdot}28}{4{\cdot}76}.$

Or if 37·128 is to be divided by 47·6, the decimal point has to be moved one place to the left so as to bring the divisor to the STANDARD FORM.

Thus, $\dfrac{37{\cdot}128}{47{\cdot}6} = \dfrac{3{\cdot}7128}{4{\cdot}76}$; here both the divisor and the dividend have been divided by 10

EXAMPLE. Divide 3496·818 by 38·1.

To bring the divisor to the standard form, move the point one place to the left; also move the point one place to the left in the dividend. We have now to divide 349·6818 by 3·81.

The new divisor is a little below 4, and the new dividend is below 350. Therefore there should be two integral figures in the quotient.

91·78 = quotient.
3·81)349·6818
3429
678
381
2971
2667
3048
3048

Working by the Italian method,
91·78
3·81)349·6818
678
2971
3048

Write the quotient over the dividend, as in ordinary division, placing the decimal point of the quotient right over the decimal point of the dividend; and then divide as if both were whole numbers.

EXAMPLES. IV.

Divide—

1. 349·377 by 3·81.
2. 34447·2101 by 7·01.
3. 332·75 by 27·5.
4. 431·376 by 81·7.
5. 84·375 by 0·375.
6. 183·688305 by 0·2315.
7. 3·5295624 by 0·0504.
8. 5·5757592 by 0·09207.

EXAMPLE. Divide ·0000036792 by 0·00219.

$$\frac{0{\cdot}0000036792}{0{\cdot}00219} = \frac{0{\cdot}0036792}{2{\cdot}19}$$

Using Italian method,
0·00168
2·19)0·0036792
1489
1752

2 goes into 3, the third decimal figure once. Write 2 cyphers in the quotient before 1, placing it over 3.

EXAMPLES. V.

Divide—

1. 10·836 by 51·6.
2. 0·429408 by 59·64.
3. 0·00036792 by ·00219.
4. 4663·4205 by 4807·65.

5. 185·688305 by ·2315. 6. 196·10652875 by 238·645.
7. 0·0000837 by 0·027. 8. 13·681197 by 0·00240021.

Example. Divide 590·4825 by 0·3275.

$$\frac{590\cdot4825}{0\cdot3275}=\frac{5904\cdot825}{3\cdot275}.$$

Using Italian method,

$$\begin{array}{r} 1803 \\ \hline 3\cdot725)5904\cdot825 \\ 26298 \\ 9825 \end{array}$$

When 5904 is divided by 3, the quotient must contain 4 integral figures.

The quotient is an integer.

EXAMPLES. VI.

Divide—

1. 2819·041 by 1·679. 2. 68·41197 by 1·20021.
3. 3738·028 by ·476. 4. 6·4096 by ·2003.
5. 221·9904 by ·3854. 6. 557·57592 by ·09207.
7. 2939·16669 by 5·41283. 8. 67254·023544 by ·007089.

Example. Divide 0·4625 by 0·000185.

$$\frac{0\cdot4625}{0\cdot000185}=\frac{4625}{1\cdot85}$$

$$\begin{array}{l} 2500. \\ 1\cdot85)\overline{4625.} \\ \underline{925}\ \text{Quotient} = 2500. \end{array}$$

Since the divisor is nearly 2, the quotient must contain 4 integral figures.

As two more figures have to be entered to the left of the decimal point, we write 2 cyphers.

EXAMPLES. VII.

Divide—

1. 3365·67 by 1·023. 2. 20349·48 by ·276.
3. 843·75 by 0·0375. 4. 556·591 by ·01813.
5. 1368·1197 by ·00240021. 6. 352·95624 by ·000504.
7. 1157·13 by 0·0387. 8. 491·7208 by 0·00872.

Example. Divide 0·065341 by 0·000475.

$$\frac{0\cdot065341}{0\cdot000475}=\frac{653\cdot41}{4\cdot75}$$

Here two cyphers have to be added to the dividend to complete the quotient.

$$\begin{array}{r} 137\cdot56 = \text{Quotient.} \\ 4\cdot75)\overline{653\cdot41,00}\phantom{\text{Quotient.}} \\ 1784\phantom{\text{Quotient.xx}} \\ 3591\phantom{\text{Quotient.x}} \\ 2660\phantom{\text{Quotient.}} \\ \underline{2850}\phantom{\text{Quotient}} \end{array}$$

EXAMPLES. VIII.

Divide—

1. 17·7089 by 4·735.
2. ·75445 by ·00625.
3. 2·45 by ·0784.
4. ·842 by ·0125.
5. 75·62 by ·064.
6. ·02 by ·0125.
7. 0·3217 by ·625.
8. ·1255 by ·1004.

---

133. Division to a certain number of decimal places.

EXAMPLE. Divide 3·39 by ·007 to three places of decimals.

Since $3{\cdot}39 \div {\cdot}007 = \frac{3{\cdot}39}{{\cdot}007} = \frac{3390}{}$, proceed thus:

$$\begin{array}{r|l} 7 & 3390{\cdot}000 \\ \cline{2-2} & 484{\cdot}285 \end{array}$$

EXAMPLES. IX.

Divide to four places in the quotient—

1. 673·42 by 525.
2. ·10357 by 751.
3. 876·543 by 1234.
4. ·07 by 1872.
5. 34·17 by 3·25.
6. 17·29 by 1·42.
7. 71·9 by 27·53.
8. 594·27 by ·047.

---

134. Simplification of Complex Decimals.

If a and b stand for any two numbers we have seen that—

$(a+b)^2 = a^2 + 2ab + b^2$ (Art. 44) $\therefore \frac{(a+b)^2}{a+b} = \frac{a^2+2ab+b^2}{a+b} = a+b.$

Similarly $(a-b)^2 = a^2 - 2ab + b^2 \therefore \frac{(a-b)^2}{a-b} = \frac{a^2-2ab+b^2}{a-b} = a-b.$

$a^2 - b^2 = (a+b)(a-b)$. Art. 48 $\therefore \frac{a^2-b^2}{a+b} = a-b;\ \frac{a^2-b^2}{a-b} = a+b.$

These formulæ may, with advantage, be applied in simplifying complex decimal expressions.

EXAMPLE 1. Simplify $\frac{3{\cdot}2 \times 3{\cdot}2 + 2 \times 3{\cdot}2 \times 1{\cdot}09 + 1{\cdot}09 \times 1{\cdot}09}{3{\cdot}2 + 1{\cdot}09}$.

$$\text{The expression} = \frac{(3{\cdot}2)^2 + 2 \times 3{\cdot}2 \times 1{\cdot}09 + (1{\cdot}09)^2}{3{\cdot}2 + 1{\cdot}09}$$

$$= \frac{a^2 + 2ab + b^2}{a+b}, \text{ where } a = 3{\cdot}2, \text{ and } b = 1{\cdot}09.$$

Since $\frac{a^2 + 2ab + b^2}{a+b} = a + b$ the expression $= 3{\cdot}2 + 1{\cdot}09 = 4{\cdot}29.$

If done by the ordinary method, this would involve three multiplications, two additions, and one division.

Similarly $\dfrac{3{\cdot}2 \times 3{\cdot}2 - 2 \times 3{\cdot}2 \times 1{\cdot}09 + 1{\cdot}09 \times 1{\cdot}09}{3{\cdot}2 - 1{\cdot}09} = 3{\cdot}2 - 1{\cdot}09 = 1{\cdot}11.$

EXAMPLE 2. Simplify $\dfrac{7{\cdot}564 \times 7{\cdot}564 - 3{\cdot}876 \times 3{\cdot}876}{7{\cdot}564 + 3{\cdot}876}$.

The expression $= \dfrac{(7{\cdot}564)^2 - (3{\cdot}876)^2}{7{\cdot}564 + 3{\cdot}876}$

$= \dfrac{a^2 - b^2}{a + b}$, where $a = 7{\cdot}564$, and $b = 3{\cdot}876$.

Since $\dfrac{a^2 - b^2}{a + b} = a - b$, the expression $= 7{\cdot}564 - 3{\cdot}876 = 3{\cdot}688$.

Similarly $\dfrac{7{\cdot}564 \times 7{\cdot}564 - 3{\cdot}876 \times 3{\cdot}876}{7{\cdot}5641 - 3{\cdot}876} = 7{\cdot}564 + 3{\cdot}876 = 11{\cdot}44.$

The other formulæ that may be found useful are—

$\dfrac{a^3 + b^3}{a^3 + b} = a^2 - ab + b^2$; or $\dfrac{a^3 + b^3}{a^2 - ab + b^2} = a + b$; and

$\dfrac{a^3 - b^3}{a - b} = a^2 + ab + b^2$; or $\dfrac{a^3 - b^3}{a^2 + ab + b^2} = a - b.$

Where none of the formulæ could be applied, the ordinary method of simplification should be used.

EXAMPLES. X.

1 *to* 12 *to be taken orally.*

Simplify—

1. $\dfrac{5{\cdot}5 + 2{\cdot}7}{5{\cdot}5 - 1{\cdot}4}$. 2. $\dfrac{10{\cdot}6 - 5{\cdot}8}{0{\cdot}8 + 0{\cdot}4}$.

3. $\dfrac{1{\cdot}6 \times 1{\cdot}6 - {\cdot}7 \times {\cdot}7}{1{\cdot}6 - {\cdot}7}$. 4. $\dfrac{1{\cdot}75 \times 1{\cdot}75 - {\cdot}87 \times {\cdot}87}{1{\cdot}75 - {\cdot}87}$.

5. $\dfrac{{\cdot}25 \times {\cdot}25 - {\cdot}13 \times {\cdot}13}{{\cdot}25 + {\cdot}13}$. 6. $\dfrac{9{\cdot}09 \times 9{\cdot}09 - 6{\cdot}789 \times 6{\cdot}789}{9{\cdot}09 + 6{\cdot}789}$.

7. $\dfrac{5{\cdot}62 \times 5{\cdot}62 + 2 \times 5{\cdot}62 \times 3{\cdot}75 + 3{\cdot}75 \times 3{\cdot}75}{5{\cdot}62 + 3{\cdot}75}$.

8. $\dfrac{0{\cdot}345 \times 0{\cdot}345 + 2 \times 0{\cdot}345 \times 1{\cdot}05 + 1{\cdot}05 \times 1{\cdot}05}{0{\cdot}345 \times 1{\cdot}05}$,

9. $\dfrac{6{\cdot}23 \times 6{\cdot}23 - 2 \times 6{\cdot}23 \times 4{\cdot}87 + 4{\cdot}87 \times 4{\cdot}87}{6{\cdot}23 - 4{\cdot}87}$.

10. $\dfrac{25{\cdot}4 \times 25{\cdot}4 - 2 \times 25{\cdot}4 \times 10{\cdot}1 + 10{\cdot}1 \times 10{\cdot}1}{25{\cdot}4 - 10{\cdot}1}$.

11. $\dfrac{{\cdot}2 \times {\cdot}2 \times {\cdot}2 + {\cdot}3 \times {\cdot}3 \times {\cdot}3}{{\cdot}2 \times {\cdot}2 - {\cdot}2 \times {\cdot}3 + {\cdot}3 \times {\cdot}3}$.

12. $$\frac{1\cdot4 \times 1\cdot4 \times 1\cdot4 - \cdot43 \times \cdot43 \times \cdot43}{1\cdot4 \times 1\cdot4 + 1\cdot4 \times \cdot43 + \cdot43 \times \cdot43}.$$

13. $$\frac{\cdot25 \times \cdot25 \times \cdot25 + \cdot2 \times \cdot2 \times \cdot2}{\cdot25 + \cdot2}.$$

14. $$\frac{\cdot6 \times \cdot6 \times \cdot6 - \cdot12 \times \cdot12 \times \cdot12}{\cdot6 - \cdot12}$$

15. $$\frac{1\cdot5 \times 1\cdot5 - 6\cdot5 \times 1\cdot5 + 3\cdot25 \times 3\cdot25}{3\cdot25 - 1\cdot5}.$$

16. $$\frac{(\cdot01+\cdot02+\cdot03)(\cdot01\times\cdot02+\cdot03\times\cdot02+\cdot01\times\cdot03)-\cdot01\times\cdot02\times\cdot03}{(\cdot01+\cdot02)(\cdot01+\cdot03)}.$$

CHAPTER XI.

DECIMALS OF CONCRETE QUANTITIES.

135. Reduction of concrete decimals from a higher to a lower denomination is done by successive multiplication.

EXAMPLE 1. Reduce £1·524 to pence.

£1·524	·76*d*.
20	4
30·48*s*.	3·04*f*.
12	Ans. 365¾*d*.
365·76*d*.	

EXAMPLES.—ORAL.

Find the value of—

1. 0·5 R.
2. 0·4 £.
3. 0·5 anna.
4. 0·6 £.
5. 0·25 R.
6. 0·05 £.
7. 0·75 R.
8. 0·15 £.
9. 0·75*s*.
10. 0·25 anna.
11. 0·125 anna.
12. 0·375*s*.

EXAMPLES. I.

Reduce—

1. £4·12345 to pence.
2. Rs. 6·87525 to pies.
3. 3·3456 mds. to palams.
4. £5·4325 to farthings.
5. 2·6083675 tons to lbs.
6. 1·526 miles to yds.
7. 3·56785 days to seconds.
8. 4·125 parahs to ollocks.

EXAMPLE 2. Find the value of Rs. 5·52875.

Rs. 5·52875
16
As. 8·46000
12
P. 5·52

Rs. 5 is not reduced to annas, and 8 as. is not reduced to pies. Cyphers at the end of a decimal are of no value.

Ans. Rs. 5-8 as. 5½ p.

EXAMPLES. II.

Find the value of—

1. 0·375 of a rupee.
2. 0·875 of a £.
3. Rs. 2·3125.
4. £3·1375.
5. £14·1875.
6. 3·825 tons.
7. 6·1425 days.
8. 8·375 garce.

EXAMPLE 3. Find the value of 2·346 of 5 tons.

2·346
5
tons 11·730
20
cwt. 14·60
4
qrs. 2·4

qrs. 2·4
28
lbs. 11·2
16
oz. 3·2

Ans. 11 tons 14 cwt. 2 qrs. 11 lbs 3·2 oz.

EXAMPLES. III.

1. 4·7315 of 4 cwt.
2. 1·5625 of 7 seers.
3. 5·5435 of 3 mds.
4. 1·31875 of Rs. 100
5. 2·2365 of 100 miles.
6. 2·890625 of Rs. 6.
7. 7·4453125 of Rs. 8.
8. 2·3515625 of 3 mds.

---

136. Reduction from a lower to a higher denomination is done by successive division.

EXAMPLE 4. Reduce 49490 lbs. to tons.

28 { 7 | 49490 lbs.
   { 4 | 7070
4 | 1767·5 qrs.
20 | 441·875 cwt.
22·09375 tons.

EXAMPLES. IV.

Reduce—

1. 12345 pies to Rs.
2. 396000 inches to miles.
3. 810 sec. to days.
4. 21 lbs. to tons.
5. 34563 pies to Rs.
6. 234560 palams to candies.
7. 990990 inches to miles.
8. 1095·75*d.* to £.

Example 5. Express 1 day 3 hrs. 9 min. 27 sec. as a decimal of its highest denomination.

60 | 27 sec.
60 | 9·45 min.
24 { 3 | 3·1575 hrs.
  8 | 1·0525
1·1315625 days.

27 sec. is divided by 60 and 9 mi added and so on.

EXAMPLES. V.

Express each as a decimal of its highest denomination—

1. Rs. 5-11-6. 2. £10-13-9. 3. £4-15-7½. 4. £1-12-2¾.
5. Rs 3-7-6. 6. 6 tons 1 cwt. 3 qrs. 14 lbs.
7. 10 mi. 2 fur. 143 yds. 8. 5 mi. 1 fur. 126 yds. 1 foot 6 in.

Example 6. Find the value of 3·0085 of £4-1-0.

3·0085 of £4-1-0 = 3·0085 × £4·05 = (3·0085 × 4·05) £.

3·0085
4·05
12·0340
·150425
£12·184425
20
s. 3·688500
12
d 8·2620
4
f. 1·048

Since £4-1*s.* = 81*s.*
3·0085
81
240·680
3·0085
s. 243·6885
12
d. 8·2620
4
f. 1·048

Ans. £12-3-8¼ nearly.

EXAMPLES. VI.

Find the value of—

1. 0·46875 of £2-10*s.* 2. 1·875 of Rs. 1-9-4.
3. 14·1875 of Rs. 29-2-8. 4. 12·75 of Rs. 32-3-4.
5. 3·953125 of £8-13-4. 6. 3·4125 of Rs. 48-5-4.
7. 5·205 of 4 cwt. 3 qrs. 21 lbs.
8. Which is greater, 0·39 of Rs. 21 or 0·41 of Rs. 20? By how much?

EXAMPLE 7. Find the value of—

0·2375 of £3-15*s.* + ·8375 of 16*s.* 8*d.* — 15·85 of 4*d.*

$$\begin{array}{r} \cdot 2375 \\ £3\text{-}15s. = 900d. \\ \hline 213{\cdot}75d. \end{array} \qquad \begin{array}{r} \cdot 8375 \\ 16s.\ 8d. = \quad 200d. \\ \hline 167{\cdot}5d. \end{array} \qquad \begin{array}{r} 15{\cdot}85 \\ 4d. \\ \hline 63{\cdot}4d. \end{array}$$

$\therefore$ Value = 213·75*d.* + 167·5*d.* — 63·4*d.*

= 381·25*d.* — 63·4*d.* = 317·85*d.* = £1 6*s.* $5\frac{3}{4}$*d.* nearly.

EXAMPLES. VII.

Find the value of—

1. 1·35 of £1-4-7 — £ 2·4375 + 3·4 of 4*s.* 7*d.*
2. 14·1875 of £2-18-4 + ·375 of a guinea — ·775 of 5*s.*
3. 0·68125 of Re. 1-4 as. + ·375 of 13 as. 4 p. + ·605 of Rs. 3-14-6.
4. 0·45 of £3-10*s.* + ·75 of 4*s.* 8*d.* + 3·245 of 3*s.* 4*d.*
5. 7·934375 of 6*s.* 8*d.* + 3·825 of 15*s.* 8*d.*
6. Rs. 47·5 + ·025 of Rs. 12-8 as. + Rs. 6·34375.
7. 4·05 of Rs. 2-8 as. — 1·84375 of Re. 1-12 as.
8. 3·825 of 15 as. 8 p. — ·01953125 of Rs. 4 — 7·934375 of Rs. 3-5-4 + 3·953125 of Rs. 8-10-8.

CHAPTER XII.

APPROXIMATION.

137. Degree of Approximation.

When I say my height is 5 ft. 6 in., I mean that is the height to the nearest inch. I may be a tenth of an inch taller or shorter; but that is neglected; and for ordinary purposes, the height is expressed with sufficient accuracy if it is given correct to the nearest inch.

The distance between two stations near one another is expressed with sufficient accuracy, if it is given in miles and furlongs, omitting yards, feet and inches; but the distance of Madras from Tinnevelly is considered accurate if given in miles as nearly as possible, omitting also the furlongs, say 450 miles.

When one wants to know the time, it is generally given in hours and minutes and he does not care for the seconds; but in calculating the time taken for a hundred yards' race the seconds and tenths of a second will have to be carefully noted.

In all these cases, only a reasonable degree of approximation is aimed at.

138. In statistical tables, the population of towns, for example, is given to the nearest thousand, i.e. omitting the hundreds, tens and units' figures. A population of 48,456 is given as 48,000 and one of 48,750 is recorded as 49,000.

When 48,456 is given as approximately equal to 48,000 the error is 456 or is less than half of a thousand; but if 48,456 be given as an approximate equivalent of 49,000 the error would be 544 or more than half of a thousand. Therefore 48,000 is nearer to 48,456 than to 49,000. Hence 48,000 is the equivalent of 48,456 correct to the nearest thousand.

In the same way, if 48,750 is made approximately equal to 49,000, the error is 250 or one-fourth of a thousand; but if made equal to 48,000, the error would be 750 or three times as great. Hence 49,000 is nearer to 48,750 than to 49,000; and therefore 49,000 is the equivalent of 48,750 correct to the nearest thousand.

139. It is usual therefore to increase the last figure retained by 1, if the first of the omitted figures be 9, 8, 7, 6, or 5, and to keep the last retained figure unaltered, if the first figure omitted be 4, 3, 2, 1, or 0.

For 57683654, the nearest approximation to the ten is 57683650; to the hundred, 57683700; to the thousand, 57684000; to the ten thousand, 57680000 and so on.

EXAMPLES.—ORAL.

What would be a reasonable approximation in estimating—

(1) The distance of the moon from the earth; and the distance of a star from the earth?

(2) The length and width of your school room, of a room in your house, of a page in your Arithmetic book?

(3) Your height, the height of a horse, the highest peak of the Western Ghats, the highest peak of the Himalayas?

(4) Your weight, and the weight of a parcel for post?

(5) The cost of a pound of tea, and of a ton of coal or firewood?

(6) The population of a town, of a city and of the Presidency?

Read off the following numbers correct to (1) the nearest ten, (2) the nearest hundred, (3) the nearest thousand :—

1. 56456. 2. 876543. 3. 1314378. 4. 878978. 5. 687564.
6. 256789. 7. 765432. 8. 1028645. 9. 2345671. 10. 4563645.

140. Standards of measurement and weight are incapable of infinite sub-division; and it is unnecessary to carry on calculations beyond a certain point.

Suppose you are required to mark off in your foot-rule graduated to the tenths of an inch, a length equal to 3·638 inches.

You will first take from the zero division a length equal to 3 inches; then along the next inch take 6 more divisions and thus get 3·6 inches. To get the 3 hundredths, you should sub-divide the next tenth of an inch into 10 equal parts, and of these take 3. You have now a length equal to 3·63 inches; and to get the remaining 8 thousandths of an inch, each hundredth part should be sub-divided into 10 equal parts and 8 of these should be taken. But this sub-division cannot possibly be done with ordinary instruments; and for all practical purposes, the hundredth of an inch may be considered as incapable of further sub-division.

So the best thing to be done under the circumstance is to count 8 thousandths of an inch roughly as 1 hundredth of an inch, and to estimate a length of 3·638 in. as approximately equal to 3·64 in.

In working out an example in metres and decimals of a metre there is no necessity at all to carry on the work so as to have in the final answer 7 or 8 decimal places. At the most the answer might contain 4 decimal figures; for this would give us the tenths of a millimetre; but beyond this, no figures are required for practical purposes.

In practise, no measurements can be made to a greater degree of accuracy than 4 decimal places or a ten thousandth part of the whole.

A length of 54·62 mm. may be taken as approximately equal to 54·6 mm.

And if the weight of a body be given as 5·5456 grams, this may be taken approximately equal to 5·546 grams.

141. If a rupee is divided equally among 5 persons, each person gets 3 annas and two pies; and the remaining two pies cannot be further divided among them; because the pie is incapable of further sub-division. The result is only approximately correct to the nearest pie, parts of a pie being omitted.

So also if I make purchases amounting to Rs. 8-15-9 and a fraction of a pie, I pay only Rs. 8-15-9 and the fraction of a pie is not paid ; that is I make payment to the nearest pie.

For practical purposes, a certain sum in Indian money is expressed with sufficient accuracy if it is correct to the nearest pie ; and, in English money if it is correct to the nearest fraction of a penny ; for in no case can a fraction of a farthing be given or received.

142. Errors of measurement. No measurements made can be quite exact.

When we say a lath is one yard long, all that we mean is that the two ends of the lath, as far as the eye can see, agree with the two ends of the yard. If we took a microscope and examined the two ends on either side, we would see that they did not exactly agree at either end.

As heat and cold affect the length of the metallic yard, the yard itself would, on examination, be found not exactly equal in length to the standard yard in London.

Measurement by scales should be made always from the zero division towards the right and not from the right to the left towards zero.

143. We considered 3·638 in. as approximately equal to 3·64 in. (Art. 140).

This involves an error of 2 thousandths ; but if the third decimal figure were omitted, and the decimal written as 3·63, it would have involved an error of 8 thousandths. Thus 3·64 is the nearest equivalent of 3·638 that can be given with two decimal figures ; and it gives the value correct to the nearest hundredth or correct to the second decimal place.

Again 3·6 is the nearest equivalent of 3·64 correct to the nearest tenth or correct to the first decimal place.

This involves an error of 4 hundredths ; but writing it as 3·7 would involve a greater error, namely, of 6 hundredths and therefore 3·6 is the more accurate approximation.

Also 1·2653 correct to the 3rd decimal place is 1·265 ;
but 1·2656 ,, ,, ,, 1·266.

The value of 8·417 Kg. correct to two decimal places is 8·42 Kg. ;
and 6·3625 Km. ,, ,, ,, is 6·36 Km.

The rule about increasing the last figure retained by 1, and not altering the last retained figure is applied also to decimals. (See Art. 139.)

144. Limits of Error.

When we say that a given length say 3·6 in. is correct to the nearest tenth, we mean that its real value lies between two limits, viz. 3·65 in.

and 3·55 in., that is, it is less than 3·65 in. and not less than 3·55 in., for 3·65 would be written 3·7 in.

3·65 is greater than or is in excess of 3·6 by 0·05 ; and

3·55 is less than or is in defect of 3·6 by 0·05.

The possible error in the value of the number is 0·05 in excess or defect ; and this may be expressed thus :

3·6 correct to the nearest tenth $= 3·6 \pm ·05$ (read plus or minus ·05).

∴ The limits of error when 3·6 is said to be correct to the nearest tenth are ± ·05.

Again by saying that ·64 is given correct to two decimal places is meant that it is less than ·645 and not less than ·635.

The possible error in this case is ·005 in excess or defect.

∴ The limits of error are ± ·005.

The real value of ·638 correct to 3 places lies between the limits ·6385 and ·6375 ; and the possible error is ·0005 in excess or defect. Hence the limits of error are ± ·0005.

145. Significant Figures. What are significant figures? When are cyphers to be considered significant and when non-significant?

Suppose the distance between Tinnevelly and Madras is given approximately to be 450 miles to the nearest *ten*.

This means that the unit taken is 10 miles and the distance is nearly 45 units. In this case, the cypher has no real meaning and is therefore said to be non-significant ; but the figures 45 which give the number of units are called significant.

But if 450 miles be the distance correct to the nearest mile the unit adopted is a mile and 450 miles represent 450 units. The cypher here has a real meaning and is therefore significant.

Again 1 gram = 0·035 oz. av. to the nearest thousandth = 35 thousandths of an ounce to the nearest thousandth.

The significant figures are 35 ; and the cyphers to the left and right of the decimal point are non-significant ; the cypher to the right of the decimal point serves to fix the place of the significant figures that follow it. Thus cyphers at the beginning of a decimal are non-significant. When the weight of 1 kilogram is given approximately equal to 35·3 oz. the unit taken is the *tenth* of an ounce, and the weight is given correct to the nearest tenth of an ounce, but when the weight of 1 kilogram is given approximately equal to 35·30 oz., the unit taken is the *hundredth* of an ounce ; the weight is given correct to the nearest hundredth and is equivalent to 3,530 hundredths of an ounce. Therefore when 1 Kg. = 35·30 oz. to the nearest hundredth, the cypher is significant.

From these examples we learn (1) that in any result integral or decimal given as approximately correct to a certain unit, the figures which express the number of such units are called significant figures; and (2) that cyphers at the end of integers or decimals may or may not be significant according to the unit taken, that is according to the degree of accuracy aimed at.

EXAMPLE. Express the following quantities correct to three significant figures:—56789, 4·345, 78·764, 0·00602.

Ans. 56800, 4·35, 78·8, 0·00602.

EXAMPLES.—ORAL.

Name the significant figures in the following :—

1. Distance of the sun from the earth 93,000,000 miles to the nearest million miles.

2. Velocity of light per second 188,000 miles to the nearest 1,000 miles.

3. Circumference of the earth 24,900 miles to the nearest 100 miles.

4. Distance between Tinnevelly and Madura 100 miles to the nearest 10 miles.

5. 1 inch = 25·4 mm. to the nearest tenth of a mm.
6. 1 Kg. = 35·300 oz. to the nearest thousandth. [1 gram.
7. The weight of a given quantity of water = 0·025 Kg. correct to
8. The length of a string = 0·450 metre correct to 1 mm.

Read off the following correct to three significant figures :—

1. 45678. 2. 50604. 3. 654789. 4. 564320.
5. 8·7654. 6. 19·75. 7. 872·983. 8. 905·345.
9. 9·6789. 10. 0·0456. 11. 0·00735. 12. 0·00856.

The following are correct to 4 significant figures; between what limits do the quantities lie?

1. Rs. 5065. 2. £5·065. 3. 50·65 m. 4. 0·5065 m.
5. 5·678 Kg 6. 56·78 Hg. 7. 567·8 Dg. 8. 5678 g

EXAMPLES. I.

Express each of the following quantities correct to three significant figures; and find its value to the nearest pie, penny or inch as the case may be.

1. (1) Rs. 6·578. (2) Rs. 65·78. (3) Rs. 0·6578. (4) Rs. 657·8.
2. (1) £87·67. (2) £8·767. (3) £876·7. (4) £0·8767.
3. (1) 864·321 mi. (2) 8·64321 mi.
(3) 0·864321 mi. (4) 0·00864321 mi.

146. Contracted Addition.

EXAMPLE 1. Find correct to 3 decimal places (to the nearest mm.) the sum of the following lengths :—

3·428574 Km., 9·123456 Km., 23·8763432 Km., and 54·3210123 Km.

3·4286 —
9·1235 —
23·8763 +
54·3210 +
90·7494
Ans. 90·749 Km.
= 90 Km. 749 mm.

One more place is kept in each addend to find what has to be carried to the last place finally added if the fourth figure is corrected. When there are several lines to be added it may be necessary to keep 2 extra figures in each. The sign + is affixed when the real value is below and the sign — when above the value taken.

EXAMPLE 2. The 4 sides of an irregular figure measured correct to the nearest hundredth are respectively—

9·46 cm., 9·36 cm., 9·38 cm., and 9·82 cm. Find the perimeter and determine the limits of error.

	Greatest possible length	Least possible length	Limits of error
9·46	9·465	9·455	± ·005
9·36	9·365	9·355	± ·005
9·38	9·385	9·375	± ·005
9·82	9·825	9·815	± ·005
38·02 cm.	38·040 cm.	38·000 cm.	± ·020 cm

The perimeter correct to two places = 38·02 ± ·02 cm., i.e. with a possible error (in excess or defect) of ± ·02 cm., i.e. of 2 mm.

EXAMPLES. II.

Find the sum of the following :—

1. Rs. 58,754, Rs. 2,37,543, Rs. 99,999, Rs. 3,75,632 to the nearest hundred. [thousand.

2. £19,876, £25,650, £789,102, £678,543, to the nearest

3. Find the greatest and least possible total of the following weights:—13·054 g., 29·756 g., 15·640 g., 92·903 g., correct to the nearest mg.

4. Four pieces of string measure respectively to the nearest cm. as follows:—5·27 m., 9·06 m., 12·50 m. and 7·24 m. Between what limits must the total length lie?

147. Contracted Subtraction.

EXAMPLE 1. Subtract (1) 8·50314 from 21·86125 correct to 3 places. (2) 13·75678 from 20·53865 correct to 4 places.

(1) 21·861 +	(2) 20·5387 −	(1) 21·86125	(2) 20·53865
8·503 +	13·7568 −	8·50314	13·75678
13·358	6·7819	13·35811	6·78187

If the decimals when corrected are both in excess or both in defect, it is unnecessary to retain more figures than are required in the answer.

EXAMPLE 2. Subtract 18·689553 from 26·6644321 correct to 3 places.

Written in full	To 4 places	To 3 places
26·6644321	26·6644 +	26·664 +
18·6895530	18·6896 −	18·690 −
7·9748791	7·9748	7·974

The Ans. 7·975 correct to 3 places is nearer to 7·9748 where one more figure than is required is retained than to 7·974 where only three figures are kept.

When one of the decimals is taken in excess and the other in defect, it is necessary to retain at least one more figure than is required in the answer.

EXAMPLES. III.

Find the difference correct to 3 places between—

1. 28·7891 m. and 12·98007 m.

2. 150·6780 g. and 78·9876 g.

3. From a weight of 500·345 g., a part equal to 29·782 g. is taken away. If the weighing be taken as correct to the nearest milligram, find the greatest and the least possible length of the remainder.

4. From a rope 7·56 m. long, a part 3·68 m. is cut off; between what limits must the remainder lie, given that the measurements are correct to the nearest cm.?

148. Contracted Multiplication.

EXAMPLE 1. Multiply 16·7895 by 3·47 correct to two places.

Here the multiplying decimal is in standard form. To secure accuracy, two more places than are required are retained in the multiplicand.

Full work.

$$\begin{array}{r|l} \multicolumn{2}{l}{16{\cdot}7895} \\ & 3{\cdot}47 \\ \hline 50{\cdot}36 & 85 \\ 6{\cdot}71 & 580 \\ 1{\cdot}17 & 5265 \\ \hline 58{\cdot}25 & 9565 \end{array}$$

Contracted work

$$\begin{array}{r|l} \multicolumn{2}{l}{16{\cdot}7\cancel{8}\cancel{9}\cancel{5}} \\ \multicolumn{2}{r}{3{\cdot}47} \\ \hline 50{\cdot}36 & 9 \\ 6{\cdot}71 & 6 \\ 1{\cdot}17 & 5 \\ \hline 58{\cdot}26 & 0 \end{array}$$

Ans. 58·26.

Note.—In the Multiplicand two more places than are required are retained.

In the Multiplier, it does not matter how many figures are retained; the process is continued till it will go no further.

Explanation.

5 × 3 = 15; carry 2, since 15 is nearer 20 than 10; score out 5.
Multiply 16·789 by 3, and add 2.
9 × 4 = 36, carry 4, since 36 is nearer 40 than 30; score out 9.
Multiply 16·78 by 4, i.e. by ·4, and add 4.
8 × 7 = 56; carry 6, since 56 is nearer 60 than 50; score out 8.
Multiply 16·7 by 7, i.e. by ·07, and add 6.

Write the first figure of each partial product one under the other. The figures to the right of the vertical line when added give the figure to be carried in arriving at the final result.

Rough Estimate of the product.—The product must be less than 17 × 3·5 or 59·5.

EXAMPLE 2. Multiply 456·7234 by 24·6812 to the units' figure.

Bring the multiplier to the standard form by dividing it by 10, thus 2·46812. Multiply the multiplicand by 10, so that the product may not be effected; thus, 4567·234.

$$\begin{array}{r|l} 45\cancel{6}\cancel{7}{\cdot} & \cancel{2}\cancel{3} \\ & \;2{\cdot}468 \\ \hline 9134{\cdot} & 5 \\ 1826{\cdot} & 9 \\ 274{\cdot} & 0 \\ 36{\cdot} & 5 \\ \hline 11271{\cdot} & 9 \end{array}$$

2 × 3 = 6, carry 1; score out 3; and then multiply 4567·2 by 2.

4 × 2 = 8, carry 1; score out 2; and then multiply 4567 by 0·4.

6 × 7 = 42, carry 4; score out 7; and multiply 456, i.e. 4560 by ·06.

8 × 6 = 48, carry 5; score out 6; and multiply 45, i.e. 4500 by ·008.

Ans. to the nearest unit = 11,272.

Rough Estimate.—The product must be less than 457 × 25, i.e. less than 11,425.

Example 3. Find the product of 0·00354067 and 0·10203 to 4 significant figures.

$0{\cdot}00354067 = 3{\cdot}54067 \div 10^3$; and $0{\cdot}10203 = 1{\cdot}0203 \div 10^1$

$\therefore$ The product $= 3{\cdot}54067 \times 1{\cdot}0203 \div 10^4$

$$\begin{array}{r|l} 3{\cdot}5\cancel{4}\cancel{0}\cancel{6} & \cancel{7} \\ & 1{\cdot}0203 \\ \hline 3{\cdot}5406 & 7 \\ {\cdot}0708 & 1 \\ {\cdot}0010 & 6 \\ \hline 3{\cdot}6125 & 4 \end{array}$$

The product $= 3{\cdot}6125 \div 10^4 = 0{\cdot}00036125$
$= 0{\cdot}0003613$ to 4 significant figures.

EXAMPLES. IV.

1. Find the following products—
   (1) 7809 × 657 correct to the nearest hundred.
   (2) 8765 × 789 to the nearest thousand.
   (3) 60507 × 4094 to the nearest hundred thousand.
   (4) 70850 × 6039 to the nearest million.

2. Multiply correct to two decimal places—
   (1) 7·02375 by 23·456; (2) 0·003157 by 12·567;
   (3) 37·57901 by 18·234; (4) 69·3025 by 0·0456.

3. Multiply to the units' figure—
   (1) 56·4567 by 18·7654; (2) 234·7896 by 19·0123.
   (3) 27·654 by 9·875; (4) 590·98 by 67·346.

4. Multiply to 4 significant figures—
   (1) ·012468 by ·23456; (2) ·003567406 by ·0405061;
   (3) 87·6205 by 3·16432; (4) 12035 by ·00024.

5. The earth's polar circumference is 40 million metres and the metre is 39·3708 inches. Calculate the circumference to the nearest mile.

What difference in miles would be made if the metre be taken to be 39·3709 inches ?

6. Find to the nearest million francs the cost of making a railway 27,345 miles long, the cost per kilometre being 303,625 francs ; 1 Km. = 0·675 mile approximately.

7. The circumference of a bicycle wheel is 7·145 ft. and it makes 6,500 revolutions in an hour. Find correct to two decimal places the distance in miles travelled in 5·25 hours.

8. If the marking on the weights can be trusted to the nearest milligram, by how much may the weight of a set of 25 one-hundred gram weights be in error ?

In the following examples the decimals are to be taken as correct to the last significant figure.

9. If 1 inch = 25·4 mm. find the length of 1·78 miles.

10. If 1 Hg. = 3·5 oz. avoir. find the weight of 7·856 Kg.

11. If on a hot day a foot of iron has expanded to 1·0032 ft. find the length of an iron rail 62·25 ft. long.

12. A ton of quartz when crushed is found to yield 3·425 oz. of gold. How many ounces can be got from crushing 20540·4 tons ?

---

149. In Contracted division, we gradually reduce the number of figures in the divisor instead of bringing down the figures from the dividend.

EXAMPLE 1. Divide 12·3656789 by ·7658 (1) as far as the second decimal figure ; (2) correct to two decimal places.

Bringing the divisor to the standard form,

$$\frac{12{\cdot}3656789}{{\cdot}7658} = \frac{123{\cdot}656789}{7{\cdot}658}$$

123 ÷ 7 will give two integers in the quotient ; and since two decimal places are required, there must be 4 figures in the quotient. To get the quotient correct to two places, we should know the third decimal figure. It is safe to retain two more figures in the dividend than are required.

I	II	III
Ordinary Method.	Contracted Method.	Italian Method.
16·147	16·147	16·147
76·58)123·656789	7·658)123·6567	7·658)123·6567
76·58	7658	47076
47076	47076	7·65)1128
45948	45948	7·6) 362
11287	7·65)1128	7) 56
7658	766	3
36298	7·6)362	
30632	306	
56669	7)56	Ans. (1) 16·14.
53606	53	(2) 16·15.
3063	3.	

Rough Estimate.—The dividend is above 123 ; and the divisor above 7 ; the quotient must be between 16 and 17.

After getting the integral figures of the quotient instead of bringing down the next figure 7 from the dividend, the figure 8 from the right of the divisor is omitted ; at the next step 5 is omitted, and at the next 6.

Observe that in multiplication, the nearest multiple of 10, that is, got from multiplying the digit omitted is added. Thus in the division by 7·65, the quotient 1 is multiplied by 8, the digit omitted ; and 8 being nearer 10 than 0, 1 is added, and thus we get 766. In the next division by 7·6, the quotient 4 is multiplied by 5, the digit omitted, and since 20 contains 2 tens, 2 is added and we get 306. In the final step, 7 times 6 being 42, 4 is added and 53 is thus got.

150. It is not necessary, however, that the divisor should be written down at every step. Each digit of the divisor may be scored out as soon as the division with it is done.

EXAMPLE 2. Divide 102·34567 by 98·765421 to 4 significant figures.

$$\frac{102\cdot34567}{98\cdot765421}=\frac{10\cdot234567}{9\cdot8765421}$$

Here the integral quotient consists of 1 figure and therefore there should be 3 decimal places. In the divisor take one more than the number of significant figures required in the quotient.

	Italian method.	
1·036	1·036	
9·87~~6~~~~5~~)10·2345	9·87~~6~~~~5~~)10·2345	3 × 6 = 18 carry 2.
98765	3580	3 × 7 + 2 = 23 ; etc.
3580	617	6 × 7 = 42, carry 4.
2963	25	6 × 8 + 4 = 52 ; etc.
617		
592		
25		

Rough Estimate.—The dividend is a little above 102 and the divisor nearly 99; the quotient must be a little above 1.

EXAMPLE 3. Divide 1·387635 by 259·6 to six places of decimals.

$$\frac{1·387635}{259·6} = \frac{0·01387635}{2·596}$$

This will give no integral quotient but 2 cyphers immediately to the right of the decimal point; and there should be 4 significant figures in the quotient to have six decimal places.

In the divisor there should be 5 figures; that is, one more than the number of significant figures, but as there are only 4, add a cypher at the end and proceed with the division as usual.

Using Italian method—

$$\begin{array}{r@{}l}
 & \overline{·005345} \text{ quotient.} \\
2·5\cancel{0}\cancel{0}\,\cancel{0}) & 0·0138763 \\
 & 8963 \\
 & 1175 \\
 & 137 \\
 & 7 \\
 & \overline{}
\end{array}$$

EXAMPLE 4. Divide 166·8 by 37·4, the dividend and the divisor being correct to the tenth.

$$\frac{166·8}{37·4} = \frac{16·68}{3·74}$$

$$\begin{array}{r@{}l}
 & \overline{4·46} \text{ quotient.} \\
3·\cancel{7}\,\cancel{4}) & 16·68 \\
 & 172 \\
 & 22 \\
 & 0
\end{array}$$

EXAMPLES.—ORAL.

In each of the following, bring the divisor to the standard form and give the quotient to one significant figure.

EXAMPLE (1) 1·8734 ÷ 54·72 = ·18734 ÷ 5·472 = ·03.
(2) 47·567 ÷ 8756 = ·047567 ÷ 8·756 = ·005.

1. ·5864 ÷ 27·56. 2. ·765 ÷ 23·45. 3. 8·72 ÷ 18·24.
4. 5·046 ÷ 0·025. 5. ·037 ÷ 130·98. 6. 73·045 ÷ 4265.
7. ·00256 ÷ 664. 8. 45·678 ÷ 5432. 9. ·004324 ÷ ·0072.
10. ·000789 ÷ ·0095. 11. 1006·35 ÷ 271·234. 12. ·0583 ÷ 39·27.

EXAMPLES. V.

Divide—

1. (1) 20·8642135 by 0·5678; (2) 18·97653421 by 0·3579; (3) 19·8070605 by 0·8765; (4) 93·4567891 by 0·9876 to two places of decimals.

2. (1) 1003·406809 by 901·2304;
(2) 2124·8092305 by 508·136002;
(3) 238·7635924 by 109·6325;
(4) 998·7654 by 89·87654 to 4 significant figures.

3. The dividend and divisor are given correct to the last significant digit; find the value of—

(1) 918·432 ÷ 296·645; (2) 70·987 ÷ 59·876;
(3) 44·765 ÷ 3·1416; (4) 2094·654 ÷ 39·3701.

4. If the circumference of the earth is 25,000 miles, and if that is 3·1416 times the diameter, find to the nearest mile, the diameter of the earth.

5. A motor-car travels 63·87 miles an hour. Find to two places of decimals the speed of the car in metres per second, given that 1,000 yards = 915 metres.

6. A thread was wound round a cylinder 24 times and the length was found to be 100·48 cm. correct to the 100th of a centimetre. What is the circumference of the cylinder and the greatest possible error in the calculation?

7. The Earth goes round the Sun once in 365·2564 days; Mercury once in 87·96926 days; and Neptune in 60181·11 days. Find to 2 significant figures, the number of our years in Mercury's year and in Neptune's year.

CHAPTER XIII.

AVERAGES.

151. The **average** of a series of quantities of the same kind is found by adding the quantities together and dividing their sum by the number of quantities.

EXAMPLE 1. A merchant's profits were for the first quarter of 1890, Rs. 1,056 10 as. 8 p.; for the second quarter, Rs. 1,345 5 as. 10 p.; for the third, Rs. 1,568 8 as. 6 p.; and for the fourth, Rs. 1,540 13 as. 4 p. Find the average of his quarterly profits.

Adding the four sums, we find the total profits to be Rs. 5,511-6-4.

Since there are 4 quarters, dividing Rs. 5,511-6-4 by 4, the average is Rs. 1,377-13-7.

152. When the quantities are nearly equal, take any one quantity as the unit, add only the differences; find the average of this sum and add the result to the unit chosen.

EXAMPLE. Find the average height of five boys whose heights respectively are 5 ft. 6·5 in., 5 ft. 6 in., 5 ft. 4 in., 5 ft. 7·5 in. and 5 ft. 8·5 in.

Taking 5 ft. 4 in. as the unit, the sum of the differences = 2·5 in. + 2 in. + 0 in. + 3·5 in. + 4·5 in. = 12·5 in. 12·5 in. ÷ 5 = 2·5 in.

Required average = 5 ft. 4 in. + 2·5 in. = 5 ft. 6·5 in.

All these steps may easily be done mentally.

EXAMPLES.—ORAL.

Give the average—

1. Age of 4 boys whose ages are respectively 16, 14, 12, 10.
2. No. of marks of 5 boys who secured respectively 22, 18, 20, 16, 14.
3. No. in each form: VI, 18; V, 12; IV, 20; III, 36; II, 24; I, 40.
4. Height of 5 boys whose heights in feet and inches are respectively 5.6, 5.4, 5.2, 5, 4.8. 5. $3x$, $5x$, $7x$. 6. $8y$, $10y$, $12y$.

The following is the record of five of the schools that entered for the Inter-School Sports, Tinnevelly, held in Sept. 1908. Give the average.

1. Half-mile race in min. and sec. 2.37; 2.27; 2.25; 2.28; 2.30.
2. Quarter-mile race in sec. 52, 51, 52·8, 53·4, 54·3.
3. Long jump in ft. and in. 21.10; 20.8; 19.4; 22·4·5; 23.4·5.
4. High jump in ft. and in. 5.6; 5.5; 5.6·5; 5.7·5; 5.8·5.
5. Putting the weight in ft. and in. 36.8, 34.4, 32.6, 35.2, 33.4.

EXAMPLES. I.

1. A copyist got for the 12 months of the year 1890, Rs. 18-7-6, Rs. 15-10-8, Rs. 16-8-4, Rs. 13-5-4, Rs. 12, Rs. 17-13-4, Rs. 15, Rs. 12-8-0, Rs. 14-2-8, Rs. 11, Rs. 10-11-5, Rs. 13-4-9 respectively. Find his average monthly income.

2. The temperature in the shade at 12 noon in a certain week for the 7 days, was 85·1, 84·6, 87·4, 86·2, 83·9, 82·1 and 81·5 respectively. Find the mean temperature at noon for the week.

3. Find to two places of decimals the averages, for completed innings for the following cricketers:—

	Total	Innings	Not out
Ratnam ...	1872	32	4
Singaram ...	1584	28	2
Ponnusawmy ...	1673	36	3
James ...	1445	24	2

4. Find to two places of decimals the average number of runs per wicket for the following bowlers :—

	Runs	Wickets		Runs	Wickets
James ...	1284	85	Doss ...	1532	48
Dharma ...	1073	66	Chelliah ...	1646	94

5. The gross earnings of a certain railway company during the first six months of the year 1899 were as follows :—

January,	Rs. 1,134,267-13-5 ;	February,	Rs. 1,098,763-10-11 ;
March,	Rs. 1,109,835-6-9 ;	April,	Rs. 1,148,239-2-6 ;
May,	Rs. 1,132,470-15-8 ;	June,	Rs. 1,087,493-12-7.

Find the average monthly income to the nearest pie : also the average daily earnings.

6. In September 1892, for the first 15 days, the daily readings of the barometer at Bombay were as follows :—

Day	Reading	Day	Reading	Day	Reading
1	29·662	6	29·724	11	29·688
2	29 591	7	29·741	12	29·696
3	29·600	8	29·760	13	29·770
4	29·673	9	29·719	14	29·862
5	29·685	10	29·684	15	29·874

Find to the nearest thousandth of an inch the mean of the daily reading ; and write down the excess or defect of the daily reading from the mean on the 5th, 8th and 15th.

7. According to the census of 1891, the population of each of the following districts was as given below :—

Ganjam ...	1,896,803	Salem ...	1,962,591
Vizagapatam...	2,802,992	Coimbatore ...	2,004,839
Godavari ...	2,078,782	Tanjore ...	2,228,114
Kistna ...	1,855,582	Tinnevelly ...	1,966,095
S. Arcot ...	2,162,851	Madura ...	2,608,404
N. Arcot ...	2,180,487	Malabar ...	2,652,565

What was the average population of a district ?

8. The following table shows the number of passengers carried on a certain railway and the amount of fares paid by them in each of the months named :—

Month	No. of passengers carried	Amount of fares paid		
		RS.	A.	P.
January ...	1,923,654	7,31,268	4	10
February ...	1,738,901	6,79,365	13	2
March ...	2,003,496	7,74,982	15	9
April ...	1,932,781	7,24,850	7	11
May ...	1,989,757	7,59,368	11	5
June ...	1,894,532	7,02,783	12	8

Find the average to the nearest pie paid by each passenger in the half-year.

9. A class of boys measuring the circumference of a half-anna coin obtained the following results in inches:—3·8, 3·76, 3·77, 3·75, 3·78, 3·74, 3·76, 3·73. Find to 3 significant figures the average of these results.

10. The weights of a boat's crew, to the nearest pound, were 10 stone 6 lbs., 10 stone 8 lbs., 11 stone 4 lbs. and 11 stone 5 lbs. What is the possible error in the average weight, determined from these figures?

11. The following results were obtained by a class on weighing an ounce weight in grams:—28·36, 28·40, 28·37, 28·38, 28·41, 28·35, 28·34, 28·39. Find the error in the average result, if 1 lb. = 453·593 g. correct to 3 places.

12. Madras Railway.

Year	Gross earnings	Working expenses
	RS.	RS.
1895 ...	11,353,713	5,408,603
1896 ...	11,096,897	5,638,124
1897 ...	11,165,314	5,463,126
1898 ...	10,720,483	5,218,578
1899 ...	10,545,071	5,495,340

Find the average net earnings for these years.

EXAMPLE 2. In a flock of sheep, 15 are worth Rs. 4-8 as. each; 10, Rs. 3-12 as. each; 12, Rs. 3-8 as. each; 23, Rs. 3-4 as. each. Find the average price of a sheep to the nearest pie.

The price of 15 sheep $= 15 \times 4\frac{1}{2}$ Rs. $=$ Rs. 67-8 as.
,, 10 ,, $= 10 \times 3\frac{3}{4}$ Rs. $=$ Rs. 37-8 as.
,, 12 ,, $= 12 \times 3\frac{1}{2}$ Rs. $=$ Rs. 42.
,, 23 ,, $= 23 \times 3\frac{1}{4}$ Rs. $=$ Rs. 74-12 as.
Adding........60 sheep.......................$=$ Rs. 221-12 as.
∴ The average price of 1 sheep...........$=$ Rs. 221-12 as. $\div$ 60.
$=$ Rs. 3-11-2.

EXAMPLES. II.

1. In a class of 35 boys, 2 of the boys are of the age of 19, 1 of 18, 3 of 16, 4 of 15·5, 5 of 14, 8 of 14·75, and the rest of 14·25. Find the average age of the boys.

2. In a school consisting of 100 boys, 25 pay Rs. 4 each; 35 pay Rs. 2-8 as. each; and the rest pay Re. 1-8 as. each. Find the average paid by each boy.

3. In a certain country containing 8,000 square miles, the average population per sq. mile is 340; and in another having an area of 6,400 sq. miles, it is 565 per sq. mile. Find the average population per sq. mile for the two countries together.

4. The average age of a school of 600 boys is 18·75 yrs. By the admission of 40 new boys, the average is reduced to 16·4375 yrs. Find the average age of the new boys.

EXAMPLE 3. In a class of 30 boys, 26 have attended during the week. Days attended by the boys: 10 for 5 days, 8 for 4·5 days, 6 for 4 days and 2 for 1 day. Find the average number of days attended by each boy in the class.

The whole number of days attended by the class
$= (10 \times 5 + 8 \times 4{\cdot}5 + 6 \times 4 + 2 \times 1)$ days $=$ 112 days.
∴ Average number of days attended $= \frac{112}{30} = 3{\cdot}7$ days nearly.

EXAMPLES. III.

1. In a class of 40 boys, 30 have attended during the week 15 for 5 days, 10 for 4·5 days, 3 for 3 days and 2 for 2·5 days. Find the average number of days attended by each boy in the class.

2. In a school of 250 boys, 200 boys have attended 5 days, 20 boys for 3·5 days, 15 for 2 days and the rest have been

absent. Find the average number of days attended by each boy in the school.

3. Find the average daily attendance in the previous question. The school had 5 working days.

4. In a school of 60 boys, in the month of September, 15 boys have been present all the working days (20); 20, 4 days; 15, 3 days; 10, 2·5 days. Find the average daily attendance.

EXAMPLE 4. A tradesman's average annual gain from 1861 to 1871, both inclusive, was Rs. 615-4 as. In 1861, he gained Rs. 387-1-4; and in 1872, he gained Rs. 747-5-4. What was his average annual gain from 1862 to 1872, both inclusive?

His total gain from 1861 to 1871=Rs. 615-4 as. × 11=Rs. 6,767-12.
He gained Rs. 387-1-4 in 1861, and Rs. 747-5-4 in 1872.
∴ His gain from 1862 to 1872 inclusive for the 11 years.
=Rs. 6,767-12 as. — Rs. 387-1-4 + Rs. 747-5-4 = Rs. 7,128.
∴ His average gain = Rs. 7,128 ÷ 11 = Rs. 648.

EXAMPLES. IV.

1. The average rainfall in a certain station for 6 yrs. is 29·8 in.; and that for 3 yrs. is 29·1 in. Find the average for the next 3 yrs.

2. The average weight of a crew of 8 men is 11 stone 6 lbs. 1 oz. each. Three of them together weigh 36 stone 4 lbs. What is the average weight of the others?

3. The average price of copper per ton for the first half-year of 1904, reckoned on the price on the first day of each month was £94-13-4. If the average price for the year was £99, find that per month of the second half-year.

4. The mean temperature from 9th to 16th February both days inclusive was 34·6°, and from 10th to 17th, it was 39·8°; the mean temperature on the 9th was 30·5°. What was it on the 17th?

ONE-HOUR PAPERS.—FIRST SERIES.

REVISION EXERCISES.

I.

1. A boy bought a coat and a cap for Rs. 6-4 as.; the cap cost Rs. 2-8 as. less than the coat. Find the cost of the coat.

2. The marks gained by a class of five girls in English are respectively 93, 85, 67, 56 and 24. Find the average for the class.

3. Find to the nearest penny the rent of 54·176 ac. of land at £2·348 an acre.

4. If from a ton of sugar, 13 cwt. 2 qrs. 14 lbs. has been sold, express the remainder as a decimal of 1 ton.

5. One gallon of water weighs 10 lbs. one litre of water weighs one kilogram, which approximately weighs 2·2046 lbs. Find the number of litres in a gallon.

II.

1. If 1 metre = 39·375 inches approximately, find the number of metres in one chain.

2. A man bought 100 sheep at Rs. 5-4 as. each. It cost him Re. 1-8 as. each for feeding them ; and 5 of them died on the way to the market. He sold the rest at Rs. 7-12 as. each. How much did he gain ?

3. Six baskets contained respectively 800, 550, 900, 870, 936 and 684 plantains. What was the average number in a basket ?

4. Five different substances weigh respectively 5·35 g., 7·48 g., 13·92 g., 9·14 g., and 8·67 g. What is their total weight in grams ?

5. If one kilometre of telegraph wire cost Rs. 468-12 as., find the cost per mile to the nearest pie. 1 Km. = 0·625 mile.

III.

1. If 3·25 tons of coal are bought at 18*s*. per ton and sold at 1*s*. 2*d*. per cwt., what is gained by the transaction ?

2. A man travelled 30 miles 6 fur. in the 1st day of his journey, in the 2nd day 26 miles 7 fur., and in the 3rd day 23 miles 3 fur. How many miles on an average did he travel a day ?

3. If a motor goes at the rate of 13·01 metres per second, what time does it take to cover a distance of 15·25 miles. 1 metre = 3·3 feet, true to the nearest tenth of a foot.

4. Find to the nearest penny the value of a bar of gold, 2·3 metres long at £1 per inch, taking one metre = 39·37079 inches.

5. A bicycle wheel makes 257 complete revolutions in going over 3 fur 13 yds. 1 ft. 6·5 in. What is its circumference ?

IV.

1. A bale of cotton weighs 3 cwt. 2 qrs. 15 lbs.; 25 such bales were bought at 9$\frac{1}{2}$*d.* per lb. and sold at 1*s.* 0$\frac{1}{2}$*d.* per lb. Find the profit made.

2. In a certain week, I spent on Sunday nothing and on the other days, Rs. 2-5-4, Rs. 1-2-8, 15 as., Rs. 2-13-11, Rs. 2 and Rs. 1-3-8. Find my average daily expense.

3. Reduce 7 m. 8 dm. 9 cm. to yds. ft. in., taking 1 m. = 1·0936 yds.

4. A map is drawn so that one metre represents one kilometre; what length represents a mile on the same scale. Ans. to two decimal places of an inch.

5. Add together ·35*£*, ·625 of 16*s.* 8*d.*, and ·03 of *£*7 10*s.*; and subtract the result from *£*1·25.

V.

1. Find in English money the value of Rs. 1,000-0-3, when a shilling = 12 as.

2. The average age of 7 boys is 15, and average age of 4 of them is 14·25. Find the average age of the others.

3. Find the value of 89·78 tons of copper at *£*67·375 per ton.

4. By how many ounces does the sum of ·00025 ton + ·0124 cwt. + ·012 qr. + ·002 lb. fall short of 5 lbs.?

5. If 10 8 metres of string are required to wrap round the handle of a cricket bat 120 times, what is its circumference in centimetres?

VI.

1. The material for a gold ring costs Rs. 25; and the cost of making it is Rs. 8-5-4. How many rings can be made for Rs. 1,033-5-4?

2. Present on Monday 155, on Tuesday 150, Wednesday 159, Thursday 151. Average attendance for the week (5 working days) 156. How many were present on Friday?

3. Add together 7·07 m., 8·48, and 9·60 m. each correct to the nearest centimetre. Find the greatest possible error in excess or defect.

4. The circumference of a bicycle wheel true to the centimetre is 6·42 metres. Find its diameter given that it is 3·1416 times the circumference.

5. How many feet and how many metres are there in ·58125 of a mile, given 1 metre = 1·093638 yds.

VII.

1. The sum of *£*28-4-0 is divided among 18 men and a certain number of women. Each man gets *£*1-3-6 and each woman 15*s.* 8*d.* Find the number of women.

2. The average age of 29 children is 7; that of the first 7 is 6·5 and that of the last 10 is 5·75. Find the average of the rest.

3. If lengths of 1·9 inches be cut off a thread one yard long, what length will be left over?

4. Find the value of 0·072 £ + ·875*s* + 8·45*d*. and subtract the result from 0·42£. Ans. to the nearest penny.

5. A train is travelling at the rate of 40·14 miles per hour. How far does it go in 0·16 hr. (to the nearest yard)?

VIII.

1. A cattle dealer paid £780 for cows and sheep; of this sum £350 was paid for 25 cows. If a cow cost seven times as much as a sheep, how many sheep did he buy?

2. A merchant's average profits from 1875 to 1885, both inclusive, were Rs. 3,691-8 as. In 1875, he gained Rs. 1,528-5-4; and in 1886, he gained Rs. 3,029-13-4. What were his average annual profits from 1876 to 1886, both inclusive?

3. A person thinks that he is riding at the rate of 15 Km. per hour; but his watch is found to be 15 seconds too fast. What is the real rate per hour (to the nearest metre)?

4. Multiply Rs. 16-6-4 by 6·7896, giving the ans. to the nearest pie.

5. How many pounds and how many Kg. are there in 0·708624 ton, 100 Kg. = 1·9684 cwt.

IX.

1. What cash must be given with 24 yds. of cloth at 8*s*. $3\frac{3}{4}d$. per yard to pay for 6 cwt. of sugar at 38*s*. per cwt.

2. The average number of marks of 25 boys is 16, that of the first twelve being 16 and that of the last twelve 15. What does the 13th boy get?

3. Express the remainder resulting from the division of 42·345875 ft. by ·625 ft. as a decimal of one yard.

4. If a boy gives 19·4 in. as the perimeter of a triangle whose sides are respectively 5·76 in., 6·24 in. and 7·48 in., what is his error? What would be the more accurate estimate?

5. It is estimated that thirty-six rupees piled on the top of one another would reach to a height of 7·35 cm. Find the thickness of a rupee in decimals of a millimetre to three places.

X.

1. If an estate be worth £2,374-16*s*. a year, and the land-tax be assessed at 1*s*. $11\frac{1}{2}d$. in the £, what will be the net annual income to the nearest penny?

2. Multiply 26·05632 by 0·45068, (1) exactly, (2) to the nearest thousandth.

3. How many bottles containing 0·067125 of a gallon can be taken from 345·675 gals.? What quantity is left over.

4. If the population of the United Kingdom according to the last census is 41,976,827 and the amount of the national debt is £759,135,471, what is the liability per head to the nearest penny?

5. Find the number of inches, correct to the nearest unit, in 9·24 metres, given 1 metre = 39·3701 inches.

XI.

1. The population of the British Empire is estimated to be forty crores, and the imperial revenue 375 crores of rupees. How much is this per head, to the nearest pie?

2. How many lengths of 0·1234 of an inch can be cut off from 89·576 inches? Find the remainder.

3. A train runs at the rate of 45·25 miles an hour; find in kilometres the distance run in 6·5 hrs., it being given 1 mile = 1·6093 Km.

4. In expressing £·0678 as correct to three significant figures, find the resulting percentage error. Give its value to the nearest penny.

5. The circumference of a circular plot of ground is 21,900 miles, correct to the nearest hundred. Find the diameter, given the circumference = diameter × 3·1416 (correct only to 4 places).

XII.

1. A student finds the length of a lath to be 135·75 cm., and wishes to have a length equal to one metre; what length must be cut off? If his measurement can be trusted to one-tenth of a millimetre, what is the possible error in the resulting metre?

2. Two straight rods 23 ft. and 12 yds. long respectively were heated. The former expanded 0·000412 of its original length; and the sum of the expansions of the two rods was ·25 in. What decimal of the original length did the second rod expand? Ans. correct to 6 decimal places.

3. An equal number of rupees, half-rupees, quarter-rupees and two-anna pieces amounted to Rs. 80-10 as. How many were ther

4. Express Rs. 4·276 as correct to 3 significant figures. Find the percentage error and give its value to the nearest pie

5. The work done in raising 1 lb. weight vertically through 1 foot is 1 foot-pound, and that done in raising 1 kilogram through 1 metre is 1 kilogram-metre. Express 1 kilogram-metre in terms of foot-pounds, if 1 metre = 1·0936 yds., and 1 kilogram = 2·20406 lbs.

CHAPTER XIV.

GREATEST COMMON MEASURE (HIGHEST COMMON FACTOR).

153. A measure. One number is said to be a measure of another number when the former divides the latter exactly, i.e. without any remainder ; thus, 3 is a measure of 12.

A measure of a number is simply its **factor.**

The greatest measure of any number is the number itself, and the least measure is unity.

154. A Common Measure of two or more given numbers is a number which divides each of the given numbers exactly.

EXAMPLE. 2, 3 and 6 are common measures of 12, 18 and 24.

A Common Measure is the same as a **Common Factor.**

155. Numbers are said to be **prime** to each other, if they have no Common Measure or factor except unity.

For example, 3 and 7 are prime to each other ; also 17 and 19 ; 24, 25 and 49.

156. The Greatest Common Measure (G.C.M.) or the **Highest Common Factor (H.C.F.)** of two or more numbers is the greatest number which divides each of the given numbers exactly.

EXAMPLE. 6 is the Greatest Common Measure of 12, 18, 24.

157. To find the G.C.M. (H.C.F.) of two or more numbers.

(1) *The method of Prime Factors.*

Resolve the given number into prime factors. The product of the common prime factors is the G.C.M. (H.C.F.)

EXAMPLE 1. Find by means of factors the H.C.F. of 108, 132 and 144.

$$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3.$$

$$132 = 2 \times 2 \times 3 \times 11 = 2^2 \times 3 \times 11.$$

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2.$$

The factors common to the three numbers are 2^2 and 3.

$\therefore$ The H.C.F. $= 2^2 \times 3 = 12$.

EXAMPLES.—ORAL.

Give the measures of 12, 16, 18, 21, 24, 25, 28, 30, 44, 52, 56, 64, 66, 72, 84, 96, 77, 92, 65, 78, 91, 100, 112, 120, 132, 144 $2x$, $4y$, $6z$.

Give the common measures of 4 and 10; 8 and 12; 12 and 15; 12 and 16; 18 and 27; 24 and 32; 20 and 45; 25 and 45; 36 and 40; 8, 12 and 16; 12, 16 and 18; 12, 18 and 24; 15, 18 and 21; 20, 30 and 40; 24, 36 and 48; 16, 40 and 48; $2a$ and $6a$; $6x$ and $8x$; $8y$ and $12y$.

Required the H.C.F. of—

1. 6 and 8.
2. 6 and 9.
3. 10 and 15.
4. 15 and 18.
5. 15 and 20.
6. 18 and 24.
7. 6, 9 and 12.
8. 12, 14 and 16.
9. 12, 15 and 18.
10. 9, 18 and 24.
11. 15, 20 and 25.
12. 14, 21 and 28.

Find by inspection the H.C.F. of—

13. $2 \times 3 \times 5 \times 7$, $2^2 \times 5 \times 3$ and $2^2 \times 7 \times 3$.
14. $3 \times 5 \times 11 \times 7$, $5 \times 2 \times 11 \times 3$ and $7 \times 2 \times 3 \times 5$.
15. $2 \times 3^2 \times 5^2 \times 7$, $3^2 \times 5 \times 11 \times 7$ and $5 \times 11 \times 2^2 \times 3$.
16. $3^2 \times 5^3 \times 7$, $3^3 \times 5^2 \times 2^2$, $2^4 \times 3^2 \times 5^2 \times 7$.

EXAMPLES. I.

Find by factors the H.C.F. of—

1. 360 and 540.
2. 825 and 1320.
3. 1452 and 2145.
4. 403, 455 and 481.
5. 639, 747 and 873.
6. 868, 1302 and 1736.
7. 1435, 2009 and 2583.
8. 1515, 2121 and 3636.

What prime factors are common measures of—

9. 612 and 693.
10. 3465 and 3675.
11. 14157 and 28413.
12. 89712 and 750684.

---

(2) *The method of Division.*

158. Finding the G.C.M. of two numbers by division is based upon the following principles:—

Any number that divides each of two numbers divides also (1) their sum, (2) their difference, (3) any multiples of either and (4) the sum of any multiples of the numbers, (5) the difference of any multiples of the numbers.

EXAMPLE (1) 13 divides 39 and 91; and 13 divides also 39+91 or 130.

(2) 13 divides 39 and 91; and 13 divides also 91 − 39 or 52.

(3) 13 divides 39 and 91; and 13 divides also 2 × 39 and 3 × 91.

(4) 13 divides 39 and 91; and 13 divides also 2 × 39 + 3 × 91 or 351 twenty-seven times.

(5) 13 divides 39 and 91; and 13 divides also 3 × 91 − 2 × 39 or 195 fifteen times.

NOTE.—It follows therefore that when a number divides each of two numbers, it also divides the remainder resulting from the division of the larger by the smaller.

EXAMPLE. 9 divides 99 and 909; dividing 909 by 99, the remainder is 18; and 9 divides 18.

159. Graphic Illustration.

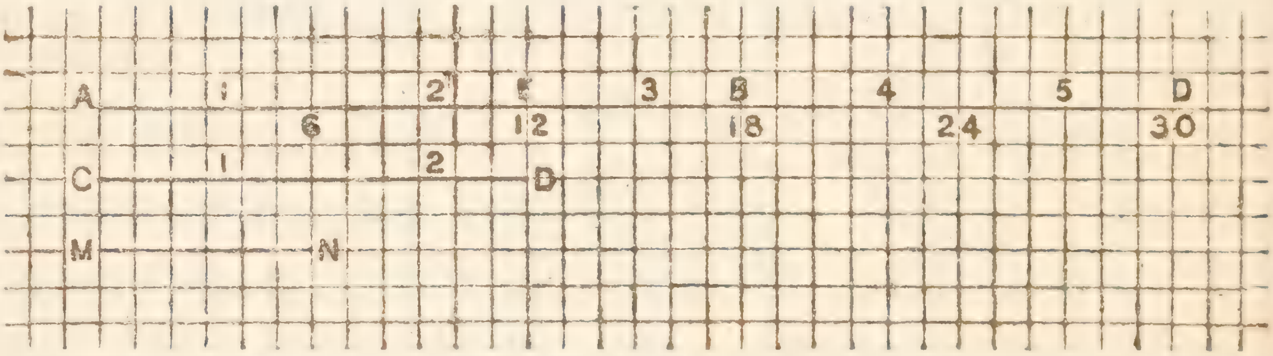


FIG. 15.

Let each division represent one unit. The lines AB and CD contain 18 and 12 units respectively; also MN contains 6 units.

From AB we can measure MN thrice and from CD we can measure MN twice.

Therefore MN measures AB and CD.

Place CD in a line and continuous with AB.

Now AD represents the sum of AB and CD and contains 30 units; and from AD we can measure MN five times.

Therefore MN measures the sum of AB and CD.

∴ 6 measures the sum of 18 and 12, i.e. 30.

The difference between AB and CD is EB; and MN measures EB once.

∴ MN measures the difference between AB and CD.

∴ 6 measures the difference between 18 and 12, i.e. 6.

In the same way, it may be shewn that MN is exactly contained in, i.e. measures any multiples of AB and CD; also the sum and difference of any multiples of AB and CD.

This may be stated generally thus: If P divides any two numbers A and B, P also divides $A \pm B$ and $mA \pm nB$, mA and nB being any multiples of A and B.

160. Suppose we wish to find by division the G.C.M. of CD and AB which contain 12 and 18 units respectively.

See Fig. 15, page 132.

Divide 18 by 12; i.e. AB by CD, the quotient is 1, and the remainder is EB which contains 6 units. The G.C.M. required is also the G.C M. of 6 and 12 units; i.e. of EB and CD.

Now divide 12 by 6; i.e. CD by EB, the quotient is exactly 2.

Therefore 6 is the G.C.M. of 12 and 18; i.e. EB is the G.C.M. of AB and CD.

Practical Exercise.

1. Take a stick a yard long, and 4 sticks 6 in., 8 in., 9 in. and 12 in. long respectively. Measure the yard by each of these and find out the common measures of the yard and the greatest length which divides the yard.

2. Mark off from your desk a length of 100 cm.; cut strips of paper of 10 cm., 20 cm., 25 cm., and 30 cm., respectively. Find out which of these divide 100 cm. exactly and the longest of them.

3. Draw on squared paper a line 4 in. long and another 3 in. long; and shew how you would use them to find the G.C.M. of 32 and 24.

4. Use squares or rectangles to find the H.C.F. of 21 and 33.

5. Illustrate graphically the method of finding the H.C.F. of—
   (1) 42 and 56. (2) 36 and 45.
   (3) 15 and 27. (4) 18 and 32.

EXAMPLE. Find the G.C.M. of 5037 and 8395.

```
5037)8395(1
     5037
     ----
     3358)5037(1
          3358
          ----
          1679)3358(2
               3358
               ----
```

Dividing 8395 by 5037, the quotient is 1, and the remainder 3358. The G.C.M. required is also the G.C.M. of 3358 and 5037. Divide 5037 by 3358; the quotient is 1, and the remainder 1679. The G.C.M. required is also the G.C.M. of 1679 and 3358. 1679 divides 3358 exactly twice. Therefore 1679 is the G.C.M. of 5037 and 8395.

The work may be more compactly arranged thus :

	5037	8395 (1
		5037
1st remr.	3358	5037 (1
		3358
2nd remr.	1679	3358 (2
		3358

or thus :

1	5037	8395	(1
	3358	5037	
	1679	3358	(2
		3358	

or using Italian method.

1	5037	8395	1
	16 79	3358	2

∴ The G.C.M. is 1679.

161. Hence the following rule to find the G.C.M. by Division :—

Divide the greater number by the less, the divisor by the remainder and repeat this process till there is no remainder. The last divisor is the G.C.M.

162. In finding the G.C.M. by division, the work may be shortened by applying the following processes :—

1. Before any division is done, take out from the given numbers any factor common to them.

But this factor must be multiplied in to the last divisor ; for being a factor of each of the given numbers, it must be a factor of the G.C.M.

2. Before any division is done any factor in one of the given numbers not contained in the other may be rejected.

3. At any stage from any remainder, a factor may be rejected that is not a factor common to the numbers whose G.C.M. is required.

4. When any two consecutive remainders contain no common factor, since the numbers are prime to each other, we need not proceed any further.

EXAMPLE. Find the G.C.M. of 13514 and 36830.

Dividing each of the numbers by the common factor 2, we get 6757 and 18415. 18415 contains the factor 5, which is evidently not a factor of 6757 and therefore 5 may be rejected. 18415 ÷ 5 = 3683.

We have now to find the G.C.M. of 3683 and 6757.

	3683	6757	1
	3074	3683	
3	609	3074	106
7	203	29	
	29	174	
		174	

Italian Method.

	3683	6757
3	609	3074
7	203	174
	29	

The quotients need not be written down.

609 is divided by 21 which is not a factor of the remainder 3074 and therefore not a factor of the two given numbers. We then proceed with 29 and 3074.

The G.C.M. of 3683 and 6757 is 29. ∴ The G.C.M. of 13514 and 36830 is 2 × 29 or 58.

EXAMPLES. II.

Find the G.C.M. of—

1. 272 and 425.
2. 728 and 455.
3. 571428 and 857142.
4. 841 and 1247.
5. 218707 and 526769.
6. 52300 and 10983.
7. 19527 and 23667.
8. 11431 and 12006.

Find the greatest number that will divide—

9. 5947 and 5985 without any remainder.
10. 1520 and 5292 ,, ,, ,,
11. 285714 and 714285 ,, ,, ,,
12. 307692 and 230769 ,, ,, ,,

---

163. To find the G.C.M. of three or more numbers.

EXAMPLE (1). Find the G.C.M. of 210, 450 and 750.

$210 = 2 \times 3 \times 5 \times 7.$
$450 = 2 \times 3^2 \times 5^2.$
$750 = 2 \times 3 \times 5^3.$
∴ The G.C.M. $= 2 \times 3 \times 5 = 30.$

If the numbers cannot be easily factorized—

First find the G.C.M. of any two ; and then find the G.C.M. of this G.C.M. and the third number. Continue the process for four or more numbers.

(2) Find the G.C.M. of 5083, 7735 and 8687.

The G.C.M. of 5083 and 7735 is 221.
And the G C.M. of 221 and 8687 is 17.
∴ The G.C.M. required is 17.

EXAMPLES. III.

Find the G.C.M. of—

1. 99, 121 and 165.
2. 115, 207 and 253.
3. 176, 1100 and 4444.
4. 837, 1134 and 1347.

5. 805, 1311 and 1978. 6. 639, 747 and 873.
7. 455, 403 and 481. 8. 3161, 36830 and 13514.

EXAMPLE. Find the greatest number which will divide 400 and 5190 leaving remainders 4 and 6 respectively.

Since the required number leaves 4 and 6 as remainders respectively when it divides 400 and 5190, it is clear that it exactly divides (400−4) or 396 and (5190−6) or 5184.

And the G.C.M. of 396 and 5184 is 36.

∴ 36 is the required number.

EXAMPLES. IV.

Find the greatest number which will divide—

1. 4669 and 5484 leaving remainders 7 and 8 respectively.
2. 6333 and 24000 ,, ,, 6 and 3 ,,
3. 940 and 2931 ,, ,, 4 and 6 ,,
4. 9761 and 8500 ,, ,, 5 and 4 ,,

Find the greatest number that will divide—

5. 78496 and 648727 leaving in each case a remainder 7.
6. 1019530 and 1231848 ,, ,, ,, 3.
7. 978, 3154 and 1955 leaving remrs. 3, 4 & 5 respectively.
8. 1331, 3100 and 4427 ,, ,, 5, 6 & 7 ,,

164. To find the G.C.M. of Concrete Quantities.

Reduce the given numbers to the same denomination and then find their G.C.M.

EXAMPLE. Find the G.C.M. of 8*s.* 2*d.* and £1-3-11.

8*s.* 2*d.* = 98*d.* and £1-3-11 = 287*d.* The G.C.M. of 98*d.* and 287*d.* is 7*d.*

EXAMPLES. V.

Find the greatest sum which exactly contains—

1. Rs. 5-5-7 and Rs. 7-7-2. 2. £4-11-3 and £5-11-3.
3. Rs. 12-11-7 and Rs. 23-10-1. 4. £2-6-0 and £3-16-8.

Find the G.C.M. of—

5. 2 mds. 7 viss 26 pals. and 5 mds. 2 viss 2 srs. 5 pals.
6. 22 days 11 hrs. 15 min. 3 sec. and 12 days 20 hrs. 8 min. 36 sec.

7. Rs. 221-0-3, Rs. 1,018-13-3 and Rs. 1,218-4-6.
8. £64-4-1, £3,000-8-4 and £3,805-10-10.

Problems on G.C.M.

EXAMPLE. Two bills which amount to £2-2-6 and £5-0-6 respectively, are to be paid in coin of one kind. What is the largest coin that can be used?

The greatest sum that will divide £2-2-6 and £5-0-6 exactly is the largest coin required.

Therefore find the G.C.M. of £2-2-6 and £5-0-6.

£2-2-6 = 510*d.* ; £5-0-6 = 1206*d.*

The G.C.M. of 510*d.* and 1206*d.* is 6*d.* Ans. 6 penny coin.

EXAMPLES. VI.

1. Find the greatest length which is a common measure of 8 yds. 2 ft. 7 in. and 11 yds. 0 ft. 11 in.

2. Find the greatest sum of money of which £205-15-3 and £594-0-9 are integral multiples.

3. A person pays three bills of Rs. 13, Rs 19-4 as. and Rs. 22-12 as. in coin of one kind; what is the largest coin used?

4. What must be the highest price of a sheep so that a person may buy an exact number of sheep for Rs. 27-8 as., Rs. 42-8 as. or Rs. 75? How many of each kind can he buy?

EXAMPLES. VII.

Miscellaneous Problems on G.C.M.

1. A fruit seller has 12,028 plantains and 12,772 mangoes and wishes to pack them in baskets containing the same number of either fruits. What is the largest number that can be put into a basket?

2. Two sums of money, Rs. 20-10 as. and Rs. 54-6 as. were distributed among some orphans, and each received the same amount. What was the highest possible sum that could have been received by an orphan?

3. In a certain factory the total of the daily wages is Rs. 68-12-2, and in another Rs. 760-13-7. If every man received the same wages, how many were employed in each factory?

4. What is the greatest weight that can be taken an exact number of times from 1 can. 12 mds. 10 pals. and 1 can. 13 mds 4 viss 13 pals.?

5. A boy has two laths whose lengths are respectively 176 cm. and 184 cm., and wishes to mark them off into divisions each of which will measure both the laths an exact number of times. What should be the greatest length between any two marks ?

6. A farmer has 492 qrs. of wheat, 744 qrs. of barley and 1038 qrs. of oats ; he wishes to put the grains without mixing them into the largest possible heaps of equal size. Find the number of heaps.

7. What is the greatest sum that can be paid exact number of times out of each of the following amounts, Rs. 71-4 as., Rs. 394-4 as. and Rs. 199-8 as. ?

8. The floor of a room 27 ft. long and 15 ft. wide is to be covered with large square tiles, none of which are to be cut. What is the biggest tile that can be used ?

9. In a yard measure one side is graduated to read tenths of an inch and is divided into 360 equal divisions ; and the other side is graduated to read eighths of an inch and is divided into 288 equal parts. What other divisions besides the zeros are in the same line ?

10. If Rs. 166-4 as. and Rs. 174-9 as. be each divided into equal portions of the same value and if the total number of portions is as small as possible, what is it and what is the value of each portion ?

11. If 295 kilograms are equivalent to 649 pounds, what is the least number of kilograms equal to an exact number of pounds ?

12. If 2,040 dollars are equal in value to £238, what is the least number of dollars equal to an exact number of sovereigns ?

CHAPTER XV.

LEAST COMMON MULTIPLE.

165. A Multiple of a number has already been defined as the number that is exactly divisible by the given number.

Thus 8 is a multiple of 2 ; and 6 is a multiple of 3.

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, etc.

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, etc.

The multiples of 4 are 4, 8, 12, 16, 20, 24, 32, 36, etc.

166. A Common Multiple of two or more given numbers is a number which is exactly divisible by each of the given numbers.

Thus 6 is a common multiple of 2 and 3.
8 is a common multiple of 2 and 4.

12 is a common multiple of 2, 3 and 4.
30 is a common multiple of 2, 3, 5 and 6.

167. The Least Common Multiple of two or more given numbers is the least number which is exactly divisible by each of the given numbers.

12, 24 and 36 are each a common multiple of 2, 3, 4 and 6 ; but the *least* of them, that is, 12 is the least common multiple of 2, 3, 4 and 6.

EXAMPLES.—ORAL.

Give all the multiples—

1. Of 2 less than 25.
2. Of 3 less than 40.
3. Of 4 less than 65.
4. Of 5 less than 96.

Give the common multiples—

1. Of 2 and 4 less than 25.
2. Of 2 and 3 less than 35.
3. Of 3 and 4 less than 65.
4. Of 2 and 5 less than 105.
5. Of 4 and 8 less than 100.
6. Of 3 and 5 less than 110.

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168. To find the L.C.M. by the method of Prime Factors.

EXAMPLE. Find by factors the L.C.M. of 1155, 1260 and 1848.

$1155 = 3 \times 5 \times 7 \times 11.$
$1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^2 \times 3^2 \times 5 \times 7.$
$1848 = 2 \times 2 \times 2 \times 3 \times 7 \times 11 = 2^3 \times 3 \times 7 \times 11.$

If the L.C.M. did not contain 2^3, it will not be divisible by 1848 ; if it did not contain 3^2, it will not be divisible by 1260 ; if it did not contain 7×11, it will not be divisible either by 1155 or by 1848 ; and lastly if it did not contain 5, it will not be divisible by either 1155 or 1260.

∴ The L.C.M. must contain not only every prime factor of the given numbers, but also the highest power of each factor which occurs in any one of them.

∴ The L.C.M. $= 2^3 \times 3^2 \times 5 \times 7 \times 11 = 27,720.$

EXAMPLES.—ORAL.

Read off the L.C.M. as the product of prime factors :—

1. $2 \times 5 \times 3,\ 2 \times 5 \times 7.$
2. $3 \times 5 \times 7,\ 2 \times 5 \times 7.$
3. $2^3 \times 3^2,\ 3^3 \times 2^2.$
4. $2^2 \times 3^3 \times 5,\ 2 \times 3^2 \times 5^3.$
5. $3^3 \times 5^2 \times 11,\ 3^2 \times 5^3 \times 7.$
6. $5^2 \times 7^2 \times 3,\ 5 \times 7 \times 3^2.$
7. $2^3 \times 3^2 \times 5,\ 3^3 \times 2^4 \times 5^2,\ 5^3 \times 2^2.$
8. $7^3 \times 11^2,\ 5^2 \times 7^2,\ 3 \times 5^3 \times 7.$

EXAMPLES. I.

Find by the method of prime factors the L.C.M. of—

1. 72, 84 and 112. 2. 108, 132 and 144.
3. 63, 99 and 132. 4. 66, 76 and 96.
5. 460, 720 and 650. 6. 385, 165 and 340.
7. 315, 385, 495 and 693. 8. 330, 726, 462 and 1155.

169. It may be easily shewn that—

(1) *The L.C.M. of two or more numbers prime to each other is their product.*

Thus the L.C.M of 3 and 5 is 15; of 8 and 7 is 56; and of 4, 5 and 9 is 180.

(2) *Of two numbers that which is a multiple of the other, is the L.C.M. of the two.*

Thus the L.C.M. of 9 and 18 is 18; of 7 and 21 is 21; also the L.C.M. of 4, 8 and 16 is 16; and of 2, 3, 4, 6 and 12 is 12.

170. To find the L.C.M. by the method of division by prime factors.

Example. Find the L.C.M. of 21, 32, 44 and 63.

Note.—It will generally be found advantageous to begin with the lowest prime number 2 as divisor, and to repeat this as often as can be done; and then to proceed with the prime numbers 3, 5, 7, etc., in the same way.

$$\begin{array}{r|rrrr}
2 & 21, & 32, & 44, & 63 \\ \hline
2 & 21, & 16, & 22, & 63 \\ \hline
3 & 21, & 8, & 11, & 63 \\ \hline
7 & 7, & 8, & 11, & 21 \\ \hline
 & 1, & 8, & 11, & 3
\end{array}$$

L.C.M. $= 2 \times 2 \times 3 \times 7 \times 8 \times 11 \times 3 = 22{,}176.$

171. The work can be shortened (*a*) by erasing at any stage in the process, a number which exactly divides one of the other numbers; (*b*) by dividing by a composite number, in which case care must be taken to divide by the highest factor common to the divisor and to any of the numbers in the dividend.

Example. Find the L.C.M. of 7, 8, 9, 10, 14, 15, 18.

$$\begin{array}{r|ccccccc}
10 & \cancel{7}, & 8, & \cancel{9}, & 10, & 14, & 15, & 18 \\ \hline
 & & 4, & & 1, & 7, & \cancel{3}, & 9
\end{array}$$

L.C.M. $= 10 \times 4 \times 7 \times 9 = 2{,}520.$

(*a*) Observe that 7 that divides 14 is erased; also 9 that divides 18.

(*b*) Note that as the composite number $10 = 2 \times 5$, 8 is divided by 2 also 14 and 18 by 2; and 15 by 5.

EXAMPLES.—ORAL.

Give the L.C.M. of—

1. 7 and 9.
2. 9 and 13.
3. 5, 4 and 17.
4. 5, 7 and 8.
5. 12 and 36.
6. 14 and 56.
7. 3, 4, 15.
8. 3, 6, 15.
9. 5, 10, 15.
10. 5, 6, 12.
11. 9, 8, 18.
12. 10, 15, 20.
13. $2x$ and $4x$.
14. $3y$ and $9y$.
15. $2z$ and $5z$.
16. $3x$ and $2y$.
17. $4x$ and $5z$.
18. $2x$, $3y$ and $4z$.

EXAMPLES. II.

Find the L.C.M. of the following (i) by dividing by prime factors, (ii) by the method explained in Art. 171.

1. 5, 6, 7, 8, 9, 10.
2. 4, 6, 8, 10, 12, 14.
3. 8, 11, 14, 17, 20.
4. 10, 15, 25, 35, 45.
5. 24, 10, 32, 45, 25.
6. 7, 8, 9, 18, 24, 72, 144.
7. 21, 22, 24, 26, 28, 30.
8. 30, 32, 35, 36, 42.
9. 24, 25, 28, 32, 60.
10. 18, 24, 40, 48, 30.
11. 18, 27, 36, 40, 45.
12. 12, 21, 33, 28, 14.

---

172. The L.C.M. of two numbers which cannot be readily resolved into factors is obtained by finding the G.C.M.

Let A and B be the two numbers and H their H.C.F.

Then $A = H \times$ some factor; call this factor a.

And $B = H \times$ some factor; call this factor b; where a and b can have no common factors.

$\therefore A = a \times H$; and $B = b \times H$.

$\therefore$ By the method of prime factors,

the L.C.M. of A and B $= a \times b \times H$.

$\therefore$ The L.C.M. of A and B $\times H = a \times b \times H \times H$.

$$= a \times H \times b \times H.$$

$$= A \times B \ldots\ldots(1)$$

$\therefore$ The L. C. M. of A and B $= \dfrac{A \times B}{H} \ldots\ldots(2)$

$$= \frac{A}{H} \times B \text{ or } A \times \frac{B}{H} \ldots\ldots(3)$$

Hence we learn that—

(*1*) *The product of the H.C.F. and L.C.M. of two numbers = the product of the two numbers.*

(*2*) *The L.C.M. of two numbers = the product of the two numbers divided by their H.C.F.*

(*3*) *The L.C.M. of two numbers = product of one of the numbers by the quotient resulting from the division of the other number by the H.C.F.*

EXAMPLE. Find the L.C.M. of 799 and 2961.

Divide either number by the H.C.F.; and multiply the quotient by the other number.

The H.C.F. of 799 and 2961 is 47. And 799 ÷ 47 = 17.

∴ L.C.M. = 17 × 2961 = 50,337.

173. The L.C.M. of three or more numbers.

Find the L.C.M. of any two, and then find the L.C.M. of the result and the third number. Proceed similarly for four or more numbers.

EXAMPLE. Find the L.C.M. of 24871, 62491 and 70499.

First find the L.C.M. of 24871 and 62491.

Thus the G.C.M. of 24871 and 62491 is 209; and 24871 ÷ 209 = 119 and 62491 ÷ 209 = 299.

24871 = 209 × 119 = 11 × 19 × 7 × 17.

62491 = 209 × 299 = 11 × 19 × 13 × 23.

∴ The L.C.M. of 24871 and 62491 = 11 × 19 × 7 × 17 × 13 × 23.

Trying these factors with the third number 70499, we find it contains 11, 13 and 17 as factors. 70499 = 11 × 13 × 17 × 29.

∴ L. C. M. required = 11 × 19 × 7 × 17 × 13 × 23 × 29 = 215, 656, 441.

EXAMPLES.—ORAL

Read off the L. C. M. of—

1. 15, 25. 2. 18, 24. 3. 16, 20. 4. 24, 36.
5. 25, 35. 6. 8, 12, 20. 7. 12, 15, 18. 8. 14, 21, 35.
9. 15, 18, 12. 10. 16, 24, 40. 11. $8x$, $10x$, $12x$ 12. $4y$, $9y$, $12y$.

EXAMPLES. III.

Find the L.C.M. of—

1. 7568 and 9504. 2. 4738 and 8234.
3. 1680 and 1920. 4. 3164 and 4228.

Required the L.C.M. of—

5. 837, 1134 and 1347. 6. 805 1311 and 1978.

7. 4290, 7392 and 9828. 8. 3315, 5005 and 7293.

Problems on L.C.M.

Example. Find the least number which when divided by 9, 12, 14 and 210 leaves in each case a remainder 1.

The L.C.M. of the numbers is 1260.

Since when divided by each of the numbers there should be a remainder 1, it is evident that the number is 1260 + 1, i.e. 1261.

EXAMPLES. IV.

1. Find the least number of plantains one must have to distribute them equally among 15, 18, 25, 32 or 40 boys.

2. What is the least number of rupees that a man must have in order that he may distribute them equally among 25, 35, 40 or 50 boys ?

3. Find the least number that is exactly divisible by 8, 15, 16, 18, 24 and 30.

4. Find the least number that is exactly divisible by each of the odd numbers from 5 to 21.

5. What is the least number of nuts a bag must contain so that they can be distributed in parcels of 3, 4, 5, 6, 7, 8 or 9 and leave 2 nuts remaining in the bag ?

Find the least number which when divided—

6. by 8, 12, 16 and 20 leaves in every case a remainder 7.
7. by 9, 14, 18 and 24 ,, ,, ,, 5.
8. by 6, 10, 15 and 18 ,, ,, ,, 3.

9. What is the smallest number of rupees which when increased by one is exactly divisible into 54 or 81 shares ?

10. Find the smallest number which is exactly divisible by 24, 36 and 84 leaving in each case a remainder 5 less than the divisor.

11. What is the smallest number which when divided by 75 leaves 65 as remainder, when divided by 36 leaves 26 as remainder and when divided by 90 leaves 80 as remainder ?

12. Find the smallest sum of money which when divided among 56 men leaves Rs. 43 as remainder, when divided among 64 women leaves Rs. 51 as remainder and when divided among 72 children leaves Rs. 59 as remainder?

174. L.C.M. of Concrete Numbers.

EXAMPLE. Find the L.C.M. of 13 as. 4 p., Rs. 1-0-8 and Rs. 1-7-4.

Reducing each to pies, we have 160 pies, 200 pies and 280 pies.

The L.C.M. of these is 5,600 pies, i.e. Rs. 29-2-8.

EXAMPLES. V.

Find the L.C.M. of—

1. Rs. 1-2-4 and Rs. 4-9-4. 2. £2-10-9 and £12-13-9.
3. 5 mds. 2v. and 14 mds. 4. 3as. 2p., 4as. 9p. and 7as. 11p.
5. 5*s.* 8*d.*, 7*s.* 1*d.* and 9*s.* 11*d.*
6. 2 ft. 3 in., 3 ft. 9 in. and 5 ft. 3 in.
7. 1 qr. 12 lbs., 2 qrs. 14 lbs. and 1 cwt. 8 lbs.
8. Rs. 5-10-8, Rs. 26-10-8 and Rs. 16.

EXAMPLE. What is the smallest sum of money for which can be bought an exact number of turkeys at Rs. 1-14-0, or of sheep at Rs. 3-7-0 or of calves at Rs. 6-9-0?

The smallest sum of money is the L.C.M. of Rs. 1-14-0, Rs. 3-7-0 and Rs. 6-9-0.

Reducing each sum to annas, we have 30 as., 55 as. and 105 as.

The L.C.M. of these is 2310 as., that is, Rs. 144-6-0, which is the smallest sum of money required.

EXAMPLES. VI.

1. What is the least sum of money for which I can buy an exact number of sheep at Rs. 3-5-0 or cows at Rs. 35-7-0 each?

2. What is the smallest sum for which I can buy an exact number of copy-books at 9 pies, 1 an. 3 p. or 1 an. 9 p. each?

3. Find the least weight, that is, a common multiple of 1 ton 7 cwt. 27 lbs., and 1 ton 16 cwt. 1 qr. 17 lbs.

4. What must be the shortest length of a tape which can be measured exactly by 3 ft. 6 in., 2 ft. 11 in. or 4 ft. 1 in.?

EXAMPLES. VII.

MISCELLANEOUS.

1. The G.C.M. of two numbers is 789, and their L.C.M. is 112,827. If one of the numbers is 8,679, find the other.

2. Six bells begin to toll at intervals of 8, 10, 12, 14, 16, and 18 seconds respectively. When will they all next toll together? How many times will each have tolled then?

3. The front-wheel of a carriage is 7 ft. 6 in. round and the hind-wheel 12 ft. 6 in. Find the smallest distance in which each will have made a complete number of turns.

4. Three men start together and walk along a road, the distance covered by each in one step being 24, 28, and 30 in. respectively. What is the smallest distance walked, if each man has reached it after taking an exact number of steps?

5. Four smiths begin simultaneously to hammer, dealing 60, 80, 90 and 100 blows respectively per minute. How many blows must be dealt before they strike together again?

6. Given a mile = 5,280 ft., a knot = 6,080 ft., and a kilometre = 3,280 ft., find the least distance which is an exact number of miles, knots and kilometres. Express your answer in miles, in knots and in Km.

7. One side of a road is planted with trees 36 yds. apart and on the other side are lamp-posts 48 yds. apart. If the first lamp-post be opposite the first tree, how far off is the next lamp-post that has a tree opposite to it?

8. If one rupee is taken away from a purse, the remainder may be distributed equally among 8, 9, 10, 11 or 12 persons. Find the sum in the purse.

9. In ascending the steps in a tower, I find if I went two steps at a time, 1 is over; if 3 at a time, 2 are over; if 4 at a time, 3 are over; if 5 at a time, 4 are over. How many steps are there?

10. A number of fruits when divided equally among 24 persons leaves a remainder 14; when divided among 21 persons leaves a remainder 11; and when divided among 36 persons leaves a remainder 26. What is the least number of fruits?

11. A sum of money estimated to be between 1,000 and 1,500 rupees is found exactly divisible among 12, 15 and 18 persons. What is the sum?

12. My purse contains between 500 and 1,000 rupees. If I had Rs. 4 more, I could change the money into currency notes of Rs. 10, Rs. 20 or Rs. 50. What is in the purse?

CHAPTER XVI.

VULGAR FRACTIONS.

175. If a rupee is taken as the unit and divided into 16 equal parts, each part is an anna and is one-sixteenth of the unit; and seven such parts are seven-sixteenths of the unit.

When the unit taken is a viss, and when it is divided into 5 equal parts, each part is a seer and is one-fifth of the unit and 3 parts are three-fifths of the unit.

In a scale where an inch is taken as the unit and is divided into 10 equal parts, 7 parts are seven-tenths of the unit.

And it has already been stated that 0·7 represents seven-tenths of the unit; the unit in this case is the abstract number 1; and the meaning is that the unit is divided into 10 equal parts and 7 of such parts are taken.

When the unit is, as in these examples, divided into any number of equal parts and one or more of these parts are taken, the result is called a **Fraction.**

Such fractions are called vulgar or common fractions to distinguish them from decimal fractions or decimals where the unit is divided into parts that are powers of ten.

176. The pupil has already learnt to express 1 farthing as $\frac{1}{4}$ of a penny. This means that a penny is divided into 4 equal parts and one of such parts is taken. If three out of the four parts are taken, we write $\frac{3}{4}$ of a penny.

In the same way the fraction denoting seven-sixteenths of the unit is written $\frac{7}{16}$; the fraction denoting three-fifths of the unit is written $\frac{3}{5}$; seven-tenths is written $\frac{7}{10}$.

Thus a fraction is expressed by two numbers placed one above the other with a line between them.

The number below the line is called the **Denominator** which shews into how many equal parts the unit is divided; and the number above the line is called the **Numerator** which shews how many of such parts are taken to form the fraction.

Thus in $\frac{7}{8}$, 7 is the numerator and 8 the denominator and in $\frac{11}{12}$, 11 is the numerator and 12 the denominator.

$\frac{7}{8}$ is read seven-eighths; also seven over eight: $\frac{11}{12}$ is read eleven-twelfths; also eleven over twelve

177. Graphic representation.

Let the straight line AB be the unit. It is divided into 24 equal parts, so that each part is one-twenty-fourth of the unit.

AP contains 5 parts and is therefore five-twenty-fourths of the unit or $\frac{5}{24}$ of AB; and AQ contains 17 parts and therefore represents $\frac{17}{24}$ of the unit.

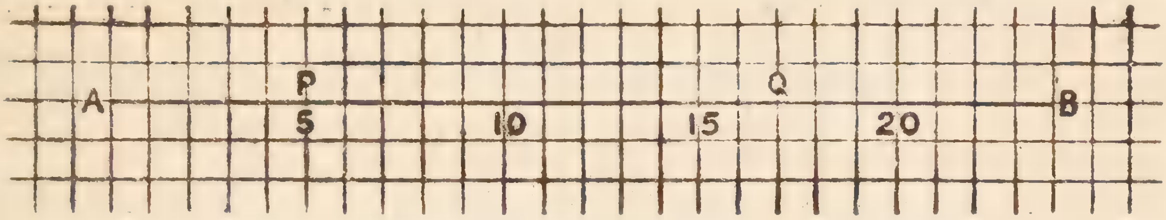

FIG. 16.

178. Let the rectangle ABCD be the unit.

Then the shaded part AEFD containing three columns represent $\frac{3}{15}$ or $\frac{1}{5}$ of the unit, and the shaded portion CBHG containing 5 columns denotes $\frac{5}{15}$ or $\frac{1}{3}$ of the unit, and the rectangle KLMN denotes $\frac{4}{15}$ of the unit. The remaining shaded portion and the unshaded portions each represents $\frac{1}{15}$ of the unit.

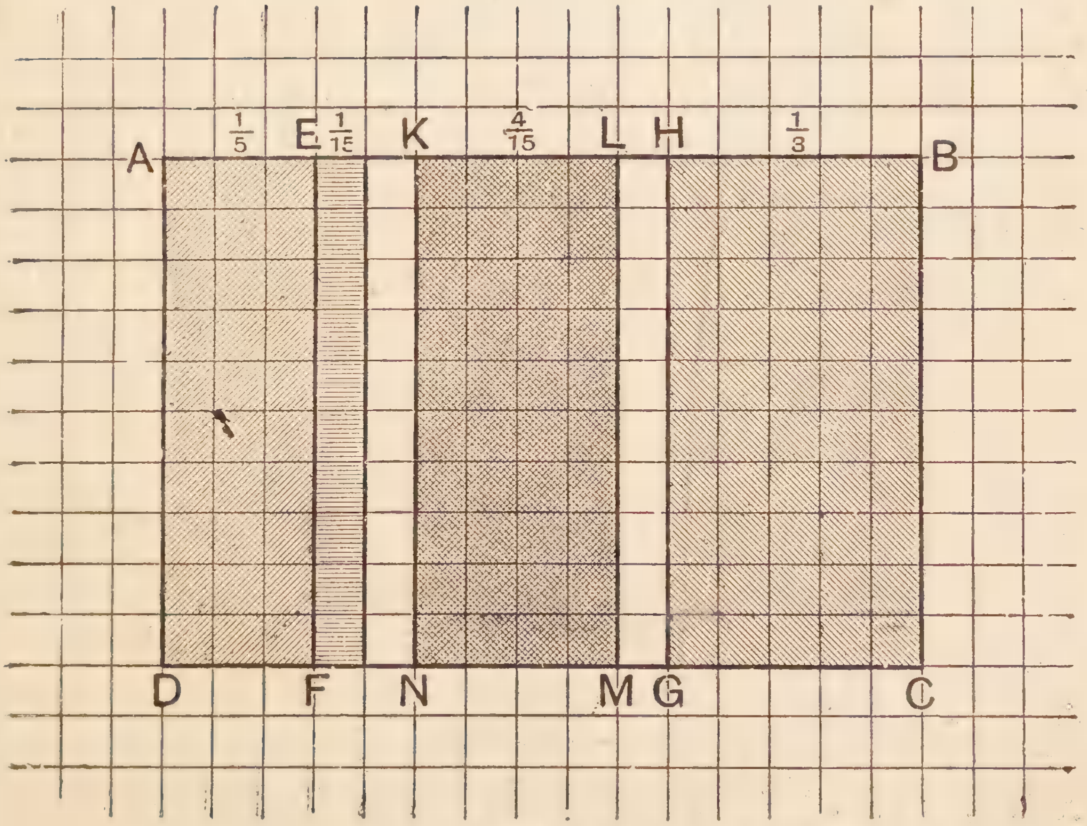

FIG. 17

Note.—The rectangle ABCD may be drawn four times, each figure containing only its respective fractional part shaded with different coloured crayons.

The teacher might ask the boys to fold a sheet of paper a number of times and elicit from them the number of parts into which it is divided and the number of such parts taken to form given fractions.

Practical Exercise.

1. Take a line XY as unit and draw lines to represent.
(1) $\frac{3}{4}$, (2) $\frac{7}{8}$, (3) $\frac{11}{12}$, (4) $\frac{5}{6}$, (5) $\frac{13}{24}$.

2. Take lines of different lengths and draw lines to represent fractions mentioned by your teacher.

3. Draw rectangles measuring (1) 4 in. by 3 in., (2) 6 cm. by 4 cm., (3) 8 in. by 9 in., (4) 16 cm. by 9 cm. Taking each rectangle as the unit, represent by shaded portions in rectangle (1) $\frac{1}{12}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{4}$; in rectangle (2) $\frac{7}{48}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{11}{24}$; in rectangle (3) $\frac{3}{72}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{7}{24}$; in rectangle (4) $\frac{19}{144}$, $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{9}$.

4. Let the pupils have exercises in paper folding to represent different fractions.

EXAMPLES.—ORAL.

Express as vulgar fractions—

1. Four-ninths. 2. Three-elevenths. 3. Seven-twelfths.
4. Nine-twenty-fifths. 5. Six-hundredths. 6. One-fivehundredth
7. Seven-thousandths. 8. Three-fivethousandths.

Read off the following fractions and give their meaning :—

1. $\frac{3}{5}$. 2. $\frac{4}{7}$. 3. $\frac{1}{9}$. 4. $\frac{3}{10}$. 5. $\frac{3}{11}$.
6. $\frac{7}{12}$. 7. $\frac{6}{15}$. 8. $\frac{11}{18}$. 9. $\frac{13}{25}$. 10. $\frac{31}{50}$.
11. $\frac{29}{64}$. 12. $\frac{13}{75}$. 13. $\frac{21}{100}$. 14. $\frac{37}{750}$. 15. $\frac{101}{1000}$.

179. Fractions of concrete quantities.

Since $\frac{1}{2}$ means that the unit is divided into 2 equal parts and one part is taken, $\frac{1}{2}$ of Re. 1 means Re. 1 is divided into two equal parts and the value of one of the parts is taken.

$\therefore$ $\frac{1}{2}$ of 1 Re. = 8 as. Similarly $\frac{1}{3}$ of £1 = 6*s*. 8*d*.

$\frac{1}{5}$ of 12*s*. 6*d*. = 2*s*. 6*d*.; and $\frac{1}{6}$ of 1 viss = 6 pals. 2 tolas.

$\therefore$ $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ ....of a unit means the division of the unit respectively by 2, 3, 4, 5, 6, 7, 8.........

EXAMPLE 1. Find the value of $\frac{7}{12}$ of £1.

$\frac{7}{12}$ of £1 means, £1 is to be divided into 12 equal parts and 7 of such parts are to be taken.

£1 ÷ 12 = 1*s*. 8*d*.; and 7 × 1*s*. 8*d*. = 11*s*. 8*d*.

∴ $\frac{7}{12}$ of £1 = 11*s*. 8*d*.

EXAMPLE 2. Find the value of $\frac{5}{22}$ of 19 cwt. 2 qrs. 16 lbs.

22 { 2 | 19 cwt. 2 qrs. 16 lbs.
  11 | 9 cwt. 3 qrs. 8 lbs.
  0 cwt. 3 qrs. 16 lbs. = Quotient by **22**.
  5

Ans. 4 cwt. 1 qr. 24 lbs. = Product by 5.

EXAMPLES.—ORAL.

Give the value of—

1. $\frac{1}{8}$ of £1. 2. $\frac{1}{6}$ of Re. 1. 3. $\frac{1}{9}$ of 1 yd. 4. $\frac{1}{7}$ of 1 qr.
5. $\frac{1}{2}$ of Rs. 2-4-6. 6. $\frac{1}{3}$ of £ 3-6-9. 7. $\frac{1}{4}$ of Rs. 4-8-4. 8. $\frac{1}{2}$ of 9*s*. 6*d*.
9. $\frac{1}{6}$ of 13 as. 6p. 10. $\frac{3}{4}$ of Re. 1. 11. $\frac{3}{4}$ of £1. 12. $\frac{7}{8}$ of Re. 1.
13. $\frac{3}{4}$ of 1 min. 14. $\frac{3}{4}$ of 1 metre. 15. $\frac{2}{5}$ of 1 Kg. 16. $\frac{7}{12}$ of R. 1.
17. $\frac{5}{16}$ of £1. 18. $\frac{9}{16}$ of 1 cwt. 19. $\frac{2}{3}$ of £ 7-5-6. 20. $\frac{3}{4}$ of Rs. 11-0-8.
21. $\frac{7}{10}$ cm. 22. $\frac{3}{100}$ m. 23. $\frac{9}{1000}$ gram. 24. $\frac{139}{1000}$ gram.

EXAMPLES. I.

Find the value of—

1. $\frac{11}{24}$ of Re. 1. 2. $\frac{5}{32}$ of £1. 3. $\frac{7}{8}$ of a guinea.
4. $\frac{5}{8}$ of a pagoda. 5. $\frac{105}{128}$ R. 6. $\frac{5}{16}$ cwt.
7. $\frac{29}{160}$ ton. 8. $\frac{3}{11}$ mile. 9. $\frac{3}{4}$ of Rs. 20-5-0.
10. $\frac{4}{5}$ of Rs. 100-10-5. 11. $\frac{5}{6}$ of Rs. 62-0-6.
12. $\frac{7}{8}$ of £75-4-8. 13. $\frac{8}{9}$ of £120-4-6. 14. $\frac{13}{15}$ of 7 tons.
15. $\frac{5}{38}$ of £78-1-2. 16. $\frac{5}{8}$ of 3 cwt. 2 qrs.

---

180. A fraction represents also the quotient of the numerator divided by the denominator. Thus $\frac{2}{3} = 2 \div 3$.

$\frac{2}{3}$ of 1 palam = 2 tolas; 2 pals. ÷ 3 = 2 tolas.

We get the same result whether we divide one palam into 3 equal parts and take 2 parts or divide 2 palams into 3 equal parts and take one part. In other words twice one-third of the unit is equivalent to one-third of two units.

181. Graphic Illustration.

Take the length AB as the unit and divide it into 3 equal parts at C and D.

AC is one-third of the unit; and AD = twice one-third of the unit.

Now take EF twice the length of AB.

Then EF denotes 2 units. Divide EF into 3 equal parts at G and H. EG is one-third of 2 units.

AD and EG are equal to one another.

∴ *Twice one-third of the unit is equivalent to one-third of 2 units.*

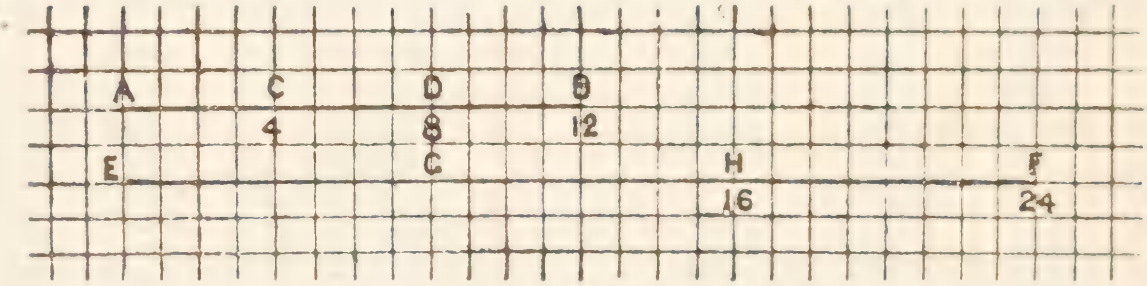

FIG. 18.

182. To shew graphically by rectangles that $\frac{7}{12}$ means 7 times the twelfth part of the unit or the twelfth part of 7 units.

Let the rectangle ABCD which contains 12 squares represent the unit. Then one square represents $\frac{1}{12}$ of the unit; and the rectangle ADEF which contains 7 squares represents 7 times $\frac{1}{12}$ of the unit. Fig. 19, page 151.

The rectangle ABGH, which contains 7 such rectangles as ABCD represents 7 units. The rectangle AMNH is $\frac{1}{12}$ of the rectangle ABGH, i.e. $\frac{1}{12}$ of 7 units.

But the rectangle ADEF = the rectangle AMNH, since each contains 7 squares.

∴ 7 times $\frac{1}{12}$ of the unit is the same as $\frac{1}{12}$ of 7 units.

Practical Exercise.

Draw figures to shew that $\frac{2}{3}$ is equivalent to $\frac{1}{3}$ of 2; $\frac{5}{6}$ is the same as $\frac{1}{6}$ of 5; $\frac{3}{8}$ is equal to $\frac{1}{8}$ of 3; $\frac{7}{9}$ is equivalent to $\frac{1}{9}$ of 7, taking (1) known lengths, (2) rectangles.

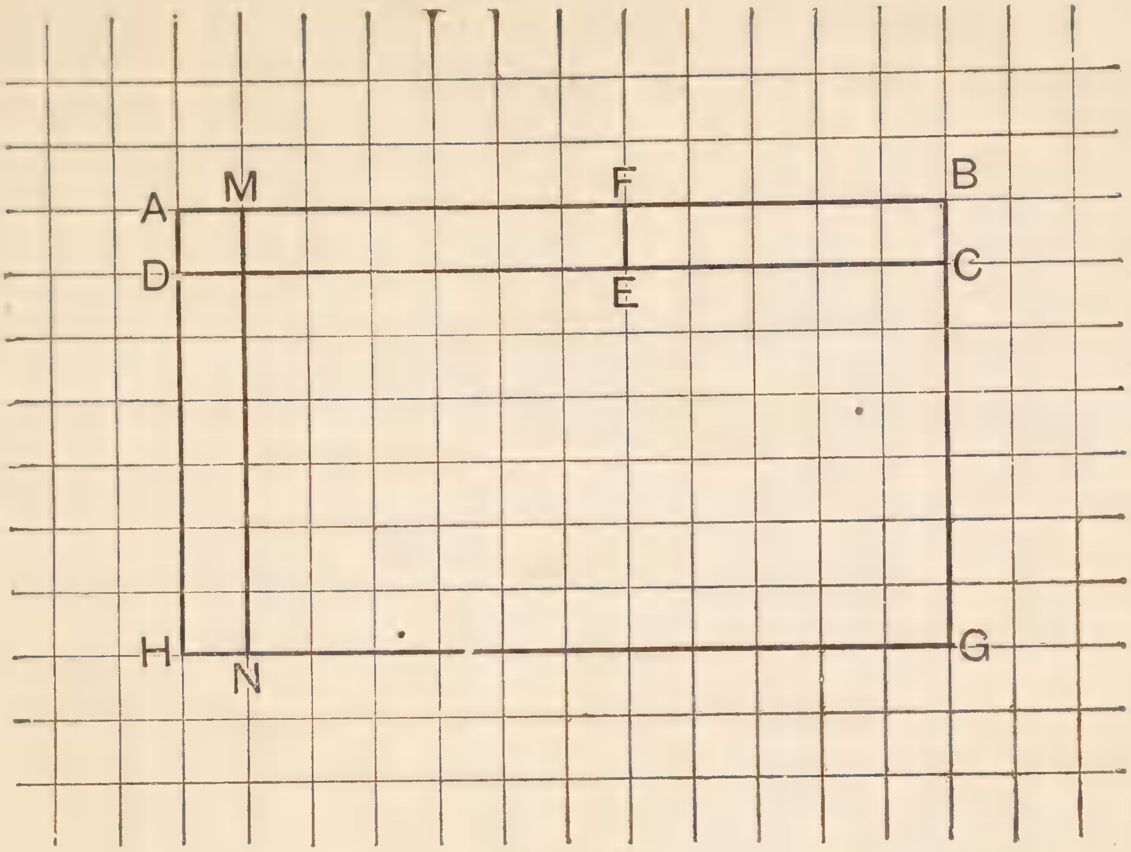

FIG. 19.

EXAMPLE. Find the value of $\frac{12}{17}$ of Rs. 9-14-8.

The value may either be found by first dividing Rs. 9-14-8 by 17 and then multiplying the quotient by 12; or first by multiplying Rs. 9-14-8 by 12 and then dividing the product by 17. Hitherto we have been adopting the first method.

Adopting the second method, Rs. 9-14-8 × 12 = Rs. 119; now dividing Rs. 119 by 17, the value is Rs. 7.

That method should be used which will save time and labour.

EXAMPLES. II.

Find the value of—

1. $\frac{8}{9}$ of £14-12-6.
2. $\frac{6}{11}$ of £9-3-4.
3. $\frac{7}{15}$ of 4 cwt. 1 qr. 4 lbs.
4. $\frac{16}{31}$ of Rs. 7-12 as.
5. $\frac{15}{16}$ of 2 days 3 hrs. 12min.
6. $\frac{16}{19}$ of 10 mds. 1 viss 20 pals.
7. $\frac{8}{23}$ of Rs. 7-10-8.
8. $\frac{24}{25}$ of 5 m. 1 fur. 66 yds. 2 ft.

---

183. Any whole number may be expressed as a fraction, having for its numerator the number itself and for its denominator unity.

Thus $3 = \frac{3}{1}$; $7 = \frac{7}{1}$; $20 = \frac{20}{1}$.

184. Equivalent Fractions Graphically Illustrated.

We may shew that $\frac{3}{4} = \frac{6}{8} = \frac{24}{32}$ as follows :—

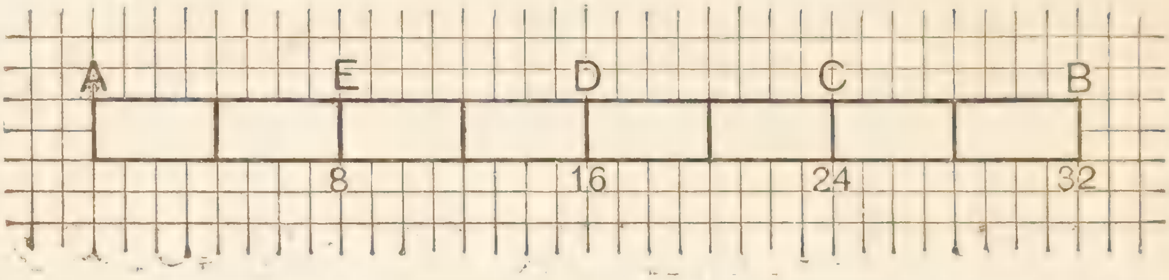

FIG. 20.

Let AB representing one yard be the unit, divide it into 4 equal parts AE, ED, DC, and CB. Then AC is $\frac{3}{4}$ of one yard.

Now divide each of the 4 equal parts into 2 equal parts. The whole yard AB is now divided into 8 equal parts ; and you see that AC contains 6 of the 8 equal parts. Therefore AC is $\frac{6}{8}$ of one yard.

Divide each part again into 4 equal parts.

AB now contains 32 equal parts ; and AC contains 24 of such parts.

$\therefore$ AC is $\frac{24}{32}$ of 1 yard. But AC is also $\frac{3}{4}$ of 1 yard and $\frac{6}{8}$ of 1 yard.

$\therefore \frac{3}{4} = \frac{6}{8} = \frac{24}{32}$.

185. To shew by means of rectangles that $\frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{120}{160}$.

Take the rectangle ABCD as the unit. Divide ABCD into 4 equal parts as shewn in the figure by thick lines. Fig. 21, page 153. Then AEFD contains 3 such parts and represents $\frac{3}{4}$ of the unit.

Divide each part into two as shewn by the dotted lines. ABCD now contains 8 equal parts and AEFD contains 6 of such parts and therefore represents $\frac{6}{8}$ of the unit.

Again the rectangle ABCD contains 16 columns of ten squares each ; and the part AEFD containing 12 columns represents $\frac{12}{16}$ of the unit.

Finally the rectangle ABCD contains 160 small squares.

$\therefore$ 1 square represents $\frac{1}{16}$ of the unit.

$\therefore$ AEFD which contains 120 squares represents $\frac{120}{160}$ of the unit.

$\therefore$ The same figure AEFD represents $\frac{3}{4}$, $\frac{6}{8}$, $\frac{12}{16}$ and $\frac{120}{160}$ of the unit.

$\therefore \frac{3}{4} = \frac{6}{8} = \frac{12}{16} = \frac{120}{160}$; therefore $\frac{3}{4}$, $\frac{6}{8}$, $\frac{12}{16}$ and $\frac{120}{160}$ are equivalent fractions.

In the same way it may be shewn that $\frac{120}{160} = \frac{12}{16} = \frac{6}{8} = \frac{3}{4}$.

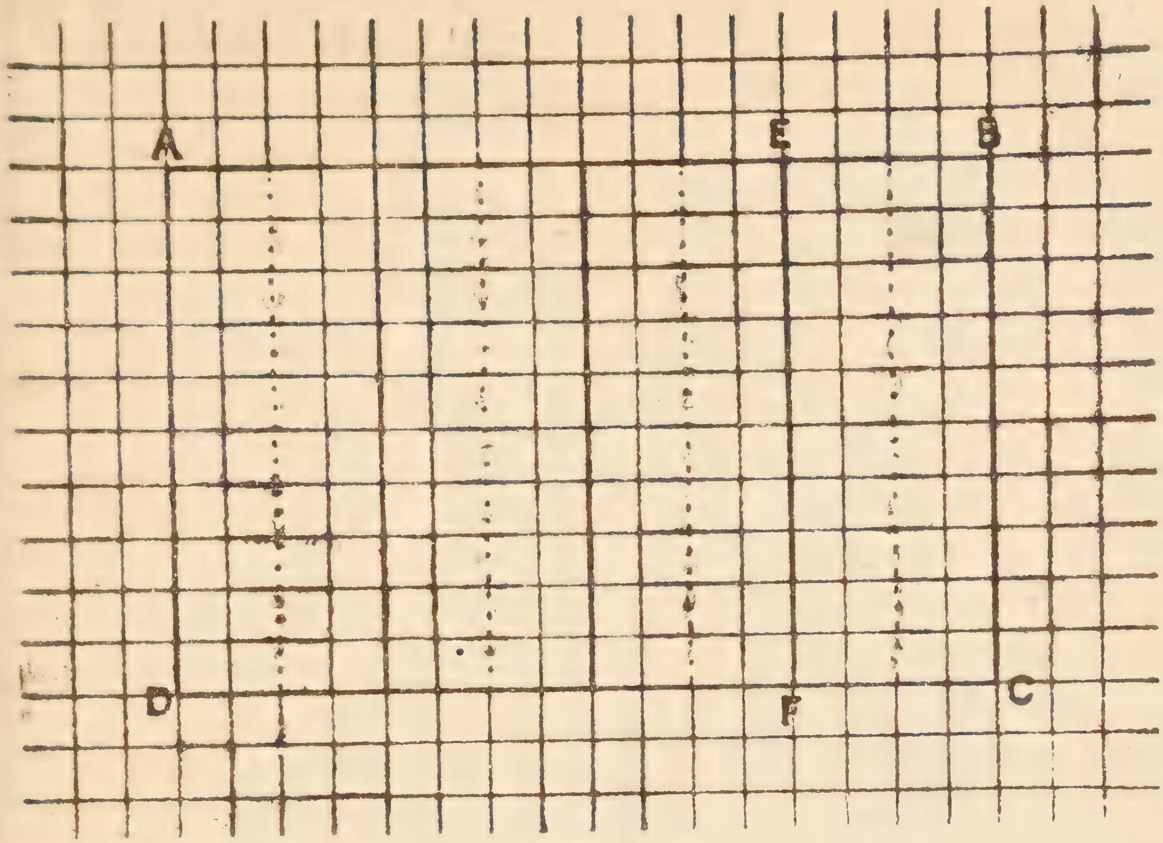

FIG. 21.

Practical Exercise.

1. Take a sheet of paper and doubling it shew that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$.

2. Take a piece of paper and fold it so as to shew that $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$.

3. Take a straight line 4 cm. long as unit and divide it to shew that $\frac{3}{4}$, $\frac{9}{12}$ and $\frac{18}{24}$ are equivalent fractions.

4. Take a straight line 6 in. in length and mark off portions to shew that $\frac{5}{8} = \frac{15}{24} = \frac{30}{48}$.

5. Draw a square and by dividing it into smaller squares, shew that $\frac{1}{4} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64}$.

6. Draw another square and use it to prove that $\frac{2}{3} = \frac{6}{9} = \frac{12}{18} = \frac{24}{36}$.

7. Draw a rectangle 6 in. by 4 in. to represent the unit, and divide it into parts to shew $\frac{1}{4} = \frac{2}{8} = \frac{6}{24}$.

8. Draw 3 rectangles each 6 cm. by 3 cm. to denote the unit, and shade portions to shew $\frac{2}{3} = \frac{6}{9} = \frac{12}{18}$.

9. Shew graphically taking rectangles as units that the following groups of fractions are equivalent :—

(1) $\frac{1}{5}, \frac{3}{15}, \frac{8}{40}$.
(2) $\frac{2}{3}, \frac{8}{12}, \frac{24}{36}$.
(3) $\frac{1}{9}, \frac{4}{36}, \frac{8}{72}$.
(4) $\frac{5}{12}, \frac{15}{36}, \frac{20}{48}$.

186. *Hence we learn* (I) *If the numerator and denominator of a fraction be multiplied by the same number, the value of the fraction is not altered.* Thus $\frac{3}{4}=\frac{6}{8}$; $\frac{5}{9}=\frac{15}{27}$; $\frac{7}{12}=\frac{49}{84}$.

Also $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}$. . . .

(II) *If the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not altered.* Thus $\frac{6}{8}=\frac{3}{4}$; $\frac{12}{18}=\frac{2}{3}$; $\frac{21}{35}=\frac{3}{5}$.

Also $\frac{24}{48}=\frac{12}{24}=\frac{6}{12}=\frac{3}{6}=\frac{1}{2}$.

187. It therefore follows that—

(1) *A whole number may be converted into a fraction with any denominator.*

(2) *A fraction may be changed into another fraction with any denominator or numerator.*

(3) *A fraction may be reduced to its lowest terms by dividing the numerator and denominator by their common factors successively or by their H.C.F.*

EXAMPLE. Express 5 as a fraction with denominator 7.

$$5=\frac{5}{1}=\frac{5\times 7}{1\times 7}=\frac{35}{7}.$$

188. A whole number may be converted into a fraction with any denominator by multiplying the number by the required denominator for the numerator of the fraction, and placing the required denominator underneath.

EXAMPLE. Convert $\frac{5}{7}$ into a fraction with denominator 49.

$$\text{Since } 49 \div 7 = 7,\ \frac{5}{7}=\frac{5\times 7}{7\times 7}=\frac{35}{49}.$$

189. To convert a fraction into an equivalent fraction with a denominator, that is, a multiple of the denominator of the given fraction :

Divide the new denominator by the denominator of the given fraction ; and multiply both the numerator and the denominator of the given fraction by the quotient.

EXAMPLE. Convert $\frac{4}{9}$ into a fraction with numerator 64.

$$\text{Since } 64 \div 4 = 16,\ \frac{4}{9}=\frac{4\times 16}{9\times 16}=\frac{64}{144}.$$

190. To convert a fraction into an equivalent fraction with a numerator, that is, a multiple of the numerator of the given fraction:

Divide the new numerator by the numerator of the given fraction; and multiply both the numerator and the denominator of the given fraction by the quotient.

EXAMPLES. III.

1. Express 21, 32, 75, 81 each as a fraction with denominator (1) 19, (2) 31.

2. Express 11, 19, 23, 31 as (1) eighths, (2) elevenths.

3. (*a*) Express (1) $\frac{9}{5}$ and (2) $\frac{7}{25}$ each as an equivalent fraction with denominator (1) 75, (2) 125, (3) 150, (4) 900.

(*b*) Convert (1) $\frac{5}{9}$; (2) $\frac{4}{7}$; (3) $\frac{1}{2}$; (4) $\frac{20}{63}$ into equivalent fractions with denominator 126.

4. Transform (1) $\frac{6}{17}$; (2) $\frac{9}{20}$ into equivalent fractions with numerator (1) 18; (2) 72; (3) 90; (4) 108.

191. A fraction that has its numerator less than its denominator is called a **Proper Fraction.**

EXAMPLE. $\frac{3}{4}$; $\frac{5}{9}$; $\frac{11}{16}$; $\frac{13}{27}$.

192. A fraction that has its numerator greater than its denominator is called an **Improper Fraction.**

EXAMPLE. $\frac{39}{13}$; $\frac{17}{15}$; $\frac{27}{23}$.

193. A number which consists of the sum of a whole number and a fraction is called a **Mixed Number.**

EXAMPLE. $(6+\frac{2}{5})$ is a mixed number.

The notation that is adopted is $6\frac{2}{5}$, which is a shortened form of $6+\frac{2}{5}$.

194. To express a mixed number as an improper fraction.

EXAMPLE. To express $6\frac{2}{5}$ as an improper fraction.

$$6=\frac{6\times 5}{1\times 5}=\frac{30}{5}=30 \text{ fifths.}$$

$$\frac{2}{5}=2 \text{ fifths.}$$

$$\text{Adding, } 6\frac{2}{5}=30 \text{ fifths}+2 \text{ fifths}=32 \text{ fifths}$$
$$=\frac{32}{5}.$$

Hence we have the following rule to express a mixed number as an improper fraction :—Multiply the whole number by the denominator of the fractional part, and to the product add the numerator of the fractional part ; the result will be the required numerator, and the denominator of the fractional part will be the required denominator.

EXAMPLE. Find the value of $31\frac{11}{12}$ units when the unit is Rs. 5.

In other words, find the value of $31\frac{11}{12}$ of Rs. 5.

Since $31\frac{11}{12} = 31 + \frac{11}{12}$, $31\frac{11}{12}$ of Rs. $5 = 31$ times Rs. $5 + \frac{11}{12}$ of Rs. 5 = Rs. 155 + Rs. 4-9-4 = Rs. 159-9-4.

Or since $31\frac{11}{12} = \dfrac{31 \times 12 + 11}{12} = \dfrac{372 + 11}{12} = \dfrac{383}{12}$, find the value of $\dfrac{383}{12}$ of the unit ; that is, multiply Rs. 5 by 383 and divide the product by 12.

EXAMPLES.—ORAL.

Express the following mixed numbers as improper fractions :—

1. $6\frac{2}{3}$. 2. $4\frac{3}{7}$. 3. $5\frac{5}{8}$. 4. $7\frac{2}{9}$. 5. $8\frac{3}{10}$. 6. $9\frac{7}{9}$.
7. $7\frac{3}{11}$. 8. $11\frac{7}{12}$. 9. $9\frac{9}{13}$. 10. $7\frac{4}{15}$. 11. $3\frac{9}{17}$. 12. $10\frac{5}{16}$.

Find the value of—

1. $2\frac{1}{2}$ of 6 p. 2. $3\frac{1}{4}$ of 3*d*. 3. $4\frac{3}{4}$ of 4 as. 4. $5\frac{11}{12}$ of 1*s*.
5. $6\frac{7}{8}$ of Re. 1. 6. $3\frac{1}{6}$ of 2 as. 7. $2\frac{3}{4}$ of one ton. 8. $3\frac{7}{8}$ of 1 viss.
9. $4\frac{5}{8}$ of 1 mile. 10. $1\frac{3}{16}$ of Rs. 2. 11. $1\frac{9}{20}$ of £4. 12. $1\frac{7}{12}$ of 15 as.

EXAMPLES. IV.

Find the value of—

1. $3\frac{5}{12}$ of £5. 2. $17\frac{7}{8}$ of 12 grams.
3. $3\frac{3}{16}$ of £258-9-4. 4. 4^{1} of Rs. 170-0-3.
5. $5\frac{\ }{15}$ of Rs. 181-15-3.
6. $2\frac{1}{7}$ of 6 tons 12 cwt. 3 qrs. 21 lbs.
7. $27\frac{1}{3}$ of 4 days 6 hrs. 20 min.
8. $5\frac{1}{6}$ of 12 can. 6 mds. 3 pals.
9. Express the following mixed numbers as improper fractions :—

(1) $54\frac{23}{45}$. (2) $78\frac{37}{85}$. (3) $29\frac{59}{101}$. (4) $75\frac{200}{403}$.
(5) $529\frac{79}{317}$. (6) $678\frac{503}{905}$. (7) $777\frac{564}{987}$. (8) $876\frac{803}{919}$.

195. Conversion of Improper Fractions into Integers or mixed numbers.

EXAMPLE. Express $\frac{39}{13}$ as a whole number.

$\frac{39}{13}$ means $39 \div 13$ [See Art. 180] $39 \div 13 = 3$.

To express $\frac{27}{8}$ inches in inches and fraction of an inch.

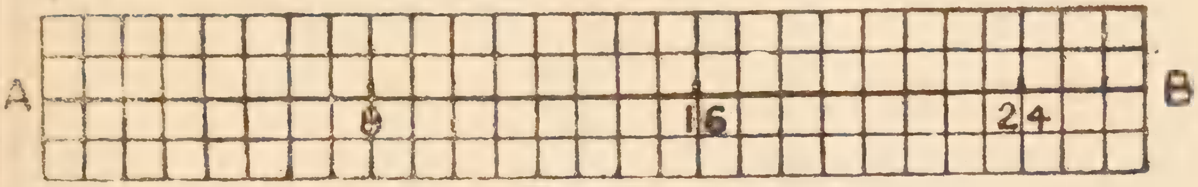

FIG. 22.

Take the length AB; divide it into 27 equal parts, each $\frac{1}{8}$ in.

Taking lengths of 8 divisions or 1 inch, we see there are 3 such lengths and 3 divisions over, i.e. there are 3 inches and $\frac{3}{8}$ of an inch left.

$\therefore$ $\frac{27}{8}$ in. $= 3$ in. $+ \frac{3}{8}$ in. $= 3\frac{3}{8}$ in.

EXAMPLE. Express $\frac{27}{7}$ as a mixed number.

$$\frac{27}{7} = 27 \text{ sevenths.}$$

$$21 \text{ sevenths} = \frac{21}{7} = 3.$$

$$6 \text{ sevenths} = \frac{6}{7}.$$

Adding, 27 sevenths or $\frac{27}{7} = 3 + \frac{6}{7} = 3\frac{6}{7}$.

We see from these examples that an improper fraction is either equal to a whole number or to a whole number + a proper fraction.

To express an improper fraction as a mixed number:

Divide the numerator by the denominator; put the quotient as the integer, the remainder as the numerator, and the original denominator as the denominator of the fraction.

EXAMPLE. Express $\frac{1021}{69}$ as a mixed number.

$1021 \div 69$ gives a quotient 14 and remainder 55.

$\therefore \frac{1021}{69} = 14\frac{55}{69}$.

EXAMPLES.—ORAL.

Express the following improper fractions as whole or mixed numbers:—

1. $\frac{11}{3}$. 2. $\frac{17}{4}$. 3. $\frac{19}{5}$. 4. $\frac{29}{6}$. 5. $\frac{13}{7}$.
6. $\frac{43}{8}$. 7. $\frac{54}{9}$. 8. $\frac{63}{10}$. 9. $\frac{79}{11}$. 10. $\frac{84}{12}$.
11. $\frac{91}{13}$. 12. $\frac{100}{14}$. 13. $\frac{90}{15}$. 14. $\frac{120}{16}$. 15. $\frac{201}{20}$.

Practical Exercise.

Verify graphically the answers to the above examples by taking straight lines or using rectangles.

EXAMPLES. V.

Reduce the following improper fractions to whole numbers or mixed numbers :—

1. $\frac{2345}{127}$. 2. $\frac{3456}{173}$. 3. $\frac{10003}{295}$. 4. $\frac{89012}{901}$.
5. $\frac{901234}{3705}$. 6. $\frac{987031}{2468}$. 7. $\frac{879323}{4567}$. 8. $\frac{8571420}{142857}$.

Reduction of Fractions to lowest terms.

196. A fraction is in its lowest terms when its numerator and denominator are prime to each other ; i.e. when they have no common factors.

EXAMPLE. $\frac{7}{8}$, $\frac{13}{35}$, $\frac{29}{72}$, and $\frac{87}{121}$.

197. A fraction may be reduced to its lowest terms by dividing its numerator and denominator successively by their common factors. (See Art. 187.)

EXAMPLE. Reduce $\frac{5184}{6912}$ to its lowest terms.

Dividing successively by the factors 8, 9, 3, 8, we have $\frac{5184}{6912} = \frac{648}{864} = \frac{72}{96} = \frac{24}{32} = \frac{3}{4}$.

198. To reduce a fraction to its lowest terms, we may also divide the numerator and the denominator by their G.C.M., the quotient resulting from the numerator is the numerator, and that resulting from the denominator is the denominator of the fraction in its lowest terms.

EXAMPLE. Reduce $\frac{95469}{359784}$ to its lowest terms.

The G.C.M. of 95469 and 359784 is 789.
Numerator $\div$ G.C.M. $= 95469 \div 789 = 121$.
Denominator $=$ G.C.M. $= 359784 \div 789 = 456$.
$\therefore$ $\frac{121}{456}$ is the simplest form of the given fraction.

EXAMPLES.—ORAL.

Reduce to lowest terms—

1. $\frac{6}{15}$. 2. $\frac{12}{20}$. 3. $\frac{10}{18}$. 4. $\frac{12}{16}$. 5. $\frac{1}{2}$.
6. $\frac{14}{21}$. 7. $\frac{14}{18}$. 8. $\frac{13}{26}$. 9. $\frac{16}{18}$. 10. $\frac{25}{30}$.
11. $\frac{14}{28}$. 12. $\frac{24}{32}$. 13. $\frac{20}{30}$. 14. $\frac{24}{36}$. 15. $\frac{30}{45}$.

EXAMPLES. VI.

Reduce to lowest terms by dividing by common factors—

1. $\frac{192}{576}$. 2. $\frac{568}{808}$. 3. $\frac{1680}{1920}$. 4. $\frac{1152}{1760}$.

5. $\frac{1065}{1515}$. 6. $\frac{3564}{8415}$. 7. $\frac{5643}{8217}$. 8. $\frac{5825}{1155}$.

Reduce to lowest terms by dividing by the H.C.F.—

9. $\frac{3444}{3556}$. 10. $\frac{11385}{16335}$. 11. $\frac{2021}{6407}$. 12. $\frac{667}{1219}$.

13. $\frac{15602}{28797}$. 14. $\frac{1955}{2001}$. 15. $\frac{12543}{71416}$. 16. $\frac{7319}{9971}$.

Express the following as mixed numbers, reducing the fractional parts to their lowest terms :—

17. $\frac{12345}{375}$. 18. $\frac{45678}{564}$. 19. $\frac{56781}{765}$. 20. $\frac{56763}{765}$.

21. $\frac{67896}{837}$. 22. $\frac{79002}{931}$. 23. $\frac{789173}{5698}$. 24. $\frac{679675}{7893}$.

---

199. Reduction of Fractions to a common denominator.

It is convenient that this new common denominator should be as small as possible ; so we find the number which is the L.C.M. of the denominators, for then each denominator can be altered into that number by multiplication.

If any of the fractions can be reduced to lowest terms, they must be simplified before expressing them as fractions having the least common denominator.

EXAMPLE. Express $\frac{1}{9}$, $\frac{5}{8}$, $\frac{7}{12}$, as fractions with a common denominator.

The L.C.M. of 9, 8 and 12 is 72.

Since $72 \div 9 = 8$, $\frac{1}{9} = \frac{1 \times 8}{9 \times 8} = \frac{8}{72}$. Since $72 \div 8 = 9$, $\frac{5}{8} = \frac{5 \times 9}{8 \times 9} = \frac{45}{72}$.

Since $72 \div 12 = 6$, $\frac{7}{12} = \frac{7 \times 6}{12 \times 6} = \frac{42}{72}$, $\frac{8}{72}$, $\frac{45}{72}$ and $\frac{42}{72}$ Ans.

EXAMPLES. VII.

Reduce to fractions having the least common denominator—

1. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$. 2. $\frac{3}{8}$, $\frac{4}{9}$, $\frac{1}{2}$, $\frac{6}{11}$, $\frac{7}{12}$.

3. $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{3}{5}$. 4. $\frac{7}{16}$, $\frac{11}{18}$, $\frac{17}{24}$, $\frac{19}{36}$.

Reduce to lowest terms where possible and then express as fractions having the least common denominator—

5. $\frac{7}{9}, \frac{10}{15}, \frac{12}{18}$. 6. $\frac{12}{15}, \frac{11}{12}, \frac{15}{20}$. 7. $\frac{11}{14}, \frac{14}{28}, \frac{7}{49}$. 8. $\frac{36}{60}, \frac{45}{75}, \frac{21}{36}$.

200. Comparison of Fractions. Fractions may be compared—

(1) By reducing them to a common denominator; then that which has the greatest numerator is the greatest fraction; and that which has the least is the least fraction.

(2) By reducing them to a common numerator; then that which has the least denominator is the greatest fraction; and that which has the greatest denominator is the least.

EXAMPLE. Which is the greater $\frac{5}{8}$ or $\frac{7}{12}$?

The L.C.M. of 8 and 12 is 24.

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}; \quad \frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}.$$

Since $\frac{15}{24}$ is greater than $\frac{14}{24}$, $\frac{5}{8}$ is greater than $\frac{7}{12}$.

Or working mentally:

$$\frac{12 \times 5 \text{ or } 7 \times 8}{8 \times 12}, \text{ i.e. } \frac{60 \text{ or } 56}{8 \times 12} \quad \frac{60}{8 \times 12}, \text{ i.e. } \frac{5}{8} \text{ is greater.}$$

EXAMPLE. Compare the values of the following fractions:—

$\frac{9}{24}$, $\frac{7}{10}$ and $\frac{10}{18}$.

The fractions in their lowest terms are $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{5}{9}$.

The L.C.M. of 8, 10 and 9 is 360.

$$\frac{3}{8} = \frac{3 \times 45}{360} = \frac{135}{360}; \quad \frac{7}{10} = \frac{7 \times 36}{360} = \frac{252}{360}; \quad \frac{5}{9} = \frac{5 \times 40}{360} = \frac{200}{360}.$$

Hence $\frac{7}{10}$ is the greatest, $\frac{10}{18}$ the next, and $\frac{9}{24}$ is the least.

201. Graphic Representation. (Fig. 23, p. 161.)

Take the rectangle ABCD containing 30 × 12 or 360 squares as the unit (360 being the L.C.M. of the denominators of the fractions).

The rectangle AEFG containing 135 squares represents $\frac{135}{360}$ or $\frac{3}{8}$ of the unit.

The rectangle ALMN containing 200 squares represents $\frac{200}{360}$ or $\frac{5}{9}$ of the unit; and the rectangle AEHK containing 252 squares represents $\frac{252}{360}$ or $\frac{7}{10}$ of the unit.

We see at a glance that the rectangle AEHK is the largest of the three, next to it comes the rectangle ALMN and the smallest is the rectangle AEFG.

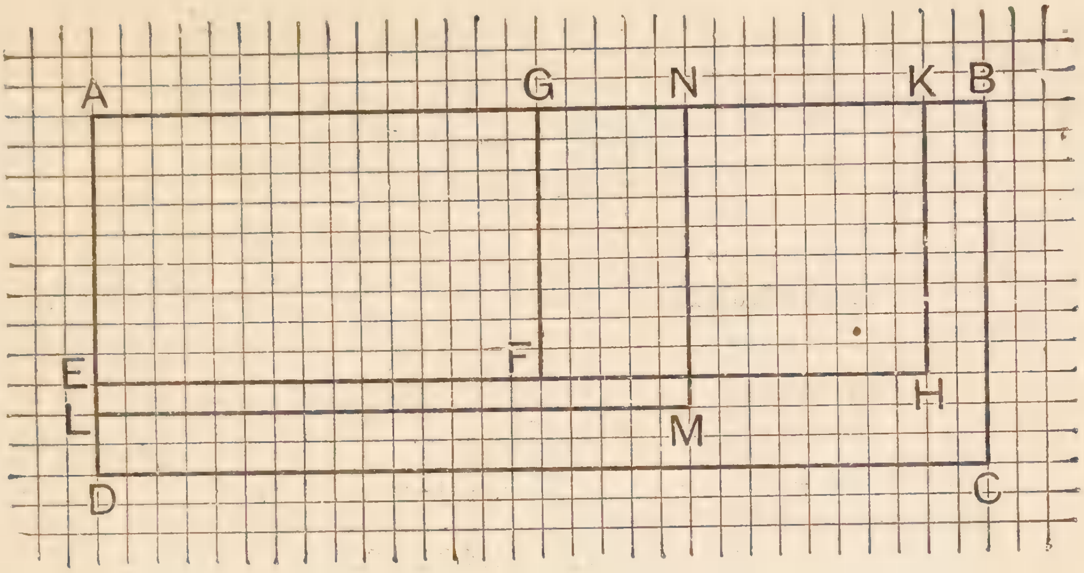

FIG. 23.

Arranged in the ascending order of magnitude, the fractions would stand thus $\frac{9}{24}$, $\frac{10}{18}$, $\frac{7}{10}$.

EXAMPLE. Which is greater $\frac{5}{32}$ or $\frac{8}{49}$?

Reduce the fractions to a common numerator.

The L.C.M. of 5 and 8 is 40.

$$\frac{5}{32}=\frac{5\times 8}{32\times 8}=\frac{40}{256}\text{; and }\frac{8}{49}=\frac{8\times 5}{49\times 5}=\frac{40}{245}.$$

$$\therefore \frac{8}{49}\text{ is greater than }\frac{5}{32}.$$

202. In cases where the L.C.M. of either the numerator or the denominator is not easily found, and even in other cases, unless the fractions given are almost equal to one another, the following simple method may be used :—

Express each fraction with 1 as numerator ; that is, divide both the numerator and denominator of each fraction by the numerator ; and then compare.

EXAMPLE. Compare (1) $\frac{3}{8}$, $\frac{7}{10}$ and $\frac{5}{9}$, (2) $\frac{21}{173}$ and $\frac{23}{187}$.

Expressing each fraction with 1 as numerator, $\frac{3}{8}$, $\frac{7}{10}$ and $\frac{5}{9}$ are respectively equal to $\frac{1}{2\frac{2}{3}}$, $\frac{1}{1\frac{3}{7}}$ and $\frac{1}{1\frac{4}{5}}$.

Of the denominators $1\frac{3}{7}$ is the least and $2\frac{2}{3}$ is the greatest.

$\therefore$ $\frac{7}{10}$ is the greatest ; and $\frac{3}{8}$ is the least.

$$(2)\ \frac{21}{173}=\frac{1}{173\div 21}=\frac{1}{8\frac{5}{21}}\text{; }\frac{23}{187}=\frac{1}{187\div 23}=\frac{1}{8\frac{3}{23}}.$$

Since $\frac{5}{21}$ is nearly $\frac{1}{4}$; and $\frac{3}{23}$ is nearly $\frac{1}{8}$, $8\frac{3}{23}$ is smaller than $8\frac{5}{21}$.

$\therefore \dfrac{1}{8\frac{3}{23}}$ is greater than $\dfrac{1}{8\frac{5}{21}}$; i.e. $\dfrac{23}{187}$ is greater than $\dfrac{21}{173}$.

EXAMPLES.—ORAL.

I. Compare from inspection—

1. $\frac{93}{100}, \frac{93}{121}, \frac{93}{99}$; 2. $\frac{87}{100}, \frac{87}{89}, \frac{87}{120}$;
3. $\frac{111}{1120}, \frac{111}{1020}, \frac{111}{1315}, \frac{111}{913}$; 4. $\frac{99}{1001}, \frac{99}{1032}, \frac{99}{807}, \frac{99}{1405}$.

II. Which is the greater—

1. $\frac{2}{3}$ or $\frac{5}{7}$? 2. $\frac{3}{4}$ or $\frac{5}{6}$? 3. $\frac{5}{6}$ or $\frac{5}{8}$? 4. $\frac{5}{9}$ or $\frac{3}{7}$?
5. $\frac{7}{8}$ or $\frac{8}{9}$? 6. $\frac{6}{7}$ or $\frac{7}{9}$? 7. $\frac{3}{9}$ or $\frac{5}{12}$? 8. $\frac{7}{12}$ or $\frac{5}{9}$?
9. $\frac{11}{10}$ or $\frac{6}{5}$? 10. $\frac{8}{11}$ or $\frac{7}{9}$? 11. $\frac{12}{11}$ or $\frac{13}{12}$? 12. $\frac{8}{12}$ or $\frac{7}{11}$?

Practical Exercise.

Draw diagrams to verify your answers to Examples II. 1 to 12.

EXAMPLES. VIII.

1. Which is greater—

(1) $\frac{5}{9}$ or $\frac{6}{13}$? (2) $\frac{13}{15}$ or $\frac{23}{25}$?
(3) $\frac{18}{18}$ or $\frac{15}{16}$? (4) $\frac{19}{32}$ or $\frac{17}{36}$?

2. Arrange in the ascending order of magnitude—

(*a*) $\frac{10}{21}, \frac{8}{20}, \frac{9}{7}, \frac{7}{8}$; (*b*) $\frac{9}{12}, \frac{12}{21}, \frac{13}{10}, \frac{5}{6}$;
(*c*) $\frac{11}{12}, \frac{12}{10}, \frac{4}{13}, \frac{7}{26}$; (*d*) $\frac{7}{8}, \frac{13}{12}, \frac{5}{7}, \frac{20}{21}$.

3. Arrange in the descending order of magnitude—

(*a*) $\frac{3}{11}, \frac{11}{12}, \frac{12}{121}, \frac{7}{24}$; (*b*) $\frac{7}{23}, \frac{8}{30}, \frac{5}{69}, \frac{6}{45}$;
(*c*) $\frac{12}{13}, \frac{13}{39}, \frac{9}{26}, \frac{10}{11}$; (*d*) $\frac{13}{14}, \frac{5}{8}, \frac{75}{100}, \frac{21}{28}$.

4. Reduce to fractions having the least common numerator and compare—

(1) $\frac{21}{22}, \frac{7}{10}, \frac{13}{16}$. (2) $\frac{101}{210}, \frac{21}{100}, \frac{303}{410}$.
(3) $\frac{5}{7}, \frac{6}{11}, \frac{7}{12}, \frac{8}{9}$, (4) $\frac{4}{5}, \frac{5}{7}, \frac{6}{13}, \frac{7}{10}$.

---

203. If the numerators of two fractions be added for a numerator and the denominators for a denominator the resulting fraction lies in value between the two fractions from which it is derived.

Thus of the fractions $\frac{3}{4}$, $\frac{2}{3}$ and $\dfrac{3+2}{4+3}$, it may be shown that the third is less than the first and greater than the second.

EXAMPLES. IX.

Shew that—

1. $\frac{2+3}{5+7}$ is greater than $\frac{2}{5}$ and less than $\frac{3}{7}$.
2. $\frac{4+5}{9+8}$ is greater than $\frac{4}{9}$ and less than $\frac{5}{8}$.
3. $\frac{7+9}{11+13}$ is greater than $\frac{7}{11}$ and less than $\frac{9}{13}$.
4. $\frac{2+4+6}{3+5+7}$ lies between the greatest and least of the fractions $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$.

---

204. A compound quantity in a lower denomination expressed as the fraction of another of a higher denomination.

EXAMPLE. What fraction is (1) 1 an. 2 p. of R. 1 ? (2) $7\frac{1}{2}$*d.* of £1 ?

(1) Since 1 an. 2 p. = 14 p., and R. 1 = 192 p.,

1 an. 2 p. is $\frac{14}{192}$ or $\frac{7}{96}$ of R. 1.

(2) Since $7\frac{1}{2}$*d.* = 30 farthings, and £1 = 960 farthings,

$7\frac{1}{2}$*d.* is $\frac{30}{960}$ or $\frac{1}{32}$ of £1.

EXAMPLES.—ORAL.

What fraction is—

1. 7*s.* of £1.
2. 1 an. 6 p. of R. 1.
3. 2*s.* 6*d.* of £1.
4. 3*f.* of 1*s.*
5. 6*d.* of £1.
6. 8 p. of R. 1.
7. 7 lb. of 1 qr.
8. 3 sec. of 1 min.
9. 1 ft. 3 in. of 1 yd.
10. 7 hrs. of 1 day.
11. 3*s.* 6*d.* of half a guinea.
12. 5*s.* 6*d.* of £1.
13. 7 cm. of 1 m.
14. 9 cg. of 1 g.
15. 23 mm. of 1 m.
16. 6 chains of 1 mi.

EXAMPLES. X.

What fraction is—

1. 15 as. 10 p. of R. 1 ?
2. 16*s.* $6\frac{1}{4}$*d.* of £1 ?
3. 5 fur. 22 yds. of 1 mi. ?
4. 3 qrs. 26 lbs. of 1 ton ?
5. 3 fur. 7 ch. 72 l. of 1 mi. ?
6. 2 hrs. 15 min. of 1 day ?
7. Rs. 4-13-9 of R. 1 ?
8. £2-8-$6\frac{3}{4}$ of £1 ?

---

205. Conversion of Decimals to Fractions.

Since $\cdot 7 \times 10 = 7$, $\cdot 7 = \frac{7}{10}$; and

since $\cdot 123 \times 1000 = 123$, $\cdot 123 = \frac{123}{1000}$ or $\frac{123}{10^3}$.

Similarly 3·27 $= 3\frac{27}{100} = \frac{327}{100}$ or $\frac{327}{10^2}$; and

34·0567 $= 34\frac{0567}{10000} = \frac{340567}{10000}$ or $\frac{340567}{10^4}$.

We see that in each case the equivalent vulgar fraction has for its numerator the number formed by the figures in the decimal and for its denominator, 1 followed by as many cyphers as there are decimal figures, or the power of 10 corresponding to the number of decimal figures.

206. The equivalent vulgar fraction should always be expressed in the simplest form by reducing it to its lowest terms.

EXAMPLE. 0·0125 $= \frac{125}{10000} = \frac{25}{2000} = \frac{5}{400} = \frac{1}{80}$.

It will be useful to remember the following equivalents :—

·5 $= \frac{1}{2}$; ·25 $= \frac{1}{4}$; ·75 $= \frac{3}{4}$.

·125 $= \frac{1}{8}$; ·325 $= \frac{3}{8}$; ·625 $= \frac{5}{8}$; ·875 $= \frac{7}{8}$.

EXAMPLE. Express 0·1875 as a fraction in its lowest terms.

0·1875 $=$ ·1$\frac{7}{8} = \frac{15}{80} = \frac{3}{16}$.

EXAMPLES.—ORAL.

Express as vulgar fractions the following decimals :—

1. (1) ·7; (2) ·23; (3) ·09; (4) ·073.
2. (1) 4·3; (2) 5·01; (3) 9·759; (4) 13·709.

Convert into equivalent vulgar fractions in their lowest terms—

3. ·4; ·5; ·6; ·8. 4. ·12; ·16; ·15; ·18.
5. ·25; ·35; ·45; ·75. 6. ·24; ·36; ·48; ·64.

Convert into vulgar fractions in their lowest terms—

7. (1) ·125; (2) ·875; (3) ·0075; (4) ·675.
8. (1) ·001875; (2) ·5675; (3) ·03725; (4) ·2375.

EXAMPLES. XI.

Express as vulgar fractions in their lowest terms :—

1. 0·56. 2. 0·104. 3. 0·984. 4. 1·03775.
5. 5·3675. 6. 0·21875. 7. 30·2825. 8. 105·6875.
9. 0·075375. 10. 0·0390625. 11. 0·048828125. 12. 1·9287109375

---

207. Conversion of Vulgar Fractions to Decimals.

Since a fraction denotes the division of the numerator by the denominator, to convert a fraction to its equivalent decimal, divide the numerator by the denominator expressing the result as a decimal.

(1) When the fraction has some power of 10 for its denominator, it is simply a case of locating the decimal point.

Thus $\frac{327}{100}$ or $\frac{327}{10^2} = 3{\cdot}27$.

Similarly, $\frac{1}{10} = {\cdot}1$; $\frac{807}{100} = \frac{807}{10^2} = 8{\cdot}07$; $\frac{3}{10000} = \frac{3}{10^4} = {\cdot}0003$.

(2) When the denominator does not contain any power of 10, reduce the fraction to its lowest terms, when this can be done; and then proceed with division.

EXAMPLE. Express (1) $\frac{11}{64}$, (2) $\frac{7}{15}$ as decimals.

$$\begin{array}{r|l} 8 & 11{\cdot}000 \\ \hline 8 & 1{\cdot}375 \\ \hline & 0{\cdot}171875 \end{array} \qquad\qquad \begin{array}{r|l} 15 & 7{\cdot}0000 \\ \hline & 0{\cdot}4666\ldots\ldots\ldots \end{array}$$

$\frac{11}{64} = 0{\cdot}171875$. $\qquad$ $\frac{7}{15} = 0{\cdot}4666\ldots\ldots\ldots$

The decimal, ·171875 is called a Terminating decimal, because there is an end to the process of division.

The decimal, ·4666 is an example of Recurring decimals. It is so called because the division does not end and the figure 6 recurs or appears again and again times without number.

208. Easy fractions can be expressed as decimals mentally by multiplying both the numerator and denominator by a suitable factor so as to make the denominator a power of 10.

Mental steps $\qquad\qquad$ Mental steps

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = {\cdot}4; \quad \frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = {\cdot}35$$

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = {\cdot}75; \quad \frac{11}{125} = \frac{11 \times 8}{125 \times 8} = \frac{88}{1000} = {\cdot}088$$

EXAMPLES.—ORAL.

Express as decimals—

1. $\frac{303}{10}$. 2. $\frac{74}{10^2}$. 3. $\frac{209}{10^2}$. 4. $\frac{13}{10^3}$. 5. $\frac{7}{10^3}$.

6. $\frac{1875}{10^4}$. 7. $\frac{307}{1000}$. 8. $\frac{5143}{10000}$. 9. $\frac{9207}{1000}$. 10. $\frac{5203}{1000}$.

11. $\frac{9178}{100}$. 12. $\frac{3014}{10}$. 13. $\frac{71}{10000}$. 14. $\frac{31}{1000}$. 15. $\frac{502}{100000}$.

Convert the following vulgar fractions to their equivalent decimals :—

$\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{19}{50}$, $\frac{9}{25}$, $\frac{13}{20}$, $\frac{17}{25}$, $\frac{39}{40}$, $\frac{37}{50}$, $\frac{23}{25}$, $\frac{9}{400}$, $\frac{17}{400}$, $\frac{13}{125}$, $\frac{1}{125}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$.

EXAMPLES. XII.

Convert the following Vulgar Fractions into their equivalent Decimals :—

1. $\frac{15}{16}$; $\frac{11}{32}$; $\frac{17}{64}$; $\frac{1}{128}$;
2. $\frac{3}{256}$; $\frac{21}{96}$; $\frac{49}{224}$; $\frac{15}{384}$.
3. $\frac{171}{720}$; $\frac{25}{512}$; $\frac{588}{78125}$; $\frac{1975}{1024}$.
4. Express as decimals to two places—$7\frac{17}{36}$; $\frac{29}{108}$; $\frac{71}{96}$; $3\frac{29}{81}$.

CHAPTER XVII.

ADDITION AND SUBTRACTION OF FRACTIONS.

209. Fractions with the same denominator.

EXAMPLE. Add together three-sixteenths of a rupee, five-sixteenths of a rupee and seven-sixteenths of a rupee.

$$\frac{3}{16} \text{ of R. } 1 + \frac{5}{16} \text{ of R. } 1 + \frac{7}{16} \text{ of R. } 1.$$

$$= 3 \text{ as.} + 5 \text{ as.} + 7 \text{ as.} = 15 \text{ as.} = \frac{15}{16} \text{ of R. } 1 = \frac{15}{16} \text{ R.}$$

$$\therefore \frac{3}{16} \text{ R.} + \frac{5}{16} \text{ R.} + \frac{7}{16} \text{ R.} = \frac{3+5+7}{16} \text{ R.} = \frac{15}{16} \text{ R.}$$

$$\therefore \frac{3}{16} + \frac{5}{16} + \frac{7}{16} = \frac{3+5+7}{16} = \frac{15}{16}.$$

$$\text{Also, } \frac{7}{16} \text{ R.} - \frac{5}{16} \text{ R.} = \frac{7-5}{16} \text{ R.} = \frac{2}{16} \text{ R.} = \frac{1}{8} \text{ R.}$$

$$\therefore \frac{7}{16} - \frac{5}{16} = \frac{7-5}{16} = \frac{2}{16} = \frac{1}{8}.$$

The addition or subtraction of fractions with the same denominator is done by taking the sum or the difference of the numerators.

EXAMPLE. (1) Add together $\frac{8}{13}, \frac{5}{13}, \frac{19}{13}$, (2) Take $\frac{5}{13}$, from $\frac{8}{13}$.

$$\text{The sum} = \frac{8+5+19}{13} = \frac{32}{13} = 2\frac{6}{13}.$$

$$\text{The difference} = \frac{8}{13} - \frac{5}{13} = \frac{8-5}{13} = \frac{3}{13}.$$

It will be useful to remember the following :—

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$; $\frac{3}{4} + \frac{1}{4} = 1$; $\frac{1}{2} + \frac{1}{2} = 1$; $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$; $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$.

EXAMPLE. Add together (1) $\frac{3}{4} + \frac{1}{2} + \frac{1}{4}$; (2) $7\frac{3}{4} + 8\frac{3}{4} + 12\frac{1}{2}$.

(1) $\frac{3}{4} + \frac{1}{4} = 1$; $1 + \frac{1}{2} = 1\frac{1}{2}$;

(2) $7 + 8 + 12 + \frac{3}{4} + \frac{3}{4} + \frac{1}{2} = 27 + 2 = 29$.

These steps should be done mentally and only the answer given out.

EXAMPLES.—ORAL.

Find the value of—

1. $\frac{1}{2}$R. + $\frac{1}{4}$R. + $\frac{1}{2}$R. 2. $\frac{1}{4}$£ + $\frac{3}{4}$£ + $\frac{1}{2}$£. 3. $\frac{1}{4}$yd. + $\frac{3}{4}$yd. + $\frac{1}{2}$yd.
4. $\frac{3}{4}$hr. + $\frac{1}{4}$hr. + $\frac{3}{4}$hr. 5. $\frac{3}{4}$lb. + $\frac{1}{2}$lb. + $\frac{3}{4}$lb. 6. $\frac{3}{4}$viss + $\frac{1}{2}$viss + $\frac{1}{4}$viss.
7. $\frac{7}{16} + \frac{9}{16} + \frac{11}{16}$. 8. $\frac{19}{32} + \frac{21}{32} + \frac{5}{32}$. 9. $\frac{17}{64} + \frac{23}{64} + \frac{29}{64}$.
10. $\frac{19}{23} - \frac{11}{23}$. 11. $\frac{100}{101} - \frac{79}{101}$. 12. $\frac{119}{120} - \frac{73}{120}$.

210. Fractions with different denominators.

EXAMPLE 1. Find the value of £$\frac{7}{12}$ + £$\frac{11}{30}$ + £$\frac{13}{16}$.

Find the value of each fraction and express the sum as the fraction of a pound.

$$£\frac{7}{12} + £\frac{11}{30} + £\frac{13}{16} = 11s.\ 8d. + 7s.\ 4d. + 16s.\ 3d. = £1\text{-}15\text{-}3 = £1\frac{61}{80}.$$

The same result may be obtained by adding them as fractions of a pound.

If these are converted into equivalent fractions with the same denominator, then we may proceed as explained in Article 209.

The L.C.M. of 12, 30 and 16 is 240.

$$£\frac{7}{12} + £\frac{11}{30} + £\frac{13}{16} = £\frac{140}{240} + £\frac{88}{240} + £\frac{195}{240}.$$

$$= £\frac{140 + 88 + 195}{240} = £\frac{423}{240} = £1\frac{183}{240} = £1\frac{61}{80}.$$

$$\therefore \frac{7}{12} + \frac{11}{30} + \frac{13}{16} = \frac{140 + 88 + 195}{240} = \frac{423}{240} = 1\frac{183}{240} = 1\frac{61}{80}.$$

211. Graphic Representation of Addition and Subtraction of Fractions.

Find the value of (1) $\frac{5}{8} + \frac{4}{7}$; (2) $\frac{5}{8} - \frac{4}{7}$.

Let the rectangle ABCD containing 56 squares represent the unit. Then 1 square represents $\frac{1}{56}$ of the unit.

The rectangle ADEF which contains 35 squares represents $\frac{35}{56}$ of the unit, that is, $\frac{5}{8}$ of the unit.

And the rectangle ABGH which contains 32 squares represents $\frac{35}{56}$ of the unit, that is, $\frac{4}{7}$ of the unit.

∴ The two rectangles together contain 67 squares and represent $\frac{35}{56} + \frac{32}{56}$ of the unit, that is, $\frac{67}{56}$ or $1\frac{11}{56}$ of the unit.

$$\therefore \frac{5}{8} + \frac{4}{7} = \frac{35}{56} + \frac{32}{56} = \frac{35 + 32}{56} = \frac{67}{56} = 1\frac{11}{56}.$$

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ADEF − ABGH = 35 squares − 32 squares = 3 squares.

$$= \frac{35}{56} \text{ unit} - \frac{32}{56} \text{ unit} = \frac{3}{56} \text{ of the unit.}$$

$$\therefore \frac{5}{8} - \frac{4}{7} = \frac{35}{56} - \frac{32}{56} = \frac{35 - 32}{56} = \frac{3}{56}.$$

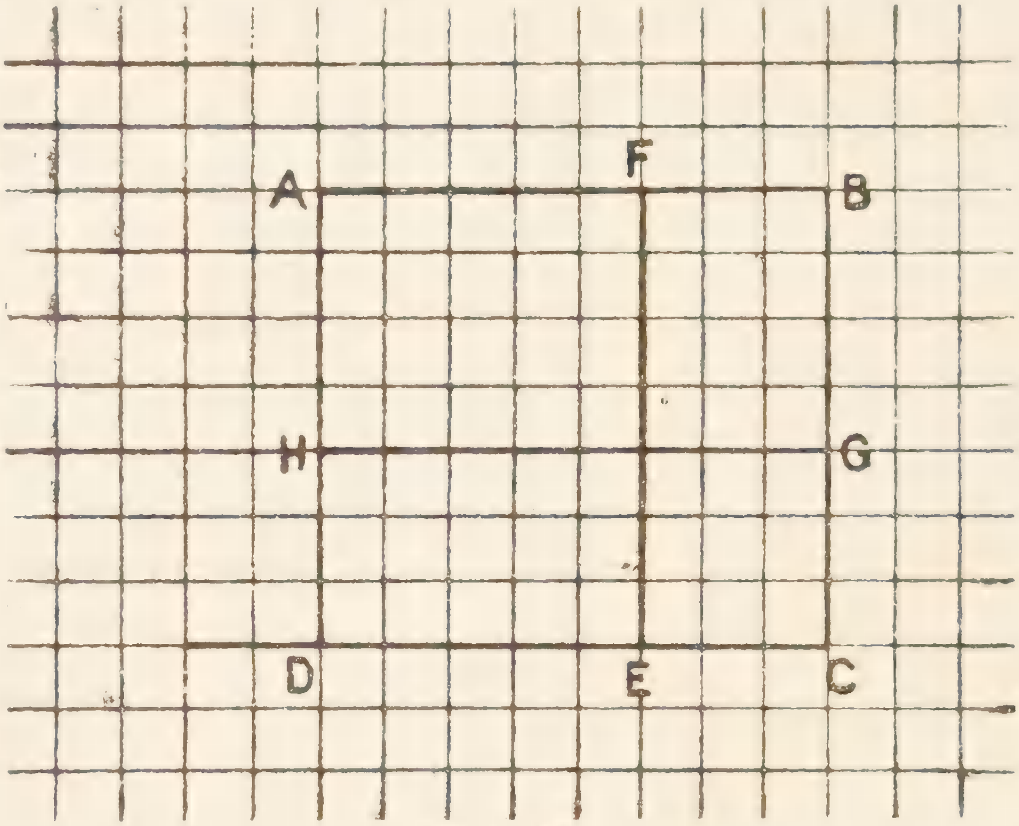

FIG. 24.

Practical Exercise.

Represent graphically on squared paper, taking (1) straight lines (2) rectangles, the sum and difference of the following fractions :—

1. $\frac{3}{4}$ and $\frac{1}{2}$. 2. $\frac{5}{8}$ and $\frac{7}{16}$. 3. $\frac{11}{12}$ and $\frac{7}{8}$. 4. $\frac{7}{16}$ and $\frac{5}{12}$.
5. $\frac{3}{8}$ and $\frac{17}{24}$. 6. $\frac{2}{5}$ and $\frac{1}{4}$. 7. $\frac{7}{15}$ and $\frac{4}{9}$. 8. $\frac{19}{24}$ and $\frac{5}{16}$.

EXAMPLE. Find the sum of $\frac{3}{4}$ and $\frac{2}{3}$; and the difference between $\frac{7}{8}$ and $\frac{5}{6}$.

Mental Steps.

$3 \times 3,\ 9 ;\ 4 \times 2,\ 8 ;\ \frac{17}{12} = 1\frac{5}{12}.$

$7 \times 6,\ 42 ;\ 5 \times 8,\ 40 ;\ \frac{2}{48} = \frac{1}{24}.$

EXAMPLES.—ORAL.

Find the value of—

1. $\frac{1}{2} + \frac{3}{4}$. 2. $\frac{1}{2} + \frac{3}{8}$. 3. $\frac{3}{4} + \frac{5}{8}$. 4. $\frac{1}{4} + \frac{2}{3}$. 5. $\frac{3}{8} + \frac{3}{4}$.
6. $\frac{3}{4} + \frac{1}{6}$. 7. $\frac{2}{3} + \frac{5}{12}$. 8. $\frac{5}{8} + \frac{3}{16}$. 9. $\frac{4}{9} + \frac{1}{6}$. 10. $\frac{4}{5} + \frac{5}{9}$.
11. $\frac{3}{8} + \frac{5}{12}$. 12. $\frac{5}{12} + \frac{7}{16}$. 13. $\frac{7}{3} - \frac{3}{4}$. 14. $\frac{3}{4} - \frac{3}{8}$. 15. $\frac{3}{4} - \frac{2}{3}$.
16. $\frac{5}{8} - \frac{1}{2}$. 17. $\frac{7}{8} - \frac{2}{3}$. 18. $\frac{7}{9} - \frac{5}{8}$. 19. $\frac{5}{8} - \frac{3}{11}$. 20. $\frac{7}{11} - \frac{5}{9}$.

EXAMPLES. I.

Find the value of examples 1 to 8 as fractions and test the correctness of your answer as compound quantities—

1. $\frac{7}{8}$ R. + $\frac{13}{16}$ R. + $\frac{11}{24}$ R.
2. $\frac{5}{12}$ R. + $\frac{7}{16}$ R. + $\frac{3}{4}$ R.
3. $\frac{9}{16}$£ + $\frac{7}{12}$£ + $\frac{13}{24}$£.
4. $\frac{9}{16}$£ + $\frac{3}{8}$£ + $\frac{1}{2}$£.

Find the value of—

5. $\frac{17}{24}$ R. — $\frac{15}{36}$ R.
6. $\frac{21}{25}$£ — $\frac{16}{45}$£.
7. $\frac{115}{1}$ mile — $\frac{19}{33}$ mile.
8. $\frac{103}{650}$£ — $\frac{17}{120}$£.
9. $\frac{3}{7} + \frac{5}{8} + \frac{2}{3} + \frac{3}{16} + \frac{4}{21}$.
10. $\frac{4}{9} + \frac{4}{21} + \frac{4}{81} + \frac{4}{15}$.
11. $\frac{13}{18} + \frac{10}{21} + \frac{9}{14} + \frac{25}{32}$.
12. $\frac{11}{12} + \frac{13}{16} + \frac{19}{24} + \frac{25}{32}$.
13. $\frac{15}{17} + \frac{15}{34} + \frac{15}{8} + \frac{15}{136}$.
14. $\frac{25}{36} + \frac{19}{30} + \frac{7}{25} + \frac{11}{18}$.
15. $\frac{13}{17} + \frac{10}{51} + \frac{2}{3} + \frac{3}{4}$.
16. $\frac{27}{70} + \frac{5}{21} + \frac{31}{50} + \frac{3}{5}$.

Subtract—

17. $\frac{31}{68}$ from $\frac{29}{51}$.
18. $\frac{11}{46}$ from $\frac{25}{69}$.
19. $\frac{101}{144}$ from $\frac{101}{112}$.
20. $\frac{203}{216}$ from $\frac{31}{32}$.

Find the sum, by simplifying the fractions before adding—

21. $\frac{27}{81} + \frac{38}{57} + \frac{91}{169} + \frac{17}{39}$.
22. $\frac{34}{119} + \frac{39}{42} + \frac{17}{28} + \frac{51}{85}$.
23. $\frac{111}{148} + \frac{175}{245} + \frac{206}{309}$.
24. $\frac{206}{369} + \frac{321}{428} + \frac{609}{1015}$.

---

212. In the addition and subtraction of mixed numbers it is convenient to take the whole numbers by themselves, and the fractions by themselves.

Example 2 (*a*). Add together as (1) vulgar fractions, (2) decimals correct to two places and thus verify the result :—

$$1\frac{5}{6} + 8\frac{10}{19} + 12\frac{17}{38} + 5\frac{5}{57}$$

$$\text{The expression} = 1 + 8 + 12 + 5 + \frac{5}{6} + \frac{10}{19} + \frac{17}{38} + \frac{5}{57}$$

$$= 26 + \frac{95 + 60 + 51 + 10}{114}$$

$$= 26 + \frac{216}{114} = 26 + 1\frac{102}{114} = 27\frac{17}{19}.$$

Expressed in decimals

1·83
8·53
12·45
5·09
27·90

$27\frac{17}{19}$ expressed as a decimal correct to two places = 27·90.

Note.—When there are improper fractions, it is simpler to express them as mixed numbers before adding them.

EXAMPLE 2 (*b*). Take $5\frac{12}{77}$ from $8\frac{15}{49}$.

$$8\frac{15}{49}-5\frac{12}{77}=8-5+\frac{15}{49}-\frac{12}{77}=3+\frac{165-84}{539}=3+\frac{81}{539}=3\frac{81}{539}.$$

EXAMPLES.—ORAL.

Find the value of—

1. $2\frac{1}{4}$ R. + $1\frac{3}{4}$ R. + $8\frac{1}{2}$ R. 2. 3 as. + $5\frac{1}{4}$ as. + $9\frac{1}{4}$ as.
3. $16\frac{1}{4}$ in. + $18\frac{1}{2}$ in. + $2\frac{3}{4}$ in. 4. £$12\frac{1}{2}$ + £$7\frac{3}{4}$ + £$3\frac{3}{4}$.
5. $7\frac{3}{4}$yd.—$3\frac{1}{4}$yd. 6 $10\frac{1}{2}$R.—$8\frac{1}{2}$R. 7. £$12\frac{3}{4}$—£$7\frac{1}{4}$. 8. $12\frac{1}{2}$R.—$9\frac{1}{4}$R.
9. $22\frac{1}{2}+10\frac{3}{4}$. 10. $2\frac{1}{4}+3\frac{5}{8}$. 11. $3\frac{1}{2}+4\frac{7}{8}$. 12. $15\frac{5}{8}+13\frac{1}{2}$.
13. $18\frac{3}{4}-6\frac{1}{2}$. 14. $19\frac{1}{2}-11\frac{1}{4}$. 15. $5\frac{1}{2}-3\frac{3}{8}$. 16. $12\frac{3}{4}-9\frac{5}{8}$.

EXAMPLES. II.

Find the value of—

1. $7+\frac{300}{21}+\frac{400}{49}+\frac{5}{3}$. 2. $3\frac{15}{16}+10\frac{7}{12}+\frac{35}{36}+2\frac{23}{64}$.
3. $\frac{13}{97}+3\frac{40}{135}+\frac{173}{15}+\frac{7}{9}$. 4. $6\frac{7}{8}+11\frac{7}{12}+4\frac{5}{24}+\frac{7}{32}$.

Find the difference between—

5. $11\frac{5}{9}$ and $8\frac{1}{12}$. 6. $20\frac{5}{24}$ and $15\frac{13}{144}$.
7. $9\frac{7}{9}$ and $9\frac{7}{10}$. 8. $31\frac{13}{21}$ and $27\frac{2}{5}$.

Add as (1) vulgar fractions and (2) decimals ; and use the one result to test the correctness of the other.

9. $101\frac{3}{8}$, $\frac{7}{25}$, $4\frac{1}{2000}$. 10. $17\frac{13}{1000}$, $\frac{123}{10}$, $\frac{4}{125}$, $\frac{5}{8}$, $\frac{5}{16}$.
11. $1\frac{1}{3}+7\frac{5}{8}+16\frac{1}{6}+2{\cdot}875$.
12. $2\frac{3}{3000}+1\frac{3}{1600}+5\frac{1}{6000}+2{\cdot}000875$.

EXAMPLE 3. From $10\frac{7}{8}$ subtract $7\frac{9}{10}$.

$$10\frac{7}{8}-7\frac{9}{10}=10-7+\frac{7}{8}-\frac{9}{10}.$$
$$=3+\frac{7}{8}-\frac{9}{10}.$$

As $\frac{9}{10}$ is greater than $\frac{7}{8}$, we proceed thus :

$$2+(1+\tfrac{7}{8})-\tfrac{9}{10}=2+\tfrac{15}{8}-\tfrac{9}{10}.$$
$$=2+\frac{75-36}{40}=2\frac{39}{40}.$$

Or thus :

$$3+\frac{7}{8}-\frac{9}{10}=3+\frac{35-36}{40}$$
$$=3-\tfrac{1}{40}$$
$$=2\tfrac{39}{40}$$

EXAMPLES.—ORAL.

Find the value of—

1. $1\frac{1}{4}-\frac{3}{4}$. 2. $1\frac{1}{2}-\frac{3}{4}$. 3. $1\frac{1}{4}-\frac{1}{2}$. 4. $8-3\frac{1}{2}$. 5. $10-6\frac{1}{4}$.
6. $15-7\frac{3}{4}$. 7. $10\frac{1}{4}-5\frac{3}{4}$. 8. $7\frac{1}{2}-5\frac{3}{4}$. 9. $7\frac{3}{8}-\frac{1}{2}$. 10. $9\frac{1}{4}-\frac{7}{8}$.
11. $15\frac{3}{8}-6\frac{3}{4}$. 12. $13\frac{1}{2}-9\frac{7}{8}$. 13. $1\frac{1}{2}-\frac{3}{16}$. 14. $2\frac{1}{4}-1\frac{7}{16}$. 15. $3\frac{1}{2}-1\frac{11}{16}$

EXAMPLES. III.

What must be added to—

1. Rs. $13\frac{5}{6}$ to make Rs. $17\frac{3}{4}$?   2. £$3\frac{46}{57}$ to make £$4\frac{17}{76}$?

3. $8\frac{3}{56}$ cwt. to make $20\frac{5}{18}$ cwt. ?

4. $33\frac{23}{35}$ mds. to make 50 mds. ?

Find the value of—

5. $20\frac{17}{18} - 15\frac{31}{32}$.   6. $9 - 7\frac{121}{125}$.

7. $3\frac{11}{192} - 1\frac{17}{144}$.   8. $5\frac{3}{14} - 4\frac{11}{18}$.

Example 4. Find the value of $7\frac{1}{8} - 5\frac{1}{12} + 3\frac{1}{15} - 4\frac{1}{20}$.

$$\text{Value} = 7 + 3 - 5 - 4 + \frac{1}{8} - \frac{1}{12} + \frac{1}{15} - \frac{1}{20}$$

$$= 1 + \frac{15 - 10 + 8 - 6}{120} = 1\frac{7}{120}.$$

EXAMPLES. IV.

Find the value of—

1. Rs. $1\frac{2}{3}$ + Rs. $3\frac{7}{8}$ − Rs. $2\frac{7}{32}$.

2. $2\frac{1}{3}$ guinea + £$1\frac{1}{6}$ − $\frac{7}{15}$ crown.

3. $\frac{5}{6}$ fur. − $\frac{3}{4}$ yd. + $\frac{1}{8}$ chain.

4. $\frac{2}{5}$ Kg. − $90\frac{3}{4}$ grams + $6\frac{7}{8}$ mg.

5. A man sold $\frac{1}{16}$ of his estate to one, $\frac{3}{8}$ to another and $\frac{5}{12}$ to a third; what part of the estate has he still left ?

6. Three men did $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{11}{24}$ respectively of a certain piece of work. What fraction of the work was still undone ?

7. A person owned $\frac{1}{8}$ of a garden, another $\frac{3}{16}$ of it, and a third man owned $\frac{3}{8}$ of it. What did the fourth own ?

8. One-third of a pole is coloured red, $\frac{1}{8}$ yellow, $\frac{3}{10}$ blue and $\frac{5}{24}$ green. What part of it is left uncoloured ?

9. Which is greater, the sum of $9\frac{2}{27}$ and $11\frac{5}{9}$ or the sum of $12\frac{3}{54}$ and $8\frac{1}{18}$?

10. What fraction added to $\frac{10}{11}$, $\frac{11}{14}$, $\frac{2}{33}$ and $\frac{41}{42}$ will make the sum of 3 ?

11. By how much does the sum of $\frac{51}{75}$, $\frac{131}{150}$, $\frac{24}{25}$, $\frac{1}{2}$ and $\frac{2}{3}$ fall short of 4 ?

12. What fraction must be added to the sum of $\frac{14}{45}$, $\frac{27}{39}$ and $\frac{2}{21}$ to make the result an integer ?

CHAPTER XVIII.

MULTIPLICATION AND DIVISION OF FRACTIONS.

Multiplication by Integers.

213. Illustration by coins.

Take four quarter-anna coins and shew that they make up one anna.

$\therefore$ Four times quarter is one.

$\therefore \frac{1}{4} \times 4 = 1.$

214. Graphic Illustration by straight lines.

EXAMPLE. Multiply $\frac{1}{2}$ inch by 3.

Take an inch as the unit and draw any length AB. Divide the unit into 6 equal parts; 1 division represents $\frac{1}{6}$ inch; and AC containing 3 divisions measures $\frac{1}{2}$ an inch. Take 3 lengths each equal to AC; now AE is 3 times AC and measures $1\frac{1}{2}$ in.

$$\therefore \frac{1}{2}\text{ in.} \times 3 = 1\frac{1}{2}\text{ in.} = \frac{3}{2}\text{ in}$$

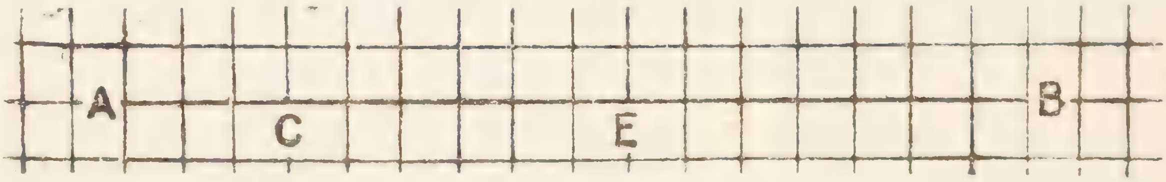

FIG. 25.

215. Graphic Illustration by Rectangles.

EXAMPLE. To multiply $\frac{3}{16}$ by 3.

Let the square ABCD represent the unit. Divide AB and AD each into 4 equal parts and thus divide ABCD into 16 equal squares by drawing parallels through the points of division both horizontally and vertically. Fig. 26, page 173.

Then the square GFMB represents $\frac{1}{16}$ of the unit and the rectangle AEFG which contains 3 such squares represents $\frac{3}{16}$ of the unit.

$\therefore$ The rectangle AHKG which is 3 times AEFG represents $\frac{3}{16} \times 3$ of the unit.

Since AHKG contains 9 squares, it is $\frac{9}{16}$ of the unit.

$$\therefore 3 \text{ times } \frac{3}{16} \text{ of the unit} = \frac{9}{16} \text{ of the unit.}$$

$$\therefore \frac{3}{16} \times 3 = \frac{9}{16} = \frac{3 \times 3}{16}.$$

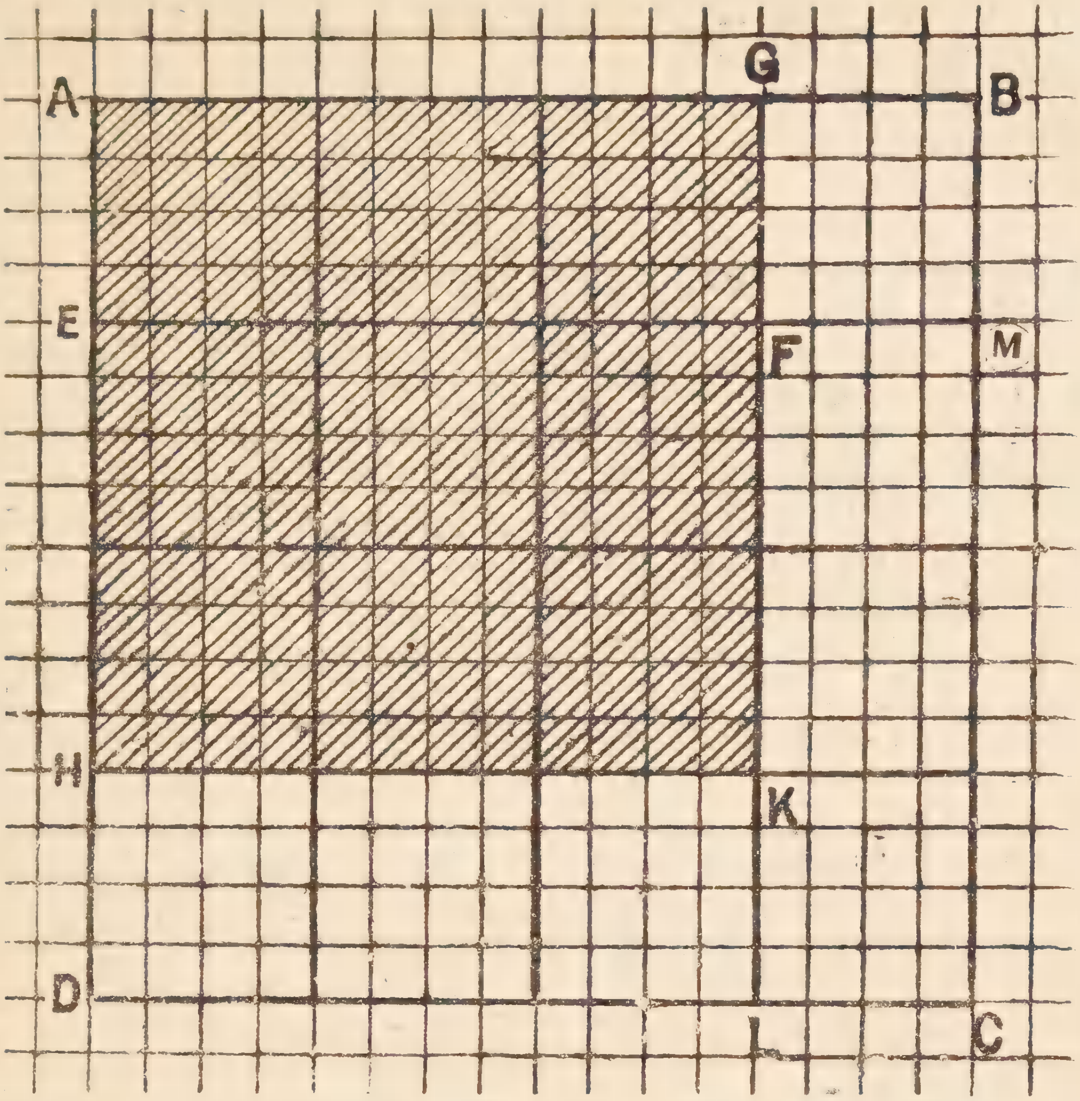

FIG. 26.

EXAMPLE. Multiply $\frac{5}{19}$ by 4.

In other words add together $\frac{5}{19}+\frac{5}{19}+\frac{5}{19}+\frac{5}{19}$.

The sum $=\frac{20}{19}$. Therefore $\frac{5}{19}\times 4=\frac{20}{19}=\frac{5\times 4}{19}$.

We thus see that to multiply a fraction by an integer, the numerator of the fraction has to be multiplied by the integer.

If $\frac{a}{b}$ denotes a fraction and c an integer, $\frac{a}{b}\times c=\frac{a\times c}{b}=\frac{ac}{b}$.

216. The product is sometimes capable of Simplification.

EXAMPLE. $\frac{3}{16}\times 4=\frac{12}{16}=\frac{3}{4}$.

In Fig. 26, the rectangle ADLG is 4 times the rectangle AEFG, i.e., 4 times $\frac{3}{16}$ unit.

But the rectangle ADLG is $\frac{3}{4}$ of the unit, since it contains 12 out of 16 squares.

$\therefore$ 4 times $\frac{3}{16}$ unit $= \frac{3}{4}$. Hence $\frac{3}{16} \times 4 = \frac{3}{4}$.

$$\therefore \frac{a}{bc} \times c = \frac{a}{b}.$$

Practical Exercise.

1. Taking an inch as the unit of length, draw a straight line 4 inches long, divide it and use it to shew that $\frac{5}{8} \times 5 = \frac{25}{8} = 3\frac{1}{8}$.

2. In the same way, draw lines of suitable lengths to prove—
   (1) $\frac{3}{4} \times 3 = 2\frac{1}{4}$. (2) $\frac{3}{5} \times 8 = 4\frac{4}{5}$. (3) $\frac{7}{8} \times 5 = 4\frac{3}{8}$.
   (4) $\frac{3}{10} \times 7 = 2\frac{1}{10}$. (5) $\frac{5}{6} \times 4 = 3\frac{1}{3}$. (6) $\frac{7}{12} \times 9 = 5\frac{1}{4}$.
   (7) $\frac{3}{8} \times 10 = 3\frac{3}{4}$. (8) $\frac{9}{16} \times 4 = 2\frac{1}{4}$.

3. Find graphically, using rectangles, the following products :—
   (1) $\frac{1}{8} \times 5$. (2) $\frac{3}{20} \times 3$. (3) $\frac{2}{15} \times 7$. (4) $\frac{3}{16} \times 5$.

4. Find the following products graphically :—
   (1) $\frac{3}{4} \times 5$. (2) $\frac{4}{5} \times 4$. (3) $\frac{5}{8} \times 6$. (4) $\frac{5}{12} \times 8$.

EXAMPLE 1. Find the value of $\frac{7}{12} \times 7$; and test the correctness of the answer by taking a rupee as the unit.

$$\frac{7}{12} \times 7 = \frac{7 \times 7}{12} = \frac{49}{12} = 4\frac{1}{12}.$$

Verification.

$\frac{7}{12}$ unit $= \frac{7}{12}$ R. $= 7 \times$ 1 a. 4 p. $=$ 9 as. 4 p.
$\therefore \frac{7}{12}$ R. $\times 7 =$ 9 as. 4 p. $\times 7 =$ Rs. 4-1-4 $=$ Rs. $4\frac{1}{12}$.
$= 4\frac{1}{12}$ of R. $1 = 4\frac{1}{12}$ of the unit.

EXAMPLE 2. Multiply $\frac{17}{18}$ by 12.

$$\frac{17}{18} \times 12 = \frac{17 \times 12}{18} = \frac{17 \times 2 \times 6}{3 \times 6} = \frac{17 \times 2}{3} = \frac{34}{3} = 11\frac{1}{3}.$$

Practical Exercise.

Construct three Multiplication Tables for each of the fractions $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{1}{16}$.

(1) from 1 to 16.
(2) 10, 20, 30,............ 100.
(3) 100, 200, 300,............ 1000.

EXAMPLES.—ORAL.

Find the product of—

1. $\frac{13}{20}$ by 8, 9, 10, 12,............................20.

2. $\frac{1}{16} \times 112$. 3. $\frac{3}{4} \times 16$. 4. $\frac{5}{8} \times 24$. 5. $\frac{7}{16} \times 96$.

6. $\frac{7}{16} \times 24$. 7. $\frac{9}{10} \times 25$. 8. $\frac{8}{13} \times 39$. 9. $\frac{13}{12} \times 15$.

EXAMPLES. I.

Multiply and verify the correctness of the answers to Examples 2 and 3 by taking £1 as the unit.

1. $\frac{53}{89}$ by (1) 17, (2) 21, (3) 30, (4) 47.
2. $\frac{11}{15}$ by (1) 6, (2) 8, (3) 10, (4) 12.
3. $\frac{19}{20}$ by (1) 15, (2) 18, (3) 24, (4) 30.
4. $\frac{29}{54}$ by (1) 18, (2) 27, (3) 36, (4) 81.

---

217. To multiply a mixed number by an integer.

EXAMPLE. Find the value of $1\frac{2}{5} \times 2$.

Graphic Illustration.

Take the straight line AB, $1\frac{2}{5}$ in. long, it contains 14 divisions, since one inch is divided into 10 equal parts.

To AB add BC a length equal to $1\frac{2}{5}$ in. as in the figure.

Now AC is *twice* AB and contains 28 divisions.

$\therefore$ AD $= \frac{28}{10}$ in. $= 2\frac{4}{5}$ in.

$\therefore 1\frac{2}{5}$ in. $\times 2 = 2\frac{4}{5}$ in.

$\therefore 1\frac{2}{5} \times 2 = 2\frac{4}{5}$.

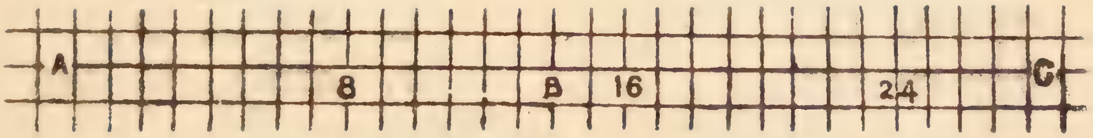

FIG. 27

Or thus: $1\frac{2}{5} \times 2 = (1 + \frac{2}{5}) \times 2 = 2 + \frac{2}{5} \times 2 = 2 + \frac{4}{5} = 2\frac{4}{5}$.

Or since $1\frac{2}{5} = \frac{7}{5}$, $1\frac{2}{5} \times 2 = \frac{7}{5} \times 2 = \frac{14}{5} = 2\frac{4}{5}$.

Practical Exercise.

Draw lines on squared paper and shew—

1. $1\frac{1}{2}$ in. $\times 3 = 4\frac{1}{2}$ in.
2. $2\frac{1}{4}$ in. $\times 5 = 11\frac{1}{4}$ in.
3. $2\frac{1}{2}$ cm. $\times 3 = 7\frac{1}{2}$ cm
4. $3\frac{7}{10}$ cm. $\times 3 = 11\frac{1}{10}$ cm.
5. $3\frac{1}{4}$ in. $\times 4 = 13$ in
6. $2\frac{7}{8}$ in. $\times 4 = 11\frac{1}{4}$ in.
7. $2\frac{5}{8}$ in. $\times 3 = 7\frac{7}{8}$ in
8. $3\frac{1}{10}$ cm. $\times 5 = 15\frac{1}{2}$ cm.

Draw rectangles on squared paper and divide them to shew that—

1. $4\frac{1}{3} \times 6 = 26$. 2. $3\frac{3}{4} \times 4 = 15$

3. $5\frac{1}{2} \times 5 = 27\frac{1}{2}$. 4. $4\frac{1}{4} \times 6 = 25\frac{1}{2}$.

EXAMPLES.—ORAL.

Find the product of—

1. Rs. $1\frac{1}{2} \times 3$. 2. Rs. $1\frac{3}{4} \times 5$. 3. £$2\frac{1}{4} \times 6$.

4. £$3\frac{1}{2} \times 7$. 5. $4\frac{1}{2}$ in. × 8. 6. $5\frac{1}{4}$ ft. × 9.

7. $3\frac{1}{3} \times 4$. 8. $4\frac{2}{3} \times 6$. 9. $12\frac{1}{4} \times 12$.

10. $9\frac{1}{2} \times 14$. 11. $7\frac{7}{8} \times 10$. 12. $9\frac{5}{12} \times 16$.

EXAMPLE. Multiply $121\frac{3}{7}$ by 8.

$$121\frac{3}{7} \times 8 = (121 \times \tfrac{3}{7}) \times 8 = 121 \times 8 + \tfrac{3}{7} \times 8 = 968 + \tfrac{24}{7}.$$
$$= 968 + 3\tfrac{3}{7} = 971\tfrac{3}{7}.$$

Or, reducing $121\frac{3}{7}$ to an improper fraction, multiply it by 8, thus :

$$121\frac{3}{7} = \frac{850}{7}\,;\ \frac{850}{7} \times 8 = \frac{850 \times 8}{7} = \frac{6800}{7} = 971\frac{3}{7}.$$

EXAMPLES. II.

Multiply and test the accuracy of the answers by taking Re. 1 as the unit in Ex. 1, and £1 as the unit in Ex. 2.

1. $10\frac{5}{8}$ by (1) 6, (2) 8, (3) 15, (4) 16.

2. $5\frac{3}{10}$ by (1) 15, (2) 25, (3) 45, (4) 60.

3. $8\frac{11}{12}$ by 151. 4. $9\frac{3}{11}$ by 111.

5. $5\frac{23}{24}$ by 723. 6. $100\frac{7}{120}$ by 420.

Division by Integers.

218. Illustrated by the division of concrete quantities. To divide $\frac{1}{4}$ of a rupee by 4.

Take a quarter of a rupee and change it into 4 one-anna coins.

Divide these among 4 boys ; each boy gets one anna ; and one anna is $\frac{1}{16}$ of a rupee.

$$\therefore \frac{1}{4} \text{ of Re. } 1 \div 4 = \frac{1}{16} \text{ of R. } 1, \text{ i.e. } \frac{1}{4 \times 4} \text{ of R } 1.$$

$$\therefore \frac{1}{4} \div 4 = \frac{1}{4 \times 4} = \frac{1}{16}.$$

EXAMPLE. Divide $\frac{3}{5}$ viss of sugar among 4 boys.

$\frac{3}{5}$ viss $= \frac{3}{5} \times 40$ pals. $= 24$ pals.

$\therefore \frac{3}{5}$ viss $\div 4 = 24$ pals. $\div 4 = 6$ pals.

Now divide one viss of sugar into 20 equal parts and take 3 such parts, you get the same result, viz. 6 pals.; that is, $\frac{3}{20}$ viss $= 6$ pals.

$\therefore \frac{3}{5} \div 4 = \frac{3}{20}$ viss.

$\therefore \frac{3}{5} \div 4 = \frac{3}{20} = \frac{3}{\times}.$

219. Illustration by paper folding.

Take any rectangular strip of paper; and divide it by creases into 4 equal parts; each part is now $\frac{1}{4}$ of the strip.

Divide each part again into 4 equal parts; now there are 16 parts; and each part is $\frac{1}{16}$ of the whole strip.

$\therefore \frac{1}{4} \div 4 = \frac{1}{16}.$

220. Graphic Illustration.—Divide $2\frac{5}{8}$ inches by 7.

Take a length AB equal to $2\frac{5}{8}$ inches. Since an inch is divided into 8 equal parts, AB contains 21 divisions.

Divide AB into 7 equal parts as in the figure; and one part contains 3 divisions, i.e. $\frac{3}{8}$ of an inch.

$\therefore 2\frac{5}{8}$ in. $\div 7 = \frac{3}{8}$ in.

$\therefore \frac{21}{} \div 7 = \frac{3}{8}$; that is, $\frac{21}{8} \div 7 = \frac{21}{8 \times 7} = \frac{3}{8}.$

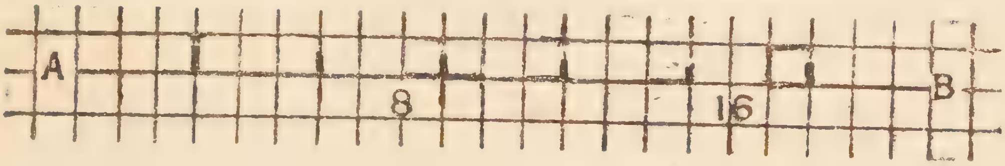

FIG. 28.

221. Illustration by Rectangles.—Divide $\frac{3}{4}$ by 6.

Take the rectangle ABCD as the unit. Divide AB into 8 equal parts and AD into 4 equal parts and thus divide ABCD into 32 equal squares each consisting of 4 small squares.

The rectangle AEFD which contains 24 such squares represents $\frac{24}{32}$ or $\frac{3}{4}$ of the unit.

Dividing AEFD by 6, we get the rectangle AGHD, a column of 4 squares.

And AGHD is $\frac{1}{8}$ of ABCD, the unit.

$\therefore \frac{3}{4}$ unit $\div 6 = \frac{1}{8}$ unit.

$\therefore \frac{3}{4} \div 6 = \frac{1}{8}$; that is, $\frac{3}{4} \div 6 = \frac{3}{4 \times 6} = \frac{1}{8}$.

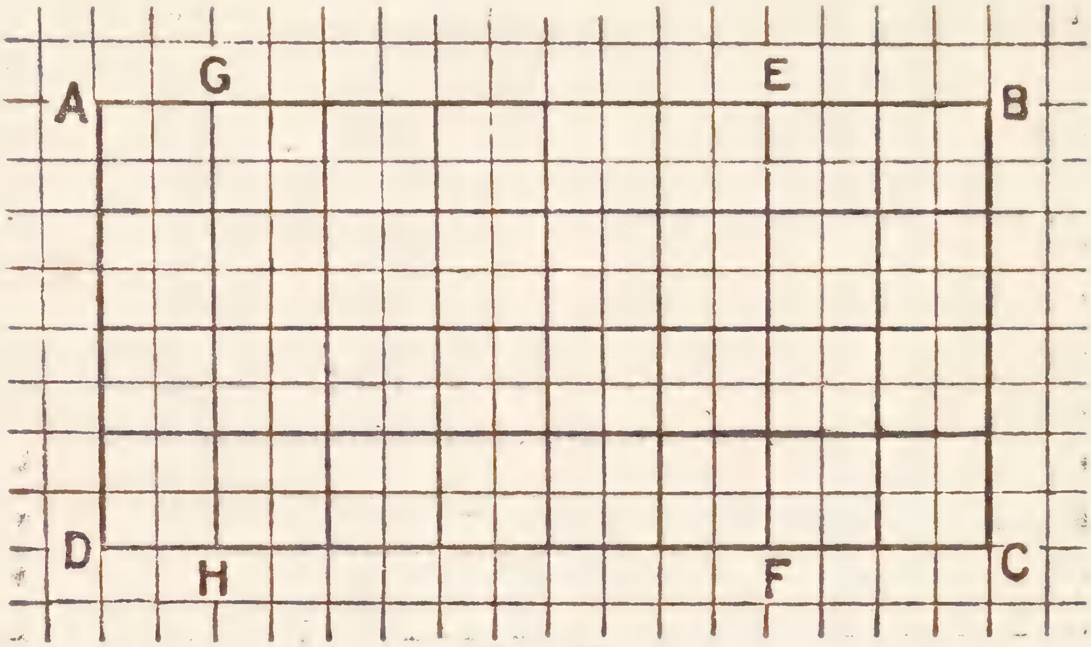

FIG. 29.

We thus see that to divide a fraction by an integer, the denominator has to be multiplied by the integer.

$$\therefore \frac{a}{b} \div c = \frac{a}{b \times c} = \frac{a}{bc}.$$

Practical Exercise.

1. Take strips of paper, divide them by creases and shew that—

(1) $\frac{1}{3} \div 4 = \frac{1}{12}$. (2) $\frac{1}{5} \div 3 = \frac{1}{15}$.

(3) $\frac{3}{4} \div 6 = \frac{1}{8}$. (4) $\frac{2}{3} \div 4 = \frac{1}{6}$.

2. Draw suitable lengths on squared paper, and find the quotient of—

(1) $2\frac{1}{2} \div 5$. (2) $3\frac{3}{4} \div 6$. (3) $2\frac{5}{8} \div 3$. (4) $5\frac{1}{4} \div 7$.

3. Divide graphically and find the quotient—

(1) $3\frac{3}{4}$ in. $\div 5$. (2) $3\frac{1}{3}$ in. $\div 5$.

(3) $7\frac{7}{8}$ in. $\div 9$. (4) $4\frac{7}{12}$ in. $\div 11$.

4. Draw the following rectangles on squared paper and find the quotient in each case—

(1) 5 cm. by 4 cm.; $\frac{2}{5} \div 4$. (2) 6 cm. by 4 cm.; $\frac{3}{8} \div 5$.

(3) 3 in. by 2 in.; $\frac{2}{3} \div 8$. (4) $1\frac{1}{2}$ in. by $1\frac{1}{4}$ in.; $\frac{3}{5} \div 6$.

5. Draw suitable rectangles and prove that—

(1) $\frac{2}{3} \div 4 = \frac{1}{6}$. (2) $\frac{3}{8} \div 6 = \frac{1}{16}$.

(3) $\frac{4}{5} \div 10 = \frac{2}{25}$. (4) $\frac{9}{10} \div 3 = \frac{3}{10}$.

EXAMPLE 1. (*a*) Divide $\frac{7}{8}$ by 9. (*b*) Divide $\frac{12}{13}$ by 6.

$$(a)\quad \frac{7}{8} \div 9 = \frac{7}{8 \times 9} = \frac{7}{72}.$$

$$(b)\quad \frac{12}{13} \div 6 = \frac{12}{13 \times 6} = \frac{2 \times 6}{13 \times 6} = \frac{2}{13}.$$

EXAMPLE 2. $72\frac{1}{2} \div 29 = \frac{145}{2} \div 29 = \frac{145}{2 \times 29} = \frac{5 \times 29}{2 \times 29} = \frac{5}{2} = 2\frac{1}{2}$.

$72\frac{1}{2} \div 29$ gives 2 quotient and $14\frac{1}{2}$ remainder; and $14\frac{1}{2} \div 29 = \frac{1}{2}$; $72\frac{1}{2} \div 29 = 2\frac{1}{2}$.

EXAMPLES.—ORAL.

Give the value of—

1. $\frac{3}{8}$ R. $\div$ 4.
2. $\frac{1}{4}$ R. $\div$ 3.
3. £ $\frac{3}{4} \div 6$.
4. £ $\frac{3}{8} \div 5$.
5. £ $\frac{3}{20} \div 9$.
6. $1\frac{1}{2}$ viss $\div$ 6.
7. $2\frac{1}{4}$ cm. $\div$ 5.
8. $7\frac{1}{2}$ yds. $\div$ 10.
9. $13\frac{1}{2}$ ft. $\div$ 6.
10. $1\frac{3}{4} \div 7$.
11. $1\frac{1}{2} \div 8$.
12. $2\frac{7}{8} \div 4$.
13. $3\frac{1}{3} \div 5$.
14. $4\frac{1}{2} \div 12$.
15. $7\frac{1}{5} \div 12$.

EXAMPLES. III.

Divide and verify your answers to examples 1, 2 and 3 by taking as the unit a guinea, a rupee and a pound (£) respectively.

1. $\frac{16}{21}$ by (1) 8, (2) 12, (3) 18, (4) 24.
2. $13\frac{1}{3}$ by (1) 15, (2) 24, (3) 30, (4) 25.
3. $25\frac{1}{5}$ by (1) 12, (2) 15, (3) 21, (4) 63.
4. $193\frac{1}{13}$ by (1) 40, (2) 56, (3) 105, (4) 112.

---

222. A fraction whose numerator and denominator are both whole numbers is called a simple fraction.

A simple fraction may be a proper or an improper fraction.

EXAMPLES. $\frac{7}{15}, \frac{8}{9}, \frac{12}{7}, \frac{16}{12}$.

223. Multiplication by Fractions: One simple fraction by another.

EXAMPLE. Multiply $\frac{2}{5}$ by $\frac{2}{3}$.

That is, $\frac{2}{5}$ of the unit is to be divided into 3 equal parts, and 2 of these parts are to be taken.

$$\tfrac{2}{5} \div 3 = \tfrac{2}{15}\text{; and } \tfrac{2}{15} \times 2 = \tfrac{4}{15}$$

$$\therefore \frac{2}{5} \times \frac{2}{3} = \frac{4}{15} = \frac{2 \times 2}{15}.$$

Graphic Illustration.

Let the rectangle ABCD represent the unit.

Divide AB into 5 equal parts and AD into 3 equal parts; and thus divide the rectangle into 15 equal squares, each square consisting of 9 small squares.

Then the rectangle ABEF which contains 10 squares represents $\frac{10}{15}$ or $\frac{2}{3}$ of ABCD the unit.

AFGH which contains 2 squares represent $\frac{1}{5}$ of ABEF; that is, it represents $\frac{1}{5}$ of $\frac{2}{3}$ of the unit.

Since AFKL is double of AFGH, AFKL represents $\frac{2}{5}$ of $\frac{2}{3}$ of the unit.

And AFKL represents also $\frac{4}{15}$ of the unit, since it contains 4 out of the 15 squares.

$$\therefore \frac{2}{5} \times \frac{2}{3} \text{ of the unit} = \frac{4}{15} \text{ of the unit.}$$

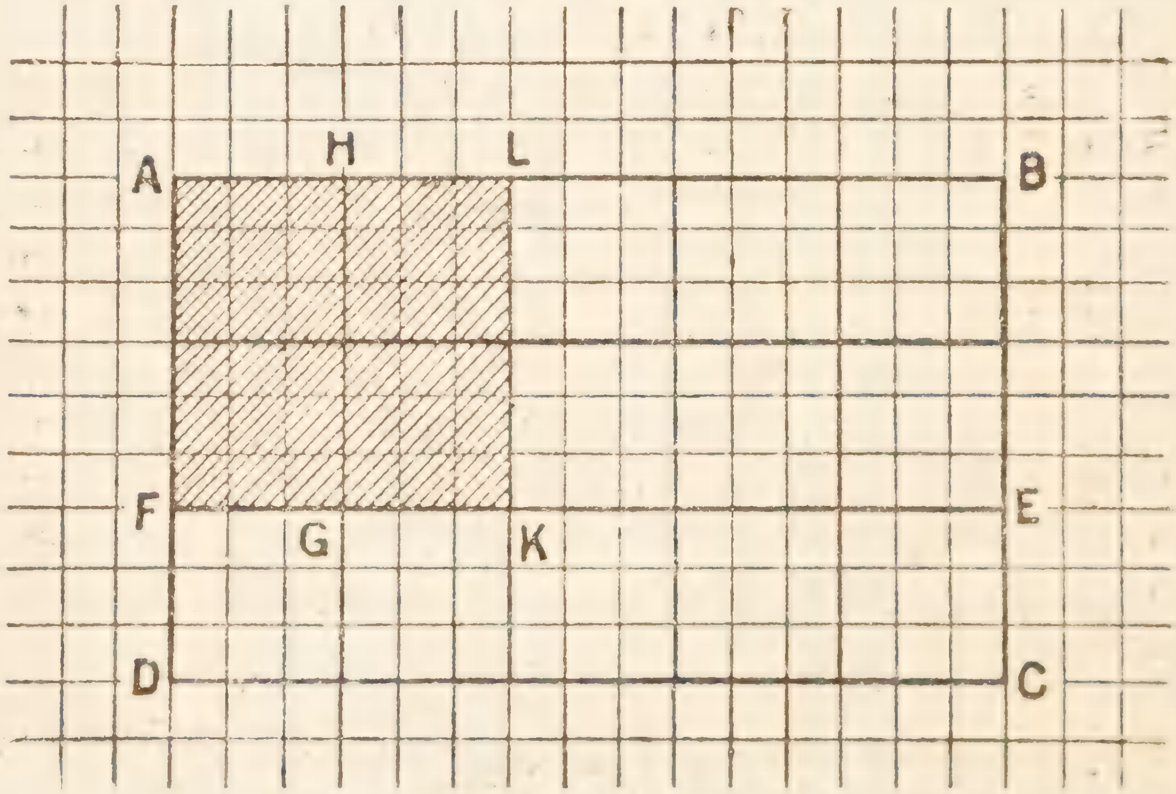

FIG. 30.

224. *Hence to multiply one fraction by another, multiply the numerators together for the new numerator, and the denominators together for the new denominator.*

$\therefore$ If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$.

EXAMPLE. $\frac{7}{15} \times \frac{8}{9} = \frac{7 \times 8}{15 \times 9} = \frac{56}{135}$.

Practical Exercise.

Shew graphically (1) by taking lengths in inches, (2) by taking rectangles—

(1) $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. (2) $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (3) $\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}$. (4) $\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$.

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225. Product in the simplest form.

EXAMPLE. Multiply $\frac{17}{21}$ by $\frac{14}{15}$.

$$\frac{17}{21} \times \frac{14}{15} = \frac{17 \times 14}{21 \times 15} = \frac{17 \times 2 \times 7}{3 \times 7 \times 15} = \frac{17 \times 2}{3 \times 15} = \frac{34}{45}.$$

To shew **graphically** that $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$.

Let the line AB 3 inches long represent the unit. AB contains 30 divisions; AC containing 25 divisions represents $\frac{5}{6}$ of the unit.

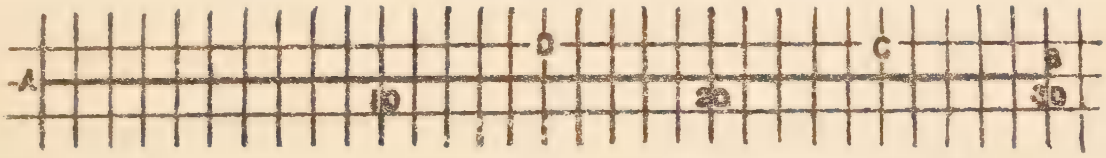

FIG. 31.

Divide AC into 5 equal parts; and take AD equal to 3 of these parts. Then AD is $\frac{3}{5}$ of AC.

But AC is $\frac{5}{6}$ of AB.

$\therefore$ AD is $\frac{3}{5}$ of $\frac{5}{6}$ of AB.

But AD $= \frac{1}{2}$ of AB, since AD contains 15 divisions.

$\therefore$ $\frac{3}{5}$ of $\frac{5}{6}$ of AB $= \frac{1}{2}$ of AB.

$\therefore$ $\frac{3}{5}$ of $\frac{5}{6}$ unit $= \frac{1}{2}$ unit.

$\therefore$ $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$.

By Rectangles.

Let the rectangle ABCD represent the unit.

Divide AD into 6 equal parts and AB into 5 equal parts.

Through the points of division draw parallels horizontally and vertically and thus divide the rectangle into 30 equal squares, each square consisting of 9 small squares.

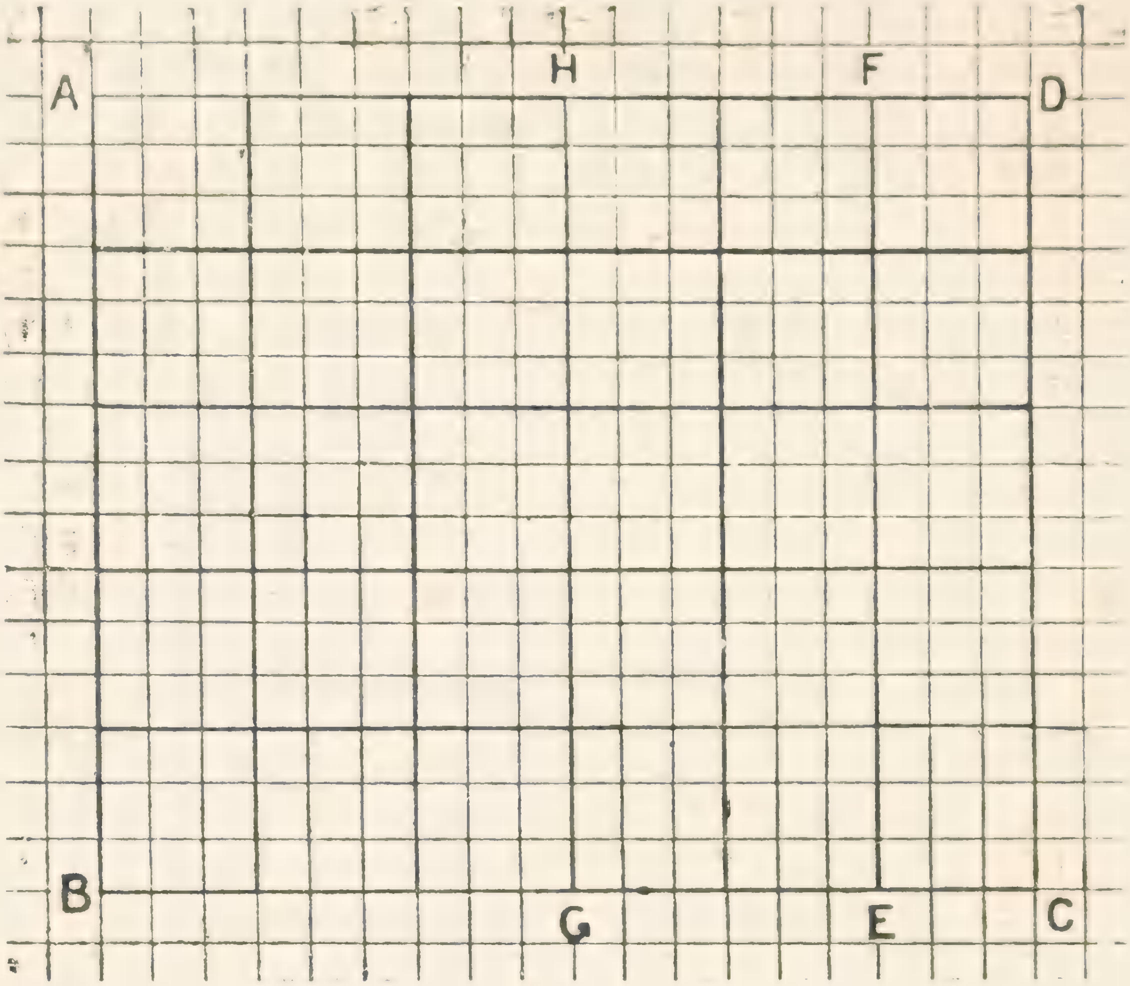

FIG. 32.

Then the rectangle ABEF which contains 25 squares is $\frac{25}{30}$ or $\frac{5}{6}$ of ABCD ; and the rectangle ABGH which contains 15 squares is $\frac{15}{25}$ or $\frac{3}{5}$ of ABEF, i.e. $\frac{3}{5}$ of $\frac{5}{6}$ of ABCD.

But the rectangle ABGH is $\frac{1}{2}$ of ABCD.

$\therefore$ $\frac{3}{5}$ of $\frac{5}{6}$ of ABCD $= \frac{1}{2}$ of ABCD ; i.e. $\frac{3}{5} \times \frac{5}{6}$ unit $= \frac{1}{2}$ unit.

$\therefore$ $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$.

Practical Exercise.

1. (*a*) Take a line 3 inches long, divide it into 12 equal parts and shew that $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

(*b*) Similarly shew that by taking a straight line—

(1) 9 in. long, that $\frac{7}{15} \times \frac{5}{6} = \frac{7}{18}$.
(2) 6 in. long, that $\frac{3}{8} \times \frac{2}{3} = \frac{1}{4}$.
(3) 4 in. long, that $\frac{2}{5} \times \frac{3}{8} = \frac{3}{20}$.

2. Draw on squared paper a rectangle—

(1) 3 in. by 2 in., and shew that $\frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$.
(2) 5 cm. by 4 cm., and shew that $\frac{3}{10} \times \frac{5}{8} = \frac{3}{16}$.
(3) 2 in. by 1·5 in., and shew that $\frac{7}{12} \times \frac{3}{4} = \frac{7}{16}$.
(4) 2·7 in. by 2·4 in., and shew that $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$.

EXAMPLES. IV.

Multiply—

1. $\frac{17}{39}$ by $\frac{10}{19}$. 2. $\frac{20}{27}$ by $\frac{13}{21}$. 3. $\frac{103}{105}$ by $\frac{26}{37}$.
4. $\frac{83}{211}$ by $\frac{19}{61}$. 5. $\frac{18}{81}$ by $\frac{20}{27}$. 6. $\frac{46}{27}$ by $\frac{12}{69}$.
7. $\frac{121}{132}$ by $\frac{12}{77}$. 8. $\frac{64}{125}$ by $\frac{150}{256}$.

226. Mixed numbers must be reduced to improper fractions before beginning to multiply.

EXAMPLE. Multiply $13\frac{3}{4}$ by $1\frac{1}{11}$.

$$13\frac{3}{4} \times 1\frac{1}{11} = \frac{55}{4} \times \frac{12}{11} = \frac{55 \times 12}{4 \times 11} = \frac{5 \times 11 \times 3 \times 4}{4 \times 11} = 5 \times 3 = 15.$$

EXAMPLES.—ORAL.

Give the products of—

1. $\frac{3}{2} \times \frac{2}{3}$. 2. $\frac{4}{5} \times \frac{5}{4}$. 3. $\frac{7}{3} \times \frac{6}{7}$. 4. $\frac{8}{7} \times \frac{7}{4}$.
5. $3\frac{3}{5} \times 10$. 6. $4\frac{1}{2} \times 12$. 7. $5\frac{3}{4} \times 12$. 8. $2\frac{7}{8} \times 16$.
9. $1\frac{1}{3} \times \frac{3}{4}$. 10. $2\frac{1}{2} \times \frac{2}{5}$. 11. $3\frac{3}{4} \times \frac{8}{15}$. 12. $3\frac{1}{3} \times \frac{6}{5}$.

Find the product of (1) $6\frac{1}{2}$ by $6\frac{1}{2}$. (2) $4\frac{1}{2}$ by $3\frac{1}{4}$.

(1) $6 \times 6 = 36$; $6 \times \frac{1}{2} = 3$; 39 ; $\frac{1}{2} \times 6 = 3$; 42 ; $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$;
Ans. $42\frac{1}{4}$.

(2) $4 \times 3 = 12$; $4 \times \frac{1}{4} = 1$; 13 ; $\frac{1}{2} \times 3 = 1\frac{1}{2}$; $14\frac{1}{2}$; $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$;
Ans. $14\frac{5}{8}$.

EXAMPLES.—ORAL.

Find the products of—

1. $5\frac{1}{2} \times 5\frac{1}{2}$. 2. $7\frac{1}{2} \times 7\frac{1}{2}$. 3. $13\frac{1}{2} \times 13\frac{1}{2}$. 4. $15\frac{1}{2} \times 15\frac{1}{2}$.
5. $8\frac{1}{4} \times 8\frac{1}{4}$. 6. $12\frac{1}{4} \times 12\frac{1}{4}$. 7. $13\frac{1}{4} \times 13\frac{1}{4}$. 8 $15\frac{1}{2} \times 3\frac{3}{4}$.
9. $7\frac{1}{2} \times 11\frac{3}{4}$. 10. $10\frac{1}{4} \times 10\frac{3}{4}$. 11. $5\frac{3}{4} \times 5\frac{3}{4}$. 12. $9\frac{3}{4} \times 9\frac{3}{4}$.

Practical Exercise.

Draw rectangles on squared paper to shew that—

1. $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$. 2. $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$. 3. $4\frac{1}{2} \times 4\frac{1}{2} = 20\frac{1}{4}$.
4. $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$. 5. $6\frac{1}{2} \times 6\frac{1}{2} = 42\frac{1}{4}$. 6. $3\frac{1}{2} \times 2\frac{1}{4} = 7\frac{7}{8}$.
7. $3\frac{3}{4} \times 2\frac{1}{2} = 9\frac{3}{8}$. 8. $3\frac{1}{2} \times 3\frac{1}{4} = 11\frac{3}{8}$.

EXAMPLES. V.

Obtain the following products :—

1. $17\frac{1}{2} \times \frac{4}{5}$. 2. $10\frac{3}{4} \times \frac{12}{13}$. 3. $18\frac{1}{3} \times \frac{9}{10}$.
4. $112\frac{1}{2} \times \frac{12}{25}$. 5. $217\frac{1}{3} \times \frac{18}{163}$. 6. $12\frac{2}{5} \times 7\frac{1}{2}$.
7. $40\frac{1}{3} \times 1\frac{10}{11}$. 8. $72\frac{4}{9} \times 1\frac{17}{163}$.

227. The word **of** is often used instead of the sign $\times$.

For 7 of a quantity means 7 times the quantity or $7 \times$ the quantity.

For $\frac{5}{7}$ of $\frac{3}{11}$ we may write $\frac{5}{7} \times \frac{3}{11}$, both having the same meaning.

228. A fraction of a fraction is called a **Compound Fraction.**

EXAMPLE. $\frac{3}{4}$ of $\frac{5}{23}$; $\frac{1}{2}$ of $\frac{3}{25}$ of $\frac{7}{8}$.

A compound fraction is simplified by multiplying together the several fractions.

Caution.—Beginners are often apt to make mistakes in simplifying expressions like these—

(*a*) $\dfrac{8}{15} \times \dfrac{15}{8}$. (*b*) $\dfrac{8+9}{8 \times 9}$.

(*a*) $\dfrac{8}{15} \times \dfrac{15}{8} = \dfrac{1}{1} = 1$. (*b*) $\dfrac{8+9}{8 \times 9} = \dfrac{17}{72}$.

EXAMPLES. VI.

Multiply together—

1. $11\frac{2}{5} \times 5\frac{1}{7} \times 4\frac{1}{2} \times \frac{1}{12}$. 2. $4\frac{1}{3} \times \frac{27}{32} \times 4\frac{7}{26}$.

3. $84 \times 3\frac{3}{7} \times 2\frac{1}{24} \times \frac{3}{28}$. 4. $\frac{43}{126} \times \frac{17}{860} \times \frac{540}{51}$.

Find the value of—

5. $\frac{1}{6}$ of $1\frac{2}{7}$ of $\frac{14}{15}$ of $\frac{20}{21}$. 6. $\frac{1}{2}$ of $\frac{7}{12}$ of $\frac{3}{5}$ of $\frac{4}{11}$ of $3\frac{1}{7}$.

7. $7\frac{1}{2}$ of $2\frac{2}{3}$ of $5\frac{1}{8}$ of $\frac{40}{41}$. 8. $\frac{3}{7}$ of $1\frac{3}{5}$ of $12\frac{1}{2}$ of $2\frac{1}{5}$ of $\frac{3}{44}$.

9. $2\frac{2}{5}$ of $1\frac{3}{4}$ of $7\frac{1}{2}$ pies $+$ $3\frac{2}{3}$ of $1\frac{10}{11}$ of $\frac{3}{14}$ of 4 pies.

10. $\frac{2}{5}$ of $1\frac{1}{9}$ of 16 mds. 7 viss—$3\frac{1}{2}$ of $\frac{2}{7}$ of 30 pals. $2\frac{1}{2}$ tolas.

11. $1\frac{1}{2}$ of 0·3 £ $+$ $\dfrac{\cdot 83}{\cdot 14}$ of 7*s.*—$3\frac{3}{4}$ of 8*d.* $+$ $\dfrac{3\cdot 45}{4\cdot 35}$ of 7*s.* 3*d.*

12. $\frac{3}{4}$ of $\frac{2}{19}$ of £11-8*s.* $+$ $\frac{2}{3}$ of ·375 of 15*s.* $+$ $\frac{2}{5}$ of $\frac{85}{198}$ of 8*s.* 3*d.*

Division by Fractions.

229. Illustration from the scales.

Take a scale with tenths of an inch marked ; and find how many three-tenths of an inch are there in 3 inches ; that is, find the value of 3 in. $\div \frac{3}{10}$ in.

The number required is found to be 10.

$\therefore$ 3 in. $\div \frac{3}{10}$ in. $= 3 \times \frac{10}{3} = 10$.

If in the scale eighths of an inch are marked, it will be seen that $3\frac{1}{2}$ in. $\div \frac{1}{4}$ in. $= 14$; and $\frac{3}{4}$ in. $\div \frac{1}{8}$ in. $= 6$.

$\therefore 3\frac{1}{2} \div \frac{1}{4} = 14$, and $\frac{3}{4} \div \frac{1}{8} = 6$.

230. Graphic Illustration to shew that $\frac{3}{4} \div \frac{1}{8} = 6$.

In fig. 29, AEFD containing 24 squares, each square consisting of 4 small squares, represents $\frac{3}{4}$ of the unit; and AGHD containing 4 such squares represents $\frac{1}{8}$ of the unit.

$\therefore$ AEFD $\div$ AGHD represents $\frac{3}{4} \div \frac{1}{8}$.

But AEFG contains AGHD 6 times.

$\therefore \frac{3}{4} \div \frac{1}{8} = 6$.

In all these cases it will be observed that to divide by a fraction, the dividend is multiplied by the inverted divisor or by the reciprocal of the divisor.

EXAMPLE. $\frac{8}{11} \div \frac{9}{13} = \frac{8}{11} \times \frac{13}{9} = \frac{104}{99} = 1\frac{5}{99}$.

Practical Exercise.

1. Use your scale to shew that—

(1) $3\frac{1}{2}$ in. $\div \frac{1}{2}$ in. $= 7$. (2) $4\frac{1}{4}$ in. $\div \frac{1}{4}$ in. $= 17$.

(3) $4\frac{1}{2}$ in. $\div \frac{3}{4}$ in. $= 6$. (4) $3\frac{1}{5}$ cm. $\div \frac{4}{5}$ cm. $= 4$.

2. Use your scales to find the quotient of—

(1) $10\frac{5}{12} \div \frac{5}{12}$. (2) $11\frac{1}{4} \div 1\frac{1}{8}$. (3) $7\frac{1}{2} \div 1\frac{1}{4}$. (4) $12\frac{3}{4} \div 3\frac{1}{4}$.

3. Draw on squared paper a rectangle—

(1) 4 in. by 3 in., divide it into 48 smaller rectangles and the quotient of $\frac{3}{4} \div \frac{1}{12}$.

(2) 3 in. by 2 in., divide it into 24 smaller rectangles and obtain the quotient of $\frac{2}{3} \div \frac{1}{6}$.

(3) 6 cm. by 5 cm. and divide it into rectangles and find the value of $\frac{2}{5} \div \frac{1}{10}$.

(4) 8 cm. by 7 cm. and use it to find $\frac{3}{8} \div \frac{3}{7}$.

4. Find graphically by drawing suitable rectangles the following quotients:—

(1) $\frac{4}{7} \div \frac{5}{8}$. (2) $\frac{5}{12} \div \frac{4}{7}$. (3) $\frac{4}{15} \div \frac{3}{4}$. (4) $\frac{7}{16} \div \frac{4}{5}$.

231. Before dividing, mixed numbers must first be reduced to improper fractions.

EXAMPLE. $2\frac{7}{10} \div 1\frac{1}{8} = \frac{27}{10} \div \frac{9}{8} = \frac{27}{10} \times \frac{8}{9} = \frac{3}{5} \times \frac{4}{1} = \frac{12}{5} = 2\frac{2}{5}$.

EXAMPLES.—ORAL.

Find the value of—

1. $10\frac{1}{2}$ as. $\div \frac{3}{4}$ anna. 2. 20 m. $\div 2\frac{1}{4}$m. 3. $13\frac{1}{2}d. \div 2\frac{3}{4}d.$

4. $13 \div 3\frac{1}{4}$. 5. $15 \div 2\frac{1}{2}$. 6. $15 \div 3\frac{3}{4}$.

7. $7\frac{1}{2} \div 2\frac{1}{2}$. 8. $9\frac{3}{4} \div 3\frac{1}{4}$. 9. $16\frac{1}{4} \div 3\frac{1}{4}$. 10. $17\frac{1}{4} \div 2\frac{1}{2}$.

EXAMPLE. Find the value of £28-16-11 $\div 3\frac{5}{6}$.

$$£28\text{-}16\text{-}11 \div 3\tfrac{5}{6} = £28\text{-}16\text{-}11 \div \tfrac{23}{6} = \frac{£28\text{-}16\text{-}11 \times 6}{23}$$

$$= £1\text{-}5\text{-}1 \times 6 = £7\text{-}10\text{-}6.$$

EXAMPLES. VII.

Find the value of—

1. Rs. 100-9-8 $\div 2\frac{3}{4}$. 2. Rs. 120-11-5 $\div 12\frac{5}{6}$.

3. £3-15-9 $\div 6\frac{1}{7}$. 4. £733-10-6 $\div 15\frac{1}{4}$.

5. If $\frac{7}{8}$ md. costs Rs. 3-8 as., find the cost of 1 md.

6. If $\frac{15}{16}$ of an acre of land is sold for Rs. 1,500-1-3, what is the cost of 1 acre ?

7. If in $1\frac{1}{2}$ hrs. a man walks 6 miles, how far does he walk in 1 hr. ?

8. If in $1\frac{3}{4}$ months, I spend Rs. 105, what do I spend per mensem?

9. Divide—

(1) $\frac{9}{16}$ by $\frac{3}{8}$. (2) $\frac{11}{12}$ by $\frac{5}{9}$.

(3) $\frac{14}{17}$ by $\frac{28}{51}$. (4) $\frac{32}{75}$ by $\frac{8}{15}$.

10. Find the value of—

(1) $276 \div 3\frac{11}{27}$. (2) $\frac{1275}{308} \div \frac{525}{462}$.

(3) $72\frac{8}{11} \div 1\frac{31}{44}$. (4) $468\frac{3}{4} \div 29\frac{19}{64}$.

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232. **A Complex Fraction** is one in which the numerator or the denominator is a fraction or both are fractions.

EXAMPLES $\frac{3\frac{1}{2}}{5}$, $\frac{7}{4\frac{1}{4}}$, $\frac{2\frac{1}{2}}{5\frac{1}{3}}$, $\frac{3\frac{1}{4} + 2\frac{3}{5}}{3\frac{1}{4} - 2\frac{3}{5}}$.

EXAMPLES. VIII.

Find the value of—

1. $\frac{2\frac{1}{4}}{1\frac{2}{7}}$ of 4 as.— $\frac{1\frac{1}{2}}{3\frac{1}{2}}$ of $\frac{7}{16}$ of Re. 1 + $\frac{1\frac{1}{7}}{2\frac{1}{4}}$ of 1 anna.

2. $\frac{2}{2\frac{1}{2}}$ of 3 can. 15 mds. + $\frac{1\frac{1}{3}}{3\frac{1}{3}}$ of 7 viss 24 pals. -- $\frac{8\frac{1}{2}}{5\frac{2}{3}}$ of 3 mds [$1\frac{1}{2}$ tolas.

3. $\frac{8}{15}$ of 3*s*. $1\frac{1}{2}$*d*. + $\frac{4\frac{1}{8}}{8\frac{1}{4}}$ of $\frac{5\frac{1}{5}}{4\frac{1}{3}}$ of 5*s*. — $\frac{\frac{2}{5}}{1\frac{3}{4}}$ of 2*s*. 11*d*.

4. $\frac{3\frac{1}{2} + 2\frac{3}{4} - 1\frac{1}{4}}{4\frac{1}{2} - 2\frac{1}{2} + 1\frac{1}{6}}$ of Rs. 52-8 as. — $\frac{14\frac{1}{8}}{21}$ of Rs. 10-8 as. + $\frac{1\frac{1}{2}}{4\frac{1}{2}}$ of Rs. 5.

233. In simplifying fractions in which the signs + or − and × or ÷ occur, first perform the process of multiplication or division unless the fractions are separated by brackets, and then add or subtract.

EXAMPLE. $(12\frac{5}{6} - 8\frac{3}{4} - 1\frac{1}{10} + \frac{8}{15}) \times 4\frac{1}{2} \times (7\frac{5}{12} - 6\frac{1}{2})$.

Were it not for the brackets, this would mean that $\frac{8}{15}$, $4\frac{1}{2}$ and $7\frac{5}{12}$ should be multiplied together; and then that the result should be added to the rest.

234. Where × and ÷ occur one after the other, as each symbol applies only to the number immediately following it, proceed to simplify by taking the signs in their order, beginning from the left.

EXAMPLE. $\frac{7}{8} \div \frac{3}{4} \times 2\frac{1}{3} = \frac{7}{8} \times \frac{4}{3} \times \frac{7}{3} = \frac{49}{18} = 2\frac{13}{18}$.

$\frac{7}{8} \div (\frac{3}{4} \div 2\frac{1}{3}) = \frac{7}{8} \times \frac{3}{4} \times \frac{3}{7} = \frac{9}{32}$.

Note.—When two quantities are connected by 'of', it is usual to consider them as a single quantity.

EXAMPLE. $\frac{7}{8} \div \frac{3}{4} \div \frac{2}{3} \text{ of } \frac{5}{7} = \frac{7}{8} \div \frac{3}{4} \div (\frac{2}{3} \text{ of } \frac{5}{7})$

$= \frac{7}{8} \div \frac{3}{4} \div \frac{10}{21} = \frac{7}{8} \times \frac{4}{3} \times \frac{21}{10} = \frac{7}{2} \times \frac{7}{10}$.

$= \frac{49}{20} = 2\frac{9}{20}$.

EXAMPLES. IX.

Simplify—

1. $3\frac{1}{6} + \frac{2}{3} \text{ of } \frac{5}{12} - 2\frac{1}{4}$.
2. $1\frac{3}{28} + \frac{3}{4} \times 2\frac{4}{7} + \frac{5}{9} \text{ of } \frac{3}{7} \text{ of } 6$.
3. $1\frac{3}{4} \times 2\frac{1}{2} + 5\frac{1}{2} \div 3\frac{1}{8}$.
4. $2\frac{1}{2} \times 11 + 1\frac{5}{8} \div 2\frac{3}{4} + 7\frac{5}{16} \text{ of } 4\frac{4}{11}$.
5. $\frac{2}{3} \times 1\frac{1}{2} \div \frac{1}{2} \times 1\frac{31}{51} \div \frac{3}{8}$.
6. $\frac{2}{3} \div 1\frac{1}{2} \times \frac{1}{2} \div 1\frac{3}{5} \times \frac{3}{8}$.
7. $2\frac{1}{2} \times 3\frac{2}{5} \div 1\frac{2}{15} \times \frac{1}{7\frac{1}{2}}$.
8. $7\frac{1}{2} \div 3\frac{3}{4} \times 1\frac{7}{15} \times 1\frac{4}{11}$.

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235. G.C.M. of Fractions.

The G.C.M. of two or more fractions is the greatest number integral or fractional, that can divide each of the given fractions *exactly*, that is, an integral number of times.

In other words the G.C.M. is the number which when inverted and multiplied by each of the fractions will produce an intgeral quotient. Hence the numerator of the G.C.M. should not only be a common measure but should be the greatest possible common measure of the numerators of the fractions and the denominator of the G.C.M. should be not only a common multiple but the least possible common multiple of the denominators of the given fractions.

∴ *The G.C.M. of fractions in their lowest terms,*

$$= \frac{\textit{G.C.M. of Numerators}}{\textit{L.C.M. of Denominators}}.$$

Example. Required the G.C.M. of $\frac{27}{26}$, $\frac{9}{13}$ and $\frac{12}{65}$.

The G.C.M. of 27, 9 and 12 is 3.

And the L.C.M. of 26, 13 and 65 is 130. ∴ The G.C.M. is $\frac{3}{130}$.

236. L.C.M. of Fractions.

The L.C.M. of two or more fractions is the least number, integral or fractional, that contains each of the given fractions an integral number of times.

If the L.C.M. is divided by each fraction, the quotient will be an integer; that is, if the L.C.M. is multiplied by each fraction inverted or by the reciprocal of each fraction, the product will be an integer.

Therefore the numerator of the L.C.M. required should be the least common multiple of the numerators and the denominator should be the greatest common measure of the denominators of the given fractions.

∴ *The L.C.M. of fractions in their lowest terms,*

$$= \frac{\textit{L.C.M. of Numerators}}{\textit{G.C.M. of Denominators}}.$$

Example. Find the L.C.M. of $1\frac{4}{9}$, $\frac{26}{27}$ and $\frac{7}{18}$.

The L.C.M. of 13, 26 and 7 is 182.

And the G.C.M. of 9, 27 and 18 is 9.

∴ the L.C.M. is $\frac{182}{9}$.

EXAMPLES. X.

1. Four men fire at a target at intervals 9, $10\frac{1}{2}$, $15\frac{3}{4}$ and $18\frac{2}{3}$ minutes respectively. After what time will they all fire together?

2. Five boys run round a circular garden, starting at the same time from the same place, and running in the same direction, can complete one round in 5, $7\frac{1}{2}$, 10, $6\frac{2}{3}$ and $8\frac{1}{3}$ minutes respectively. When will they be together next?

3. Find the least sum of money I should have that I may buy books costing Rs. $2\frac{1}{2}$, Rs. $3\frac{3}{4}$, Rs. 5 or Rs. $6\frac{2}{3}$ each.

4. What is the smallest whole number which is exactly divisible by $3\frac{1}{2}$, 15 and $17\frac{1}{2}$?

Find the G.C.M. and L.C.M. of—

5. $\frac{8}{9}, \frac{12}{44}, \frac{7}{18}, \frac{27}{66}.$ 6. $\frac{51}{76}, \frac{111}{152}, \frac{78}{76}.$

7. $\frac{24}{32}, \frac{7}{12}, 2\frac{1}{6}, 5\frac{1}{2}.$ 8. $\frac{11}{12}, \frac{29}{30}, \frac{17}{18}, \frac{49}{112}.$

CHAPTER XIX.

ONE QUANTITY AS A FRACTION OF ANOTHER, DECIMALIZATION OF MONEY, FRACTIONS AS RATIOS, REPRESENTATIVE FRACTION.

237. To express a given quantity as the fraction of another, reduce both the quantities to the same denomination and divide the former by the latter. The result is an abstract number; for we then find how many times or parts of times the one contains the other.

EXAMPLE 1. Express $\frac{3}{4}$ of Rs. 4-8 as. as the decimal of $\frac{2}{3}$ of Rs. 6-12 as.

$\frac{3}{4}$ of Rs. 4-8 as. = Rs. 3-6 as. = 27 two annas.
$\frac{2}{3}$ of Rs. 6-12 as. = Rs. 4-8 as. = 36 two annas.
The decimal reqd. $= \frac{27}{36} = \frac{3}{4} = {\cdot}75.$
Or, proceed thus :
Rs. 3-6 as. = Rs. 3·375. Rs. 4-8 as. = Rs. 4·5.

$$\therefore \text{ Decimal reqd.} = \frac{3{\cdot}375}{4{\cdot}5} = {\cdot}75.$$

EXAMPLE 2. What fraction of 14 cwt. 3 qrs. 14 lbs. is equal to $\frac{3}{5}$ of a ton ?

The question in other words is—Reduce $\frac{3}{5}$ of a ton to the fraction of 14 cwt. 3 qrs. 14 lbs.

$$\text{The fraction required} = \frac{\frac{3}{5} \text{ of a ton}}{14 \text{ cwt. } 3 \text{ qrs. } 14 \text{ lbs.}}$$

$$= \frac{12 \text{ cwt.}}{14\frac{7}{8} \text{ cwt.}} = \frac{12}{\frac{119}{8}} = \frac{96}{119}.$$

EXAMPLES.—ORAL.

Express the first of these quantities as the fraction of the second—

1. 3*s*. 4*d*. of £1. 2. 2*s*. 6*d*. of £1. 3. 10 as. 8 p of R. 1.
4. 13*s*. 4*d*. of £1. 5. 11 as. of R. 1-6 as. 6. 8 in. of 2 yds.
7. 2*s*. 6*d*. of $3\frac{3}{4}$*s*. 8. $7\frac{1}{2}$ as. of $11\frac{1}{4}$ as. 9. $7\frac{1}{2}$*s*. of $12\frac{1}{2}$*s*.

EXAMPLES. I.

I. Express the first of these as a decimal of the second working decimally—

1. 12*s*. 3*d*. ; £1. 2. 3*s*. 6*d*. ; £5.
3. 18*s*. $4\frac{1}{2}$*d*. ; £1,000. 4. 2*s*. 6*d*. ; $\frac{5}{12}$ of £1,000.
5. $\frac{12}{25}$ of £20 ; £100. 6. 13 as. 6 p. ; Rs. 1,000.
7. £5-18-9 ; £10-8-4. 8. Rs. 483-12-6 ; Rs. 1,290-1-4.

II. 1. What fraction of a sovereign is $(4\frac{14}{21} - 10\frac{44}{55} + 9\frac{3}{45} - \frac{52}{117})$ of a penny ?

2. What part of $\frac{1}{3}$ of Rs. 15-12-6 is $\frac{2}{3}$ of $\frac{5}{6}$ of Rs. 4-8-0 ?

3. What part of $1\frac{1}{5}$ of £12-7-6 is equal to $1\frac{1}{3}$ of £2-9-6 ?

4. What fraction is $\frac{3}{5}$ of a ton of 14 cwt. 3 qrs. ?

III. Work in decimals—

1. What decimal of a ton is 7 oz. ?

2. Express 2 cwt. 1 qr. 22 lbs. as the decimal of a ton (to 4 places).

3. What decimal fraction of a mile is 681 yds. 0 ft. $4\frac{8}{25}$ in. ?

4. Express 5 as. 7 p. as the decimal of a rupee (to 4 places).

EXAMPLE. What part of Rs. 4-13-0 must be added to $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of Rs. 14-8-0 to make the result Rs. 5 ?

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of Rs. 14-8 as. = $\frac{1}{4}$ of Rs. 14-8 as. = Rs. 3-10 as.

The question in other words is : Reduce (Rs. 5 – Rs. 3-10 as.) to the fraction of Rs. 4-13 as.

Rs. 5 – Rs. 3-10 as. = Re. 1-6 as. = 22 as.
Rs. 4-13 as. = 77 as.

$\therefore$ The fraction required $= \frac{22}{77} = \frac{2}{7}$.

EXAMPLES. II.

1. What decimal of half a sovereign together with 3*s.* will give half a guinea ?

2. What fraction of 2 cwt. 2 qrs. together with 3 qrs. 14 lbs. is equal to 1 ton 10 cwt. ?

3. What decimal of 20 mds. must be added to ·125 of 3 mds. 6 viss to get 5 mds. ?

4. What part of Rs. 12-8 as. must be subtracted from Rs. 30-12 as. to get $7\frac{1}{2}$ pagodas ?

FRACTIONS AS RATIOS.

238. Take two rectangles A and B divided into equal squares as in figure 33.

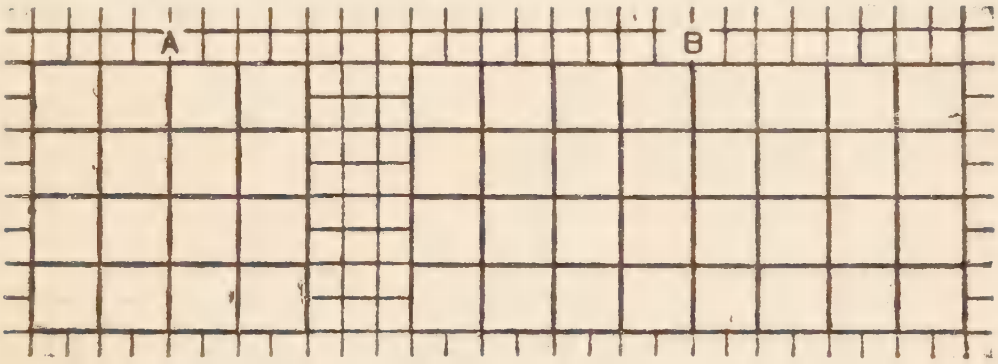

FIG. 33.

Comparing A and B, since A has 16 squares and B 32 squares, $\frac{B}{A}=\frac{32}{16}=2$; and $\frac{A}{B}=\frac{16}{32}=\frac{1}{2}$. B is double of A, and A is half of B.

Here A and B are compared one with the other by considering how many times or parts of times the one is of the other.

239. Take AB, 3 inches long, and mark off AC one inch in length.

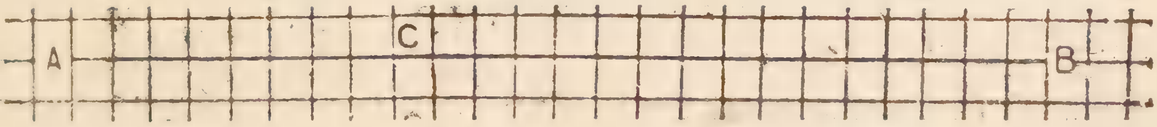

FIG. 34

Comparing AB and AC, AB is 3 times AC ; and AC is $\frac{1}{3}$ of AB ; comparing AB and CB, CB is $\frac{2}{3}$ of AB, and AB is $1\frac{1}{2}$ times CB.

The relation that exists thus between any two quantities of the same kind is called **Ratio.**

The ratio of one quantity A to another B of the same kind may be expressed as a fraction whose numerator is A and denominator B and is written $\frac{A}{B}$.

A ratio thus points out the relation between the numerator and the denominator of a fraction.

In figure 34, the ratio of AB to AC $= \frac{3 \text{ in.}}{1 \text{ in.}} = 3$; of AC to AB $= \frac{1 \text{ in.}}{3 \text{ in.}} = \frac{1}{3}$, and of CB to AB $= \frac{2 \text{ in.}}{3 \text{ in.}} = \frac{2}{3}$.

The ratio in each case is an abstract number, either integral or fractional, because a ratio expresses the number of times or parts of times one quantity contains another of the same kind.

The ratio of A to B may also be written A : B; and is read 'A is to B.'

The sign : may be taken as the sign of division ÷ with the line between the points omitted.

The ratio of A to B $= A : B = A \div B = \frac{A}{B}$.

EXAMPLES. 1. The ratio of Rs. 10 to Rs. 15 $= \frac{\text{Rs. } 10}{\text{Rs. } 15} = \frac{2}{3}$.

2. The ratio of $3\frac{1}{3}$ to 15 $= \frac{3\frac{1}{3}}{15} = \frac{10}{3 \times 15} = \frac{2}{3 \times 3} = \frac{2}{9}$.

Practical Exercise.

1. Take a line AB 2 cm. long and another CD 6 cm. long; and find the ratio AB to CD and of CD to AB.

2. Draw on squared paper a straight line AB 4 inches long and divide it unequally at the points C and D. Find the ratio of the parts to one another and of each of the parts to the whole line.

3. Draw on squared paper two straight lines AB and CD each 3 inches long; let AC contain 6 divisions and CF 8 divisions.

Find the ratio of AC to AB, CF to AB or CD, AC to CF and CB to FD.

4. Draw two rectangles A and B each 5 cm. by 4 cm. and divide into squares equal to one another. Use them to find the ratio of $\frac{2}{5}$ to $\frac{3}{4}$.

EXAMPLES.—ORAL.

Find the values of the following ratios :—

1. 9 days : 6 days.　2. 20 mds. : 25 mds.　3. 21 lbs. : 28 lbs.
4. 27 wks. : 45 wks.　5. £$3\frac{1}{2}$: £$5\frac{1}{4}$.　6. £$7\frac{1}{2}$: £$12\frac{1}{2}$.
7. 9 : $1\frac{1}{2}$.　8. $1\frac{1}{2} : 7\frac{1}{2}$.　9. $1\frac{3}{4}$: $5\frac{1}{4}$.

---

240. If the concrete numbers forming a ratio be of the same kind, but not of the same denomination, they must be reduced to one and the same **Denomination** before finding their ratio.

Thus the ratio between 5*s*. and 11*d*. is not $\frac{5}{11}$ but $\frac{5 \times 12}{11}$ or $\frac{60}{11}$; for 5*s*. and 11*d*. though of the same kind are not of the same denomination ; therefore 5*s*. should be reduced to pence or 11*d*. to shilling.

EXAMPLE. Express as a decimal the ratio of £2-10*s*. to £3-2-6.

£2-10*s*. = 20 half-crowns ; and £3-2-6 = 25 half-crowns.

∴ The ratio $= \frac{20}{25} = \frac{4}{5} = \cdot 8$.

Or since £2-10*s*. = £2·5 ; and £3-2-6 = £3·125 the ratio $= \frac{2\cdot 5}{3\cdot 125} = \cdot 8$.

EXAMPLES.—ORAL.

Express as a fraction the ratio of—

1. £1 to 1 guinea.　2. 6 in. to 1 yd.　3. 6 as. to Rs. 3.
4. 6 fur. to 2 mi.　5. 5 mm. to 7 cm.　6. 600 m. to 4 Km.
7. 500 g. to 2 Kg.　8 3 hrs. to 2 days.　9. 50 links to 4 fur.

EXAMPLES. III.

Express as fractions the following ratios :—

1. Rs. 2-13-4 : Rs. 11-5-4.　2. £4-12-8 : £5-15-10.
3. 3 mds. 6 viss : 5 mds. 5 viss.
4. 4 hrs. 33 min. : 3 hrs. 30 min.

Express as a decimal the ratio of—

5. £1-2-6 to £5.　6. Rs. 1-13-4 to Rs. 2-10-8.
7. 5 cwt. 3 qrs. to 1 ton.　8. 5 yds. 1 ft. 6 in. to $2\frac{1}{2}$ miles.

Express as a fraction the ratio of—

9. $12\frac{5}{6}$ of 1 qr. $3\frac{1}{2}$ lbs. to 1 ton 2 cwt.
10. $5\frac{1}{2}$ of 5 days 20 hrs. to 31 wks. 3 days.

11. $1\frac{1}{2}$ of $1\frac{1}{3}$ of $3\frac{1}{2}$ mds. to 2 can. 2 mds.

12. $2\frac{2}{3}$ of £1-3-9 to £3-16*s*.

Representative Fraction.

241. When you look at a map of India or of any other country, you know that the real length or breadth of India or of that country is not what you see there, but that it is very much reduced. That is, a very long distance is represented by a short length.

If the distance of one mile is represented on a map by a line an inch long, the map is drawn to a **scale** of 1 inch to the mile. That is, one inch distance on the map represents one mile of the real distance. Therefore any distance on the map is $\frac{1}{1760 \times 36}$ or $\frac{1}{63360}$ of the actual distance; and the scale shews the ratio of the distance on the map to the real distance.

The fraction denoting this ratio is called the **Representative Fraction:** and the letters **R.F.** are used to signify it.

Thus in a map drawn to the scale of 1 inch to 40 miles, the **R.F.** $= \frac{1 \text{ in.}}{40 \text{ mi.}} = \frac{1}{40 \times 1760 \times 36} = \frac{1}{2534400}$; and in a map drawn to the scale of 1·5 in. to 100 yds., the **R.F.** $= \frac{1\frac{1}{2}}{100 \times 36} = \frac{1}{2400}$.

EXAMPLES.—ORAL.

Drawings are made to the following scales; what is the R.F. in each case:—

(1) 1 in. = 1 yd. (2) 0·1 in. = 1 ft. (3) 1 in. = 7·5 ft.
(4) $1\frac{1}{2}$ in. = 1 chain. (5) 1 link = 1 fur. (6) 0·5 in. = 10 ft.
(7) 1 ft. = 1 mile. (8) 1 dm. = 1 m. (9) 1 cm. = 1 km.

A drawing is made so that $\frac{1}{2}''$ on it represents $1'$; what would be the following lengths on the drawing?

(1) $3'\ 6''$. (2) $4'\ 8''$. (3) $10'\ 3''$. (4) $6'\ 5''$.

What would the above lengths have been, if the drawing had been $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{12}$ the actual size?

Practical Exercise.

1. Measure the distances between any two towns indicated by the teacher on the map provided; and find the real distances. The scale will be found on the map.

2. A square has a side of 18 ft. Construct it on squared pape to the scale of 1 in. to 1 ft. and measure the diagonal.

3. Construct a square whose side is 15 in., the R.F. being $\frac{1}{12}$; and measure the diagonal.

4. Measure the length and breadth of your class room and make a plan of the floor. Scale 10 ft. = 1 in.

5. Make a plan of a rectangular plot of ground whose adjacent sides are 250 yds. and 100 yds. Scale 100 yds. to 1 inch.

6. A rectangular block of granite measures 36 cm. by 12 cm. Make a drawing of it; scale 1 cm. = 1 mm.

EXAMPLES. IV.

Drawings are made to the following scales; find the R.F. in each case:—

1. $1\frac{1}{2}$ in. = 1 mile. 2. $\frac{1}{4}$ in. = 2 fur.

3. 2·5 in. = 20 poles. 4. 2·5 in. = 1 Km. (1m. = 39·37 in.)

5. The distance between two towns on a map is 3·5 in. The actual distance is 70 miles. Find the R.F. of the map.

6. Find the Representative Fraction of a map where a distance of 3 fur. 9 ch. is represented by a line 2·574 in. long.

7. The R.F. of a map is $\frac{1}{2500}$; what length on the map will represent 10 miles?

8. The R.F. of a map is $\frac{1}{528000}$; what length on the map will represent 150 miles?

9. The R.F. of a map is $\frac{1}{36000}$; the distance between two towns is 2·5 in. What is the actual distance?

10. On a survey map drawn to a scale of $\frac{1}{1584000}$, the length of a field measures ·03 of an inch. What is the length of the field?

11. The R.F. of a map of France is $\frac{1}{600000}$ and the distance between two towns is 2·5 cm. How many English miles apart are the towns? 1 m. = 39·37 in.

12. A drawing is made to the scale of 6 in. to the mile. Another scale drawing is made with the same Representative Fraction. What length of line in this represents a distance of 5·28 Km.?

DECIMALIZATION OF MONEY.

242. To Decimalize Shillings.

Since 1*s*. = $\pounds\frac{1}{20}$ = ·05 of £1, any number of shillings below 20 (£1) can be decimalized by multiplying ·05 by that number; i.e. by multiplying the number of shillings by 5 and prefixing the decimal point.

Thus 7*s*. = ·35£; 13*s*. = ·65£; 19*s*. = ·95£.

Conversely, dividing by 5 the first two figures of a decimal of £1, will give the number of shillings.

Thus ·75£ = 15*s*.; ·90£ = 18*s*.; ·55£ = 11*s*.

243. To Decimalize Farthings.

Since 1*s*. = ·05£.
$\frac{1}{2}$*s*. or 6*d*. = ·025£.
1*d*. = $\cdot004\frac{1}{6}$£.
1*f*. or $\frac{1}{4}$*d*. = **$\cdot001\frac{1}{24}$£.**

Since 48 farthings make 1 shilling, any number of farthings below 48 can be decimalized correct to three places by multiplying $\cdot001\frac{1}{24}$ by that number.

Thus, $3\frac{1}{2}$*d*. or 14*f*. = $\cdot014\ \frac{14}{24}$£ = ·015£.
$7\frac{3}{4}$*d*. or 31*f*. = $\cdot031\ \frac{31}{24}$£ = $\cdot032\frac{7}{24}$£ = ·032£.
$11\frac{1}{4}$*d*. or 45*f*. = $\cdot045\ \frac{45}{24}$£ = $\cdot046\frac{21}{24}$£ = ·047£.
= $\cdot046\ \ \frac{7}{8}$£ = ·046875£ (exactly).

Thus any number of farthings can be decimalized to three places by adding to that number $\frac{1}{24}$ of itself.

244. Combining both these processes any number of shillings and pence can be decimalized either approximately or exactly.

But as ·001£ is less than 1 farthing, decimalization to the third figure corrected, will often be sufficient.

Thus 3*s*. $5\frac{1}{2}$*d*. = ($\cdot15 + \cdot001\frac{1}{24} \times 22$) £ = (·15 + ·023) £ = ·173£ to [three places.
7*s*. $6\frac{3}{4}$*d*. = ($\cdot35 + \cdot001\frac{1}{24} \times 27$) £ = ($\cdot35 + \cdot028\frac{1}{8}$) £ = ·378125£
£5-9-$11\frac{1}{2}$ = ($5\cdot45 + \cdot046\frac{46}{24}$) £ = ($5\cdot45 + \cdot047\frac{11}{12}$) £ = 5·497916£

Note.—After some practice the intermediate steps can be done mentally.

EXAMPLES.—ORAL.

Decimalize—3*s*., 5*s*., 7*s*., 9*s*., 12*s*., 13*s*., 14*s*., 16*s*., 19*s*.

Find the number of shillings in—·25£, ·35£, ·55£, ·65£, 70£, ·85£.

Decimalize—5*s*. 6*d*., 9*s*. 6*d*., 12*s*. 6*d*., 15*s*. 6*d*., 17*s*. 6*d*., 19*s*. 6*d*.
[Ex. 18*s*. 6*d* = (·90 + ·025) £ = ·925£.]

Express as shillings and six pence—·375£, ·425£, ·575£, ·675£, ·725£, ·825£. [Ex. ·325£ = (·30 + ·025) £ = 6*s*. 6*d*.]

Decimalize to three places—1¾*d*., 2¼*d*., 3½*d*., 4¾*d*., 5¾*d*., 6½*d*., 7¼*d*., 8¾*d*., 9½*d*., 10¼*d*., 11¾*d*.

EXAMPLES. V.

Decimalize to three places, or accurately where you can—

1. 10*s*. 5*d*.
2. 12*s*. 4½*d*.
3. 15*s*. 2¾*d*.
4. 16*s*. 1¼*d*.
5. 13*s*. 8*d*.
6. 17*s*. 6½*d*.
7. £1-11-7.
8. £2-10-9¾.
9. £5-11-8¼.
10. £15-9-7½.
11. £29-3-11¾.
12. £76-19-10½.

EXAMPLES.—ORAL.

Decimalize to three places—

1. 3*s*. 1¼*d*.
2. 5*s*. 1½*d*.
3. 7*s*. 3¾*d*.
4. 9*s*. 4½*d*.
5. 10*s*. 5¼*d*.
6. 12*s*. 3*d*.
7. 13*s*. 4¾*d*.
8. 15*s*. 4*d*.
9. 15*s*. 2¾*d*.
10. 16*s*. 7*d*.
11. 17*s*. 3½*d*.
12. 18*s*. 9*d*.
13. £1-5-2¾.
14. £2-6-3¼.
15. £3-4-5½.
16. £5-5-3¾.

---

245. To convert sums of money expressed in decimals of a £ in £. *s*. *d*.

We have already seen that when the first two decimal figures are divided by 5, the quotient represents the number of shillings and since any number of farthings can be decimalized to three places by adding to that number $\frac{1}{24}$ of itself, the remaining figures up to the third place represent the number of farthings increased by $\frac{1}{24}$ of that number.

We, therefore, find the number of farthings by subtracting $\frac{1}{24}$ of the number.

Thus ·041£ = 0*s*. + 41*f*. − $\frac{41}{24}$*f*. = 41*f*. − 2*f*. nearly = 39*f*. = 9¾*d*.

EXAMPLES. 1. ·378£ = 7*s*. + (28 − 1)*f*. = 7*s*. 27*f*. = 7*s*. 6¾*d*.
2. £3·440 = £3-8*s*. + (40 − 2)*f*. = £3-8-9½.

Since we first divide by 5 and then divide by 4 to reduce the farthings to pence, this rule is sometimes called the **rule of 5 and 4.**

Note.—After some practice, the student can do the steps mentally.

EXAMPLES.—ORAL.

Convert into £ *s. d.*

1. £·123. 2. £·375. 3. £·469 4. £·734.
5. £·678. 6. £2·345. 7. £9·876. 8. £5·963.
9. £15·679. 10. £21·234. 11. £19·456. 12. £36·782

Read off correct to three places and express as £ *s. d.*

1. £·2457. 2. £·3079. 3. £·4563. 4. £·67546.
5. £7·8916. 6. £6·9875. 7. £13·4568. 8. £24·3454.

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246. To decimalize pies.

1 farthing $= \frac{1}{960}$ £ $= \cdot 00\frac{1}{24}$ £.

1 pie $= \frac{1}{92}$ R. $= \frac{5}{960}$ R. $= 5 \times \cdot 001\frac{1}{24}$ R.

Any number of pies can be decimalized by multiplying that number by 5 times $\cdot 001\frac{1}{24}$.

Thus 11 pies $= 11 \times 5 \times \cdot 001\frac{1}{24}$ R. $= \cdot 055\frac{55}{24}$ R. $= \cdot 057$ to three places. 6 as. 9 p. $= 81 \times 5 \times \cdot 001\frac{1}{24}$ R. $= \cdot 405\frac{405}{24}$ R. $= 421\frac{7}{8}$ R. $=$ ·421375 R.

Thus any number of pies can be expressed as thousandths of a rupee, i.e. (to three places of decimals) by multiplying the number by 5 and adding to the product $\frac{1}{24}$ of itself.

247. Conversely to convert a decimal of a rupee to pies, when only three places are known, divide by 5 and the quotient less $\frac{1}{24}$ of itself is the number of pies.

EXAMPLE. ·344 R. $= (69 - \frac{1}{24}$ of $69)$ p. $= (69 - 3)$ p. $= 66$ p. $= 5$ as. 6 p. 3·223 R. = R 3 + 45 p. − 2 p. = Rs. 3-3-7.

Note.—Only approximate results can be got by this method.

EXAMPLES. VI.

I. Decimalize to three places—

1. 2 as. 3 p. 2. 5 as. 5 p. 3. 4 as. 9 p.
4. 7 as. 8 p. 5. 9 as. 10 p. 6. 10 as. 11 p.
7. Rs. 1-6-9. 8. Rs. 3-5-8. 9. Rs. 9-13-4.
10. Rs. 6-14-7. 11. Rs. 13-14-10. 12. Rs. 15-15-11.

II. Express as Rs. a. p.

1. ·875R. 2. ·364R. 3. ·475R.
4. ·603R. 5. ·786R. 6. ·589R.
7. Rs. 1·468 8. Rs. 3·752. 9. Rs. 5·321.
10. Rs. 8·765. 11. Rs. 10·842. 12. Rs. 13·974.

CHAPTER XX.

THE UNITARY METHOD.

248. Given the value of a certain number of units, the method of finding first the value of 1 unit and then using it to find the value of any multiple of the same unit, is called the **Unitary Method.**

EXAMPLE 1. If 20 maunds of sugar cost Rs. 50, find the cost of (*a*) 15 mds., (*b*) x mds.

The cost of 1 md. is evidently one-twentieth of the cost of 20 mds. ; i.e. $\frac{1}{20}$ of Rs. 50.

(*a*) The cost of 20 mds. of sugar = Rs. 50.
∴ The cost of 1 md. of sugar = Rs. $50 \times \frac{1}{20}$ = Rs. 2-8 as.
∴ The cost of 15 mds. = Rs. $50 \times \frac{15}{20}$ = Rs. $50 \times \frac{3}{4}$.
= Rs. 37-8 as.

(*b*) The cost of x mds. = Rs. $50 \times \frac{1}{20} \times x$ = Rs. $2\frac{1}{2}x$.

EXAMPLE 2. If $\frac{7}{12}$ of a yard of silk is sold for 14 as., find (1) the selling price of $\frac{3}{4}$ of a yard; (2) how many yards can be bought for Rs. 18.

(1) The selling price of $\frac{7}{12}$ yd. is 14 as.
∴ ,, $\frac{1}{12}$ yd. is $14 \times \frac{1}{7}$ as.
∴ ,, $\frac{12}{12}$ yd. or 1 yd. is $14 \times \frac{12}{7}$ as.
∴ ,, $\frac{3}{4}$ yd. is $14 \times \frac{12}{7} \times \frac{3}{4}$ as., or 18 as., i.e. R. 1-2 as.

(2) For R. 1-2 as. or R. $\frac{9}{8}$ can be bought 1 yard.
∴ for R. 1*.................................$\frac{8}{9}$ yd.
∴ for Rs. 18.................................$\frac{8}{9} \times 18$ yds. or 16 yds

EXAMPLES.—ORAL.

1. A motor-car travels 60 miles in 1 hr. What time does it take to travel 1 mile, (2) 40 miles, (3) 90 miles at the same rate?

2. A stone-mason earns Rs. 5 in 8 days. How much does he earn in 1 day? in 5 days? in 12 days? in x days?

3. 24 candles cost R. 1-4 as. : find the cost of 6, 8, x.

4. If one gross of pencils costs Rs. 3, find the cost of (1) 2 dozen, (2) 9 pencils.

5. If 1 dozen Ayre's tennis balls can be bought for Rs. 10-8 as., how many can be had for Rs. 2-10 as.?

6. If 10 measures of quicklime cost 3 as. 4 p., how many measures can be bought for Rs. 2-8 as.?

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* The intermediate step is omitted.

7. If 7 tons of coal cost £7-17-6, what will 8 tons cost?

8. If 10 lbs. of tea cost 12*s*. 6*d*., how many pounds can be bought for (1) 7*s*. 6*d*.? (2) £1?

9. If 3 persons are given board and lodging in a hotel for £17, how many can be taken at the same rates for £153.

10. If 13 copies of a book cost Rs. 12-8 as., how many cost Rs. 100?

EXAMPLES. I.

1. If the second-class railway fare for 99 miles is Rs. 3-1-6; what is it per mile?

2. A train goes at the rate of 45 miles per hour; what is the rate per second in feet?

3. The load in a truck is not to exceed 19 mds.; how many trucks would be required to carry goods weighing 19 can. 19 mds.?

4. Four seers of mutton cost R. 1-12 as.; what will 1 viss cost?

5. A person earns Rs. 45 in 6 weeks, how long will it take him to earn Rs. 120 at the same rate?

6. Find the cost of $15\frac{3}{8}$ yds. of twill, if $10\frac{1}{4}$ yds. cost Rs. $2\frac{3}{4}$.

7. A servant is hired on Rs. 5 a month of 30 days. What should be paid to him for June and July?

8. If 15 kilograms are equivalent to 33 lbs., how many kilograms are equal to 187 lbs.

9. The distance between Madras Beach and Tinnevelly Town is 448 miles by the S.I.R. What distance will this be on a map drawn to the scale of 24 miles to the inch?

10. If on another map the scale is 2 inches to the mile, what distance will be represented by 5·5 inches?

11. If 3 metres are equal to 9 ft. 10 in., how many metres are there in 10 yds. 1 ft. 6 in.?

12. If 1 kilometre $= \frac{5}{8}$ mile, how many kilometres are there in $17\frac{1}{2}$ miles?

13. If 1 kilogram $= 2\frac{1}{5}$ lbs., find in kilograms the weight of a ton and a half.

14. If 1 litre $= 1\frac{3}{4}$ pints, find the number of litres in a gal.

15. If 5 turns of a screw drive it in ·3 of an inch, to what depth will it be driven by 12 turns; and how many turns will drive it in 1·05 inches?

16. If ·875 of the contents of a purse be Rs. 63-5-3, what does it contain; and what is $\frac{3}{4}$ of it?

17. If $\frac{11}{25}$ of an estate be sold for Rs. 1,210, what will $\frac{2}{3}$ sell for at the same rate?

18. Having walked 7 miles, I find that $\frac{10}{21}$ of the journey is finished. How far have I yet to go?

19. Two partners A and B share the profits of a business; A gets Rs. 3,800, which is $\frac{19}{40}$ of the total profits. What does B get for his share?

20. If 1 metre = 39·3 inches, find the price of 3 yds. of silk which sells at 5*s.* $5\frac{1}{2}$*d.* per metre.

EXAMPLE. If 40 men can do a piece of work in 5 hrs., how long will (1) 1 man, (2) 8 men, (3) x men take to do it?

40 men can do the work in 5 hrs.

∴ 1 man ,, ,, ,, 40×5 hrs. or 200 hrs.

∴ 8 men ,, ,, ,, $\frac{40 \times 5}{8}$ hrs. or 25 hrs.

∴ x men ,, ,, ,, $\frac{40 \times 5}{x}$ hrs. or $\frac{200}{x}$ hrs.

EXAMPLES.—ORAL.

1. Five boys can do a piece of work in 8 days, in what time will (1) one boy do it? (2) 20 boys do it? (3) x boys do it?

2. Twenty men, can reap a field in 6 hrs., how long will (1) man (2) 12 men, (3) y men take to do the same?

3. Four pumps can empty a well in 6 hrs., how long will (1) two pumps, (2) six pumps, (3) $3x$ pumps empty it?

4. If 8 men can load a truck in 3 hrs., in what time will (1) 12 men, (2) 6 men load it?

5. A cistern is filled by a tap in 15 min., how long will it take to fill it, if the tap is half open?

6. A certain quantity of hay lasts 12 days for 15 horses, how long will the same quantity last for 20 horses?

7. If 12 men can bale out the water of a well in 5 hrs., how many should be employed to do the work in 3 hrs.?

8. If 21 masons can build a wall in 13 days, how many should be engaged to build it in 7 days?

9. A train going at 40 miles an hour, reaches a station in $2\frac{1}{2}$ hrs. How soon would it be reached, if the train went at 50 miles an hour?

10. A garrison of 600 men has provisions for 20 days; how many men should be withdrawn to make the provisions last for 30 days?

EXAMPLES. II.

1. A person walks from Palamcottah to Tuticorin at the rate of 4 miles an hour, and takes 9 hours. How long would he take if (1) he cycled at 12 miles an hour, (2) he travelled by train at 18 miles an hour?

2. If 87 men can do a piece of work in 38 days, how many men can do it in 57 days?

3. In how many days would 28 cows eat a supply of fodder that would last 21 days for 12 cows?

4. A garrison of 1,000 men has provisions for 15 days; how long will the provisions last, if it is (1) reduced by 400 men, (2) reinforced by 500 men?

5. By travelling in a train at 22 miles an hour, I finish my journey in 18 hours. How soon could the journey be finished in a motor at the rate of 45 miles an hour?

6. If when paddy is selling at Rs. 10-8 as. per *kotta*, rice is sold at $3\frac{3}{4}$ measures a rupee, at what price is paddy sold when a rupee will buy 4 measures?

7. A person by saving Rs. 500 every half-year can purchase an estate at the end of $4\frac{1}{2}$ years. How much sooner would he be able to buy it, if he laid by Rs. 750 every half-year?

8. A train goes from A to B in 6 hrs. 45 min. at the rate of 44 miles, and some time after another train goes at the rate of 45 miles an hour. What time is saved by taking this train?

Problems involving Fractions.

EXAMPLE. In a certain garden $\frac{1}{4}$ are mango trees, $\frac{1}{3}$ guavas and $\frac{1}{12}$ cocoanut trees; and there are 60 trees besides. How many trees are there in the garden and how many of each kind ?

$\frac{1}{4} + \frac{1}{3} + \frac{1}{12}$ or $\frac{2}{3}$ are mangoes, guavas and cocoanuts.
∴ The remaining $1 - \frac{2}{3}$ or $\frac{1}{3}$ are other trees.
Since $\frac{1}{3}$ of the total number of trees = 60,
the total ... ... $= 60 \times 3 = 180$.
$\frac{1}{4}$ of 180 or 45 are mango trees.
$\frac{1}{3}$ of 180 or 60 are guavas ; and $\frac{1}{12}$ of 180 or 15 are cocoanut trees.

EXAMPLES. III.

1. In a certain village, $\frac{1}{5}$ of the inhabitants can read, $\frac{1}{4}$ can read and write, and the remaining 550 can neither read nor write. Find the total number and the number in each class.

2. In a school, $\frac{3}{5}$ of the whole number are Christians, $\frac{2}{15}$ Mahomedans, $\frac{3}{25}$ Hindus and $\frac{1}{35}$ Europeans ; and there are 62 besides. How many are there in each class of people ?

3. A regiment returning from foreign service has only $\frac{11}{27}$ of its original number of men ; it has lost $\frac{2}{9}$ of its original number in battle, $\frac{1}{3}$ by sickness and 36 have deserted. What was the original number ?

4. A person in his will left 0·4 of his property to be given to his son, ·05 to his widow and ·175 to his daughter and the balance £3,000 to a hospital. What did the widow receive ?

Example. A person possessing $\frac{5}{16}$ of a ship sells $\frac{3}{8}$ of his share ; what is the value of the part of the ship still left, if the ship be worth Rs. 1,60,000 ?

He sells $\frac{3}{8}$ of $\frac{5}{16}$ or $\frac{15}{128}$ of the ship.
He still owns $\frac{5}{16} - \frac{15}{128}$ or $\frac{25}{128}$ of the ship.
Since the whole ship is worth Rs. 1,60,000,
$\frac{25}{128}$ of the ship is worth Rs. $1{,}60{,}000 \times \frac{25}{128}$ or Rs. 31,250.

EXAMPLES. IV.

1. Two-thirds of a cistern is full of water ; and $\frac{3}{4}$ of what it contains is drawn off. How much of the cistern is now full ?

2. A is the owner of a cotton mill to the extent of $\frac{7}{15}$ of it ; and he is obliged to sell $\frac{9}{14}$ of his share to B. If the

whole property is estimated at a lakh and a half, find the value of A's remaining share and B's share.

3. A man spends $\frac{11}{18}$ of his income; and then finds that $\frac{3}{14}$ of the remainder is Rs. 40. Find how much he saves.

4. A boy has 2 as. 9 p., having spent ·4 of what he had in buying oranges and ·45 of the remainder in mangoes. What sum did he start with?

EXAMPLE. The difference in the values of the two shares into which a certain property is divided is £48·575 and one share is ·51 of the whole. Find the value of each share.

Since one share is ·51 of the whole.
The other is 1 — ·51 or ·49 of the whole.
The difference between the two is ·51 — ·49 or ·02.
Since ·02 of the property = £48·575

$$\text{the whole of it} = \frac{£48{\cdot}575}{{\cdot}02} = £2428{\cdot}75.$$

$\therefore$ ·51 of it = £2428·75 × ·51 = £1238·6625 = £1238-13-3.
$\therefore$ ·49...... = £2428·75 × ·49 = £1190·0875 = £1190-1-9.

EXAMPLES. V.

1. The difference in the values of the two shares into which an estate is divided is £153·5, and one share is ·75 of the whole. Find the value of the whole and of each share.

2. A person left the whole of his property to his widow and two sons, each of the latter receiving ·375 of the property. If a son's share is worth Rs. 1,250 more than the widow's, what is the value of the whole and the share of each?

3. A gentleman leaves the whole of his property to his son and two daughters, each of the latter receiving ·32 of the property. If the son's share is worth 2,000 rupees more than that of either of the daughters, what is the value of the whole property and the share of each?

4. A man bequeaths $\frac{5}{12}$ of his estate to one son, $\frac{7}{12}$ of the remainder to another son, and the balance to his widow. The children's shares differ by Rs. 1,320. Find the widow's share.

EXAMPLES. VI.

MISCELLANEOUS.

1. A person's property consists of $\frac{11}{24}$ lands, $\frac{1}{3}$ houses and the rest 20 lakhs of rupees in a Bank. Find the value of the house property.

2. Out of a cistern which is $\frac{2}{3}$ full, 20 gals. are drawn; the cistern is then found to be $\frac{3}{5}$ full. How much does the cistern hold?

3. A man went to a shop and after buying 20 chairs at Rs. 2-4 as. each, finds that $\frac{5}{8}$ of his money remains; what had he before entering the shop?

4. A piece of cloth, when measured with a yard measure which is 0·6 of an inch too short, appears to be $88\frac{1}{2}$ yards long. What is its true length?

5. A has shares in an estate to the amount of ·25 of it and ·04 of it. B has shares in the same to the amount of ·35 of it. Find the difference between the values of their shares when ·125 of the estate is worth Rs. 3,600

6. In an election $\frac{1}{4}$ of the whole number of voters did not turn up, $\frac{3}{7}$ voted for one candidate and the rest for the other. The successful candidate was returned by a majority of 21 votes. How many did the other secure?

7. A merchant invests one-third of his capital in trading in leather and loses ·25 of the amount invested; the rest of his capital he invested in trading in indigo and gains ·2 of the amount invested. On the whole he gains Rs. 3,000. What is the amount of his capital?

8. A's farm contains 100·46875 ac., which lacks ·28125 ac. of being $\frac{1}{4}$ as large as B's; B's farm exceeds 6 times C's by 5·725 acres; find the extent of the three farms.

CHAPTER XXI.

PROBLEMS ON RATES AND TAXES.

249. In England **Rates** are payments made to Local Authorities, such as County Council or Town Council for different local purposes; and these are calculated on the annual value or rent of a property. This annual value is called the *rateable* value. Taxes are fixed by the Parliament. Rates and Taxes are reckoned at so much in the £.

250. In India the rates levied by Local Bodies, such as the Municipal Council of a town are called taxes.

That which is levied on the annual rental value of a house is called *house tax* and that levied on the net income of a person derived from the exercise of his profession is called *professional* tax There are other taxes, such as water and drainage tax, lighting tax, and land tax.

Income-tax is paid to Government and is generally calculated on the gross income.

Persons whose annual incomes are below Rs. 1,000 are exempt from income-tax. On incomes of Rs. 1,000 and above up to Rs. 2,000 the tax is 4 pies in the rupee ; and on incomes of Rs. 2,000 and above that tax is 5 pies in the rupee.

251. Gross income of a person is the entire income that he gets.

Net income is the income that a person has after deducting all his charges and expenses.

Gross income = net income + expenses.

Net income = gross income — expenses.

252. When a person has not got enough to pay all the money he owes, he becomes a *bankrupt* or an *insolvent.*

Liabilities are the amount of debts which a person has to pay and **assets** are the property of a bankrupt.

The persons to whom he owes money are called **creditors.**

When the assets are all turned into cash and divided by the liabilities the **dividend** or rate per £ or R. is obtained.

If a bankrupt's assets are Rs. 12,000 and his liabilities Rs. 48,000 the dividend is $\frac{1}{4}$, i.e. 4 as. in the rupee ; for every rupee, 4 as. will be paid to the creditors.

EXAMPLE 1. A gentleman's gross income is Rs. 10,000 ; and he pays income-tax at the rate of 5 pies in the rupee. Find the amount of his income-tax and his net income.

The income-tax on Re. 1 = Re. $\frac{5}{192}$.

∴ The income-tax on Rs. 10,000 = Rs. $\frac{5}{192} \times 10000$.

= Rs. $\frac{5}{12} \times 625$ = Rs. $\frac{3125}{12}$.

= Rs. 260-6-8.

∴ His net income = Rs. 10,000 — Rs. 260-6-8.

= Rs. 9,739-9-4.

EXAMPLES.—ORAL.

Find the income-tax on the following monthly salaries :—

At 4 pies in the rupee—

(1) Rs. 90. (2) Rs. 95. (3) Rs. 100.
(4) Rs. 120. (5) Rs. 160. (6) Rs. 180.

At 5 pies in the rupee—

(1) Rs. 200. (2) Rs. 250. (3) Rs. 300.
(4) Rs. 400. (5) Rs. 500. (6) Rs. 600.

EXAMPLES. I.

1. A Government servant whose monthly salary is Rs. 2,000, has his income-tax deducted at the rate of 5 pies in the R. What does he receive ?

2. In Arbuthnot's insolvency, the first dividend declared was 10 pies in the rupee. What does a creditor receive who had invested Rs. 5,500 ? If the second dividend gave 8 pies in the rupee, what was his loss ?

3. A person contributes out of his income Rs. 75-12 as. for religious purposes, which gives a rate of 9 pies in the rupee. Next year, his income remaining the same, he gives only Rs. 50-8 as. What is this in the rupee ?

4. An alteration in the income-tax diminishes the amount of tax which is £37-17-7½, when the tax is 9½*d*. in the £, by £14-19-0¾. What is the diminished rate of tax ?

EXAMPLE 2. I pay income-tax at the rate of 4 pies in the rupee. Find my monthly income, if I pay yearly Rs. 23-12 as.

When I pay 4 pies tax my income is Re. 1.
∴ When I pay 1 pie tax my income is Re. ¼. [× ¼, i.e. Rs. 1,140.
∴ When I pay Rs. 23-12 as. or 4,560 p. tax my income is Rs. 4,560
∴ My monthly income is Rs. 1,140 ÷ 12 or Rs. 95.

EXAMPLES. II.

1. I pay monthly Rs. 4 as. 11 for charity. Find my yearly income, if this is at the rate of 9 pies in the rupee.

2. A person pays Rs. 10-5-7 yearly for house-tax. If the tax is levied at the rate of 12 as. for every Rs. 10, find the annual rent of the house.

3. A bankrupt's assets enable him to pay a rate of 3*s.* 9*d.* in the £. If his liabilities are £50,000, what are his assets worth ?

4. A bankrupt's assets are valued at $27\frac{1}{2}$ lakhs of rupees. This would give his creditors a dividend of $1\frac{1}{2}$ as. in the rupee. Find his liabilities.

5. One-sixth of a man's income is exempted from income-tax. If the tax he pays at 4 pies in the rupee comes to Rs. 20, find his total income.

6. One-fifth of a man's income is exempt from taxation and the tax he pays at 5 pies in the rupee amounts to Rs. 143-3-8. What is his income ?

7. A person pays income-tax on one-half of his income at 5 pies in the rupee and on the other half at 4 pies in the rupee. The total tax paid is Rs. 253-2 as. What is his income ?

8. A person pays a tax of 5*d.* in the £ on one-half of his income, and of 4*d.* in the £ on the other half. Find his income, if he has to pay £63-5-$7\frac{1}{2}$ in all.

EXAMPLE. 3. After paying 4 pies in the rupee for income-tax, and spending $\frac{2}{47}$ of the remainder on purpose of charity, a gentleman has Rs. 801-2-9 left. Find his gross income.

He pays 4 pies or $\frac{1}{48}$ Re. for income-tax.
He has then $(1 - \frac{1}{48})$ Re. or $\frac{47}{48}$ Re. left.
He spends $\frac{47}{48}$ of $\frac{2}{47}$ or $\frac{1}{24}$ Re. on purposes of charity.
He has now $(\frac{47}{48} - \frac{1}{24})$ Re. or $\frac{15}{16}$ Re. remaining.
For every Re. $\frac{15}{16}$ that is left, his gross income is Re. 1.
∴ For every Re. 1..............................Re. $\frac{16}{15}$.
∴ For every Rs. 801-2-9........................Rs. $\frac{801\text{-}2\text{-}9 \times 16}{15}$
i.e. Rs. 854-9-4.

EXAMPLES. III.

1. A civilian paying an income-tax of 5 pies in the rupee has Rs. 2,434-14-4 left per mensem. Find his annual gross income.

2. A man after paying 10*d.* in the £ for income-tax and other taxes amounting to 8*d.* in the £, has £1,541-13-4 left. What is his income ?

3. A person after paying one anna in the rupee for taxes, and spending $\frac{1}{16}$ of the remainder for other purposes, finds that Rs. 5,175 remains. What is his income ?

4. A person after paying out of his income a tax of 8 pies in the rupee and 75 of the remainder for his expenses, has Rs. 260-3-0 remaining. What is his gross income ?

EXAMPLES. IV.

MISCELLANEOUS.

1. If the income-tax on £300-2-8 is £9-7-7, what is the rate per £?

2. What is the income corresponding to an income-tax of 15 guineas at the rate of 7*d.* in the £ ?

3. A person pays Rs. 26-0-8 monthly for income-tax If the tax is levied at 5 pies in the rupee, find his annual net income.

4. A person pays as taxes 8*d.* in the £ on half of his income and 6*d.* in the £ on the other half ; and £7-15-10 is thus paid. Find his income.

5. If a man's income is reduced from £750 to £734-7-6 by payment of income-tax, how much does he pay in the £ ?

6. A person who pays Rs. 25 for income-tax has a net income of Rs. 1,175. What is the rate of the tax ?

7. A person's net salary, after paying a tax of 8*d.* in the £ is £580. What would he get if he had to pay no income-tax ?

8. A person's income was Rs. 1,410 a year, before income-tax was levied. The tax being 4 pies in the rupee, how much must his salary now be so that he may still get Rs. 1,410 as net income ?

9. One-sixth of a person's income is exempted from income-tax ; and the tax he pays at 4 pies in the rupee is Rs. 2-1-4 per mensem. Find his annual gross income.

10. After a rent has been reduced $\frac{1}{5}$, a rate of $3\frac{3}{4}$*d.* in the £ amounts to £12. What was the amount of the rent before the reduction ?

11. A landlord after allowing his agent $\frac{1}{20}$ of the gross rental of his estate, and paying an income-tax of 9*d.* in the £ on the remainder, has £2,456-12-5 left. What was the gross rental ?

12. If a rate of 6*d.* in the £ on the rateable value of an estate amounts to £91-5-0, and the rates are calculated on $\frac{39}{50}$ of the gross value, find the gross value of the estate.

ONE-HOUR PAPERS—SECOND SERIES.

REVISION EXERCISES.

I.

1. Resolve into prime factors 32,736 and express it as the product of three consecutive numbers.

2. A sovereign weighs $\frac{160}{623}$ of an oz. Troy. How many sovereigns can be coined out of a piece of metal weighing 1 lb. Troy? This metal being composed of $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy, what is the value to the nearest penny of an ounce of pure gold? (12 oz. = 1 lb.)

3. A railway has a journey of 65 miles to perform and ought to finish it in 3 hrs. If its starting is delayed by a quarter of an hour, how many miles per hour must it increase her speed so as to arrive at the proper time?

4. A man's net income is Rs. 973-15-4 after paying an income-tax of 5 pies in the rupee. What would have been his net income, if the tax had been 4 pies in the rupee?

5. A person spent for his household as much during the first 6 months of the year 1908 as he did during the latter 6 months. If the average daily expenditure for the first six months was Rs. 3-13-4, find the average daily expenditure for the latter six months.

II.

1. An empty cask weighs 10 lbs. and when filled with water 54 Kg. How many gallons does it hold, it being given that the capacity of a kilogram of water is one litre and one litre contains $1\frac{3}{4}$ pints. 1 Kg. = 2·2 lbs.

2. A pound of paraffin is made into 9 candles, each of which lasts for 5 hrs. How long would each last if they were made 8 to the pound?

3. The diametre of a half anna being one and one-tenth of an inch, how many in a continuous straight line would reach from Palamcottah to Tinnevelly, a distance of $2\frac{3}{4}$ miles? What would be their total value?

4. The population of two countries are respectively 44,260,606 and 43,331,654; and their areas are 162, 655 and 121,377 square miles respectively. Find the average population of the square mile of both the countries together to the nearest integer.

5. If a steamer makes the passage from New York to Liverpool, say 2,760 miles in 9 days 14 hrs., and a train goes from London to Edinburgh, say 405 miles in 18 hrs., compare the rates of the steamer and train.

III.

1. The following rule is sometimes given :—To convert ounces to grams, multiply by $\frac{85}{3}$; find the error it being given 1 Kg.=2·205 lbs.

2. If the 10th, 100th and 1000th part of a pound be called a florin, a cent, and a mil respectively, and a man's weekly wages are £2-9fl. 7 cents 5 mils upon which he pays an income-tax of 5 cents in the £, find his yearly net income and convert the result into £ *s*. *d*. to the nearest penny.

3. If the diameter of the forewheel of a carriage be 2 ft. 3 in. and that of the hindwheel 3 ft. 6 in., find how many times (to the nearest integer) the one will revolve oftener than the other in a distance of 10 miles, having given the circumference of a circle is to the diameter as 3·1416 to 1.

4. At a siege it was found that a certain length of trench was dug by the soldiers and navvies in 4 days, but that when only half the navvies were present, it required 7 days to dig the same length. What part of the work was done by the soldiers ?

5. A book of 300 pages is $\frac{3}{4}$ of an inch thick without the cover. Calculate the height of 2,000 reams of the same paper piled one upon the other.

IV.

1. Resolve into prime factors 25,200,409,600 and 67,760 ; find which of these is a square and the H.C.F. and L.C.M. of the square root and the other two numbers.

2 Of 32 selected candidates for the Civil Service, 3 were above 20 years of age, 4 above 21, 12 above 22 and 23 respectively and 1 above 24. What was the average age of the men ?

3. If 21 lamps are required to light a road 1,600 yds. long, how many placed at the same distance apart will be required for a road 4 miles long ?

4. After paying income-tax at a certain rate per rupee, a man had Rs. 608-11-7 left. If his original income had been Rs. 48 more, his net income would have been Rs. 655-7-7. Find the income of the man, and the rate of the tax per rupee.

5. A can do as much work in 2 days as B in 3 days. If A and B together do a piece in 12 days, find how long each will take to do it by himself.

V.

1. If the average step of a horse be $2\frac{3}{4}$ ft. and that of a man $2\frac{1}{2}$ ft., what is the shortest distance in which each can take an integral number of steps and find the number of steps taken by each.

2. If a man spends as much in 4 months as he earns in 3, how much will he lay by in a year from an income of £150?

3. A person left a sum of money which was divided equally among 43 poor people, such that after a deduction of 6*d.* in the £, each received £2-14-2. What sum did he leave?

4. Express the ratio of 12 can. 10 mds. 1 viss : 20 can. 16 mds. 7 viss as a fraction in its lowest terms.

5. A steamer makes a voyage in 72 days sailing on an average 9 knots per hour. How long will another be making the same voyage whose average rate of sailing is 8 knots per hour?

VI.

1. Find the sum of money which shall be the same fraction of £61-9-1, that 2 cwt. 2 qrs. 10 lbs. is of 36 cwt. 1 qr.

2. The South Indian Railway runs the following trains:—

Boatmail, Madras Beach to Tuticorin, 446 miles in 22 hrs. 30 min.
Mail, ,, ,, ,, ,, 25 hrs. 10 min.
Madura Express, Madras Egmore to Madura, 344 miles in 17 hrs. 35 min.

Find the average speed per hour for the three trains to the nearest mile.

3. A has twice as much money as B. They play together for a certain stake. At the end of the first game B wins from A one-third of A's money. What fraction of the sum B now has must A win back in the second game that they may have exactly equal sums?

4. A bankrupt had valued his flock of sheep at Rs. 3 per head; but after one-half of the number was sold at Rs. $2\frac{3}{4}$ and the other half at Rs. $3\frac{1}{2}$, he realized Rs. 32 more than he had expected. Find the number of sheep.

5. Standard gold is worth £3-7-$10\frac{1}{2}$ per oz. Find the least number of ounces that can be coined into an exact number of sovereigns.

VII.

1. Express a speed of 25 miles an hour in feet per second. Shew that the result may be obtained approximately by multiplying the number of miles by $1\frac{1}{2}$; and find the percentage error.

2. Find two numbers between 200 and 500 which are divisible by each of the numbers 12, 16, and 18.

3. A schoolboy spends $\frac{2}{5}$ of his pocket-money in the 1st week, $\frac{3}{5}$ of the remainder in the 2nd and 4 as. in the 3rd week, and has nothing left in the 4th. What is his monthly allowance?

4. The tax on tobacco at the rate of 2 as. 6 p. per lb. produced in a certain year Rs. 52,345. Find to the nearest ton the weight of tobacco taxed.

5. The average score in a certain innings for a team of eleven was 40, that for the first six was 30; what was the average for the other five?

VIII.

1. A sum of £261-13-4 is to be levied by a rate in a parish which yields a rental of £7,850; what is the rate per £?

2. A man counts 22 telegraph posts in one minute. If they are 44 yds. apart, at what rate per hour is the train moving?

3. What is the smallest sum of money which will exactly contain each of the following sums:—1 a. 3 p., 1 a. 6 p., 2 as. 4 p., 3 as. 6 p.?

4. In 400 yrs. 97 contain 366 days and the rest 365 days. Find the average length of a year in days, hrs., min., sec. If the earth completes its orbit round the sun in 31556925·51 seconds, find the difference.

5. The distance between two wickets was marked out as 22 yds.; but the yard measure was $\frac{1}{10}$ of an inch too short. What is the actual distance?

IX.

1. The polar diameter of the earth is 7899·114 miles, and its ratio to the equatorial diameter is $\frac{298{\cdot}33}{299{\cdot}33}$; determine the latter omitting fractions of a mile.

2. Find the contents of the smallest flask that can be filled by an exact number of any one of the following measures $\frac{1}{2}$ pint, $\frac{1}{2}$ gal., 3 gals., 5 gals., 9 gals.

3. A bankrupt owes £78; his assets are £53; if the creditors get only 7*s*. 6*d*. in the £, what was the amount of expenses incurred in settling the claims?

4. In a school of 106 children, there were 17 aged 6 yrs., 26 aged 7½ yrs., 35 of 9¼ yrs., and 20 of 10 yrs. Find the average age of the remaining children, given that the average age of all the children is 8½ yrs.

5. A diamond worth £60 is cut into two pieces, the ratio of their weights being 2 : 3. Find their value, the value of diamond varying as the square of its weight.

X.

1. Test the accuracy of the following rule:—To convert pounds to kilograms, multiply by $\frac{5}{11}$, it being given 1 Kg. = 2·205 lbs.

2. The weight of a Roman coin is 60 grains, and it contains $\frac{29}{30}$ths pure silver and the rest alloy. Find the value of 333 of these coins, when the value of 480 grains of standard silver is 5*s*. 2*d*., the value of the alloy being neglected.

3. If a man's income is reduced from £750 to £734-7-6 by income-tax, how much does he pay in the £?

4. The wheels of a carriage are 11 ft. and 7 ft. 6 in. in circumference. How many times will each wheel have performed a complete number of revolutions simultaneously in a mile?

5. In a cricket match, the scores in four successive innings are each $\frac{1}{4}$ less than in the preceding innings, and the side which has had the first innings wins by 50. What are the scores in each innings?

XI.

1. The following rule is sometimes given to divide by 3·14159. 'Multiply by 7, divide by 11, then by 2, and add one-eighth of one-thousandth of the result.' Find to the 1st significant figure the error made in obtaining 10 ÷ 3·14159 by this rule.

2. The scale of a map is 1 : 125000. Find in kilometres the distance represented by a centimetre on the map, and in decimals of an inch, the distance on the map representing one mile.

3. A, B and C having respectively Rs. $5\frac{5}{8}$, Rs. $3\frac{7}{12}$ and Rs. $8\frac{11}{24}$ subscribe $\frac{5}{24}$, $\frac{3}{8}$ and $\frac{1}{4}$ of their money. What does the subscription amount to?

4. If 1 litre = $1\frac{3}{4}$ pints, find the cost of $1\frac{1}{2}$ pints of beer, at 6*s*. a litre.

5. A train is running at the rate of 45 miles an hour. Find the number of telegraph posts, 55 yds. apart that can be counted in a minute, by a traveller.

XII.

1. Find the cost of making a fence 3 fur. 11 po. $3\frac{1}{4}$ yds. at £183-6-8 a mile.

2. What is the smallest sum of money that can purchase an exact number of exercise books at 9 pies each or copy books at 1 a. 8 p. each or drawing books at 4 as. 6 p. each?

3. A person pays one-tenth of his income to a Life Assurance Company; and this amount is free from income-tax. On the remaining nine-tenths he pays tax at 5 pies in the rupee. If the tax he pays amounts to Rs. 5-13-9 a month, what is his gross annual income?

4. A carpenter first cut off $\frac{1}{3}$ of a plank and then $\frac{2}{3}$ of the remainder. If this was found to be 4 ft. 10 in., find the entire length.

5. From 1 lb. av. of bronze, 48 pennies are coined and bronze contains $\frac{1}{100}$th part by weight of zinc. Find the total weight of zinc in the bronze coinage of the value of £98,765.

XIII.

1. Bought 6 cwt. 3 qrs. 21 lbs. of sugar at Rs. 28 a cwt.; for which I am to pay $\frac{2}{3}$ in cash and the remainder in tea at 3 as. a lb.; how much tea have I to give?

2. If a population is now ten millions, and the births being 1 in 20, the deaths are 1 in 30, what will be the population at the end of 5 yrs.?

3. The price of diamonds per carat varies as the square of the weight; if a diamond of 2 carats is worth £32, what is the value of one of 3 carats?

4. A and B are out shooting; A fires 5 shots to B's 3, but A kills only one in 3 shots, while B kills 1 in 2. When B has missed 36 shots, which of the two has killed most?

5. What must be the gross produce of an estate that after paying $\frac{1}{10}$ as income-tax and a rate of 2*s.* $1\frac{1}{2}$*d.* in the £ on the residue, there may remain £2,574 per annum?

XIV.

1. If a national debt of 512 million pounds has an eighth part of its then existing amount paid off every year, how soon will it be reduced to less than half its original amount?

2. A bankrupt can pay 6*s.* 8*d.* in the £; if his assets were £500 more, he could pay 7*s.* 4*d.* Find his debts and assets.

3. One-fourth of an estate belongs to A, $\frac{2}{9}$ of it to B and the remainder to C which is worth Rs. 2,500 more than A's share. What is the value of the whole estate?

4. If one kilometre = $\frac{5}{8}$ of a mile, find the number of kilometres in 72 miles.

5. A man in a train observes 35 telegraph posts in a minute. If they are 44 yds. apart, find the rate of his travelling in miles per hour.

XV.

1. Owing to leakage, the quantity of wine in a cask is less at the end of each year by $\frac{1}{10}$th part of what it was at the beginning of that year. How much will be lost in the course of 5 years from a cask which now contains 100 gals.?

2. Three varieties of milk A, B, C are used in the manufacture of butter. Find which produces the most butter per gallon.

A, 132 gals. produces $22\frac{4}{5}$ lbs. of butter.
B, $749\frac{1}{2}$ gals. ,, 139 lbs. ,,
C, 632 gals. ,, 112 lbs. ,,

3. If a model of the earth were made with a diameter of 20 yards, express as the decimal of an inch to the first significant figure, the height of a mountain 25,000 ft. high, taking the earth's diameter as 7,900 miles.

4. A gentleman saves Rs. 75 monthly. He spends $\frac{1}{2}$ of his income and sends $\frac{2}{3}$ of the remainder to England towards the cost of the education of his children. Find how much he earns annually.

5. A sovereign may not weigh more than 123·47447 grains, and has to be withdrawn from circulation when it weighs 122·5 grains. A heap of sovereigns weighs 384 tolas, correct to the nearest tola. State the greatest and least possible number in the heap. (A tola contains 180 grains.)

CHAPTER XXII.

MENTAL ARITHMETIC.

253. We have already seen how the value of a dozen articles in Indian or English money may be calculated mentally Art. 89; and hence the value of any number of dozens may be found.

When the value is required for **any number of articles,** express the given number as so many dozens and fraction of a dozen.

EXAMPLE 1. Find the value of 88 articles at £ 3-3-6 each.

Mental process.

88 = 84 + 4 = 7 dozens + $\frac{1}{3}$ of a dozen.

The value of 1 dozen at £3-3-6 = £36 + 42*s*. = £38 2*s*.
∴ The............7 .... = £38 2*s*. × 7 = £266 14*s*.
And...........$\frac{1}{3}$ .... = $\frac{1}{3}$ of £38 2*s*. = £12 14*s*.
∴ .... 88 or $7\frac{1}{3}$ .... = £266 14*s*. + £12 14*s*.
= £279 8*s*.

254. To find the value in Indian money of any number of articles that is a multiple of 16.

Reckon each anna in the given value as a rupee, and get the value of 16 (Art. 90) and then multiply.

EXAMPLE 2. (*a*) Find the value of 16 articles at 12 as. 6 p. each.
12 as. 6 p. = $12\frac{1}{2}$ as.
∴ Required value = Rs. $12\frac{1}{2}$, i.e. Rs. 12-8 as.

(*b*) Find the value of 128 articles at Rs. 1-10-9 each.
Value of 16 articles = Rs. 16 + Rs. $10\frac{3}{4}$ = Rs. $26\frac{3}{4}$.
∴ Value of 128 of 16 × 8 = Rs. $26\frac{3}{4}$ × 8 = Rs. 214.

255. To find the value in English money of any multiple of 20 articles.

EXAMPLE 3. The value of 20 articles at 13*s*. 9*d*. $= £13\frac{3}{4} = £13$ 15*s*.

The value of any multiple of 20 may be obtained by first finding the value of 20 articles and then proceeding as shewn in example 2 above.

256. Special rules for 100.

1. To find the value in Indian Money of 100 articles.

EXAMPLE 4. Cost of 100 pencils at $4\frac{1}{2}$ pies each $= 4\frac{1}{2}$ half-rupees $+ 4 \times 4\frac{1}{2}$ p. = Rs. $2\frac{1}{4} + 18$ p. = Rs. 2-5-6.

For cost of 100 pencils at $4\frac{1}{2}$ p. each = cost of $(96 + 4)$ pencils at $4\frac{1}{2}$ p. each $= 4\frac{1}{2}\,(96 + 4)$ pies $= 4\frac{1}{2}$ (96 pies + 4 pies) $= 4\frac{1}{2}$ (8 as. + 4 p.) $= 4\frac{1}{2}$ ($\frac{1}{2}$ a rupee + 4 p.) = Rs. $2\frac{1}{4} + 4 \times 4\frac{1}{2}$ p. = Rs. 2 4 as. + 18 p. = Rs. 2-5-6

Cost of 100 things at 1 an. 7 p. each = 19 half-rupees + 76 p. = Rs. 9-14-4.

Reduce when necessary the given value to pies; take for answer as many half-rupees as there are pies, and to this add four times the number of pies.

2. To find in English Money the value of 100 articles.

EXAMPLE 5. Find the value of 100 things at $7\frac{3}{4}$*d*.

$7\frac{3}{4}$*d*. = 31 farthings.

The required value of 100 things at $7\frac{3}{4}$*d*. each = the value of $(96 + 4)$ things at 31f. each $= 31\,(96 + 4)$ f. = 31 (96f. + 4f.) = 31 (2*s*. + 1*d*.) = 62*s*. + 31*d*. = £3-4-7.

Reduce the given value to farthings, and take for answer as many pence as there are farthings and twice as many shillings.

EXAMPLE 6. If the value is given in shillings, the value of 100 things is found by taking £5 for every shilling in the given value.

The value of 100 things at 17*s*. 6*d*. $= £5 \times 7\frac{1}{2} = £87$ 10*s*.

For the value = 17*s*. 6*d*. × 100 $= 17\frac{1}{2}$*s*. × 100 $= 17\frac{1}{2}$*s*. × 20 × 5 $= £17\frac{1}{2} \times 5 = £87$ 10*s*.

257. To find the value of any number of articles at an even number of shillings each.

EXAMPLE 7. Find the value of 69 articles at 14*s*. each.

The value of 69 articles at 14*s*. each $= £69 \times \frac{14}{20} = £69 \times \frac{7}{10}$

$$= £\,\frac{69 \times 7}{10} = £\,\frac{483}{10} = £48\tfrac{3}{10} = £48 \text{ } 6s.$$

$69 \times 7 = 483$.

Value required $= 2 \times 3s. + £48 = £48$ 6*s*.

Multiply the given number of articles by half the given number of shillings; then twice the first right hand figure will be the number of shillings and the other figure or figures will be pounds.

258. To find the cost when the value is given nearly an integral number of R. or £.

EXAMPLE 8. Find the cost of 32 books at Rs. 2-15-9 each.

Rs. 2-15-9 = Rs. 3 less 3 pies = Rs. 3 − $\frac{1}{4}$ anna.

∴ The cost of 32 books at 2-15-9 = 32 (R. 3 − $\frac{1}{4}$ anna).
= Rs. 96 − 8 as.
= Rs. 95 8 as.

The cost of 40 articles at £5-19-11 is the cost at £6 each less 40 pence; for £5-19-11 = £6 − 1 penny.

The cost of 78 things at Rs. 3-13-4 = 78 (Rs. 4 − $\frac{1}{6}$ R.); for Rs. 3-13-4 = Rs. 4 − 2 as. 8 p. = Rs. 4 -- $\frac{1}{6}$ of a rupee.

259. The total cost being given, to find the number of articles by expressing the cost of each article as a fraction of R. 1 or of £1.

EXAMPLE 9. If an article cost 5 as. 4 p., find how many can be bought for Rs. 30-10-8.

5 as. 4 p. = $\frac{1}{3}$ of a rupee.

In Rs. 30, there are 90 one-thirds of a rupee; and in 10 as. 8 p there are 2 one-thirds of a rupee.

∴ Number of articles = 92.

MENTAL ARITHMETIC.

EXAMPLES.—ORAL.

1. To be worked as shewn in example 1.

Find the value at (*a*) 9 as. 8 p. each, (*b*) 12*s*. 6*d*. each, (*c*) Rs. 1-3-4 each, (*d*) £1-2-2 each of (1) 30, (2) 39, (3) 40, (4) 54, (5) 66, (6) 76, (7) 87, (8) 90, (9) 100 things.

2. To be worked as shewn in example 2 (*a*).

Find the value of 16 articles at (1) 6 as. 6 p. (2) 7 as. 9 p. (3) 8 as. 3 p. (4) 9 as. 6 p. (5) 10 as. 3 p. (6) 12 as. 9 p. (7) 14 as. 6 p. (8) 15 as. 3 p. (9) Rs. 1-2-6. (10) Rs. 2-3-9. (11) Rs. 3-1-3. (12) Rs. 3-4-9 each.

3. To be worked as shewn in example 2 (*b*).

Find the value at Rs. 2-13-6 each of (1) 48, (2) 64, (3) 80, (4) 112, (5) 128, (6) 160, (7) 192, (8) 240 articles.

4. To be worked as shewn in example 3.

Find the value of 20 articles at (1) 13*s*. 6*d*. (2) 15*s*. 3*d*. (3) 16*s*. 9*d*. (4) 18*s*. 3*d*. (5) 19*s*. 6*d*. (6) £1-2-3. (7) £1-10-9. (8) £1-12-6 each.

5. To be worked as directed in example 3.

Find the value of (1) 60, (2) 80, (3) 120, (4) 180, (5) 220, (6) 260 articles at £1-4-3 each.

Examples 6—9 are to be worked as shewn in models 4—7.

6. Find the value of 100 things at (1) 7 p. (2) 8½ p. (3) 9½ p. (4) 10½ p. (5) 11½ p. (6) 1 an. 5 p. (7) 1 an. 10 p. (8) 2 as. 1½ p. each.

7. Find the cost of 100 articles at (1) 5¼*d*. (2) 6¾*d*. (3) 7½*d*. (4) 8¼*d*. (5) 9¾*d*. (6) 10½*d*. (7) 11¼*d*. each.

8. Find the value of 100 things at (1) 7*s*. (2) 8*s*. 3*d*. (3) 9*s*. 6*d*. (4) 11*s*. 9*d*. (5) 14*s*. 6*d*. (6) 17*s*. 3*d* (7) 18*s*. 6*d*. (8) 19*s*. 3*d*. each.

9. Find the cost of (*a*) 74. (*b*) 97. (*c*) 190 articles at (1) 12*s*. (2) 14*s*. (3) 16*s*. (4) 18*s*. each.

10. To be worked as shewn in example 8.

Find the value of 30 articles at (1) Rs. 1-15-9. (2) Rs. 1-15-11. (3) Rs. 2-15-10. (4) Rs. 2-10-8. (5) Rs. 2-13-4. (6) Rs. 3-15-9. (7) Rs. 4-15-11 each.

11. To be worked as shewn in example 8.

Find the value of 50 things at (1) 19*s*. 10*d*. (2) 19*s*. 11*d*. (3) £1-19-6. (4) £1-19-8. (5) £1-19-10. (6) £2-13-4. (7) £2-19-9. (8) £2-19-11 each.

12. To be worked as shewn in example 9.

If an article costs 2 as. 8 p. how many can be bought for (1) Rs. 1-10-8. (2) Rs. 2-2-8. (3) Rs. 3-13-4. (4) Rs. 4-10-8. (5) Rs. 5-13-4. (6) Rs. 7-10-8.

13. To be worked as shewn in example 9.

If an article costs 1*s*. 8*d*. how many can be bought (1) £1-10*s*. (2) £1-11-8. (3) £2-3-4. (4) £2-16-8. (5) £3-8-4. (6) £3-15*s*.

14. If the cost of 100 articles be Rs. 10, find the cost of (1) 125, (2) 175, (3) 250, (4) 160, (5) 190, (6) 230 articles of the same kind.

CHAPTER XXIII.

PRACTICE.

260. Practice is a short method of finding the value of any number of articles or quantity of goods by means of Aliquot parts.

Aliquot parts of any quantity are such as will divide it an exact number of times; thus an Aliquot part is nothing but a measure.

2*s*. 6*d*. is an Aliquot part of 10*s*.; for 2*s*. 6*d*. divides 10*s*. four times and is one-fourth of it. Also 5 as. 4 p. is an Aliquot part of Re. 1 for 5 as. 4 p. = $\frac{1}{3}$ of Re. 1.

EXAMPLES.—ORAL.

1. Let the pupils construct Tables of Aliquot parts of an anna, of a rupee, of a shilling and of a pound.

2. Express as Aliquot parts—

(1) Of 6 as.; 1 a. 6 p., 6 p., 8 p., 9 p.
(2) Of 8 as.; 2 as. 8 p., 1 a. 4 p., 8 p., 6 p.
(3) Of 4*s*.; 4*d*., 6*d*., 1*s*. 4*d*.
(4) Of 1*s*. 3*d*.; 3*d*., 5*d*., 1½*d*., 7½*d*.

3. Of what is—

2*s*. 6*d*. (1) $\frac{1}{3}$, (2) $\frac{1}{4}$, (3) $\frac{1}{5}$, (4) $\frac{1}{6}$?
1*s*. 6*d*. (1) $\frac{1}{8}$, (2) $\frac{1}{6}$, (3) $\frac{1}{10}$, (4) $\frac{1}{12}$?
3 as. 6 p. (1) $\frac{1}{6}$, (2) $\frac{1}{10}$, (3) $\frac{1}{12}$, (4) $\frac{1}{16}$?
5 as. 4 p. (1) $\frac{1}{3}$, (2) $\frac{1}{4}$, (3) $\frac{1}{9}$, (4) $\frac{1}{12}$?

EXAMPLE. Divide into Aliquot parts—

14 as. 4 p. 13*s*. 9*d*.

14 as. 4 p.
8 as. = $\frac{1}{2}$ of Re. 1.
4 as. = $\frac{1}{2}$ of 8 as.
2 as. = $\frac{1}{2}$ of 4 as.
4 p. = $\frac{1}{6}$ of 2 as.

13*s*. 9*d*.
10*s*. = $\frac{1}{2}$ of £ 1.
2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*.
1*s*. 3*d*. = $\frac{1}{2}$ of 2*s*. 6*d*.

EXAMPLES.—ORAL.

I. Break into convenient Aliquot parts—

1. 5 as. 2 p. 2. 5 as. 8 p. 3. 7 as. 10 p. 4. 8 as. 4 p.
5. 8 as. 10 p. 6. 9 as. 5 p. 7. 9 as. 11 p. 8. 10 as. 3 p.
9. 14 as. 3 p. 10. 14 as. 8 p. 11. 15 as. 5 p. 12. 15 as. 10 p.

II. Divide into Aliquot parts—

1. 10*s*. 9*d*. 2. 12*s*. 8*d*. 3. 13*s*. 4*d*. 4. 15*s*. 9*d*.
5. 8¾*d*. 6. 10½*d*. 7. 11¾*d*. 8. 7*s*. 9¾*d*.
9. 13*s*. 5½*d*. 10. 16*s*. 7¾*d*. 11. 17*s*. 4¼*d*. 12. 19*s*. 8¾*d*.

261. Practice may be either **Simple** or **Compound** according as the quantity whose value weight, etc., is to be found in Simple or Compound.

In **Simple Practice** the given quantity is in the same denomination as the unit whose value is given.

87 things at 9 as. 10 p. each ; 38 cwt. at £2-6-8 a cwt. ; Rs. 87 at 1*s*. 4*d*. a rupee.

Thus Simple Practice is only a process of finding by means of Aliquot parts the cost of a simple quantity, when the value of the unit is given.

Before the student begins to work out a sum in Practice, he must be able to split the given amount into its Aliquot parts.

262. An Example in Simple Practice is worked as follows :—

EXAMPLE 1. Find the value of 180 jack-fruits at 5 as. 9 p. each.

	RS.	A.	P.	
	180	0	0	= the value of 180 fruits at Re. 1 each.
4 as. = ¼ of 1 Re.	45	0	0	= .... 4 as. ,,
1 anna = ¼ of 4 as.	11	4	0	= .... 1 an. ,,
6 pies = ½ of 1 an.	5	10	0	= .... 6 p. ,,
3 pies = ½ of 6 p.	2	13	0	= .... 3 p. ,,
	Rs. 64	11	0	= .... 5 as. 9 p. each.

We do not take into account Rs. 180, for that is the value at Re. 1 each, which is not proposed in the question.

EXAMPLES.—ORAL.

Find the values of the following number of things :—

1. (1) 16, (2) 24, (3) 40, (4) 60 at 3½*d*.
2. (1) 10, (2) 15, (3) 25, (4) 35 at 1 an. 4 p.
3. (1) 14, (2) 16, (3) 18, (4) 20 at 1*s*. 6*d*.
4. 18, at (1) 2 as. 8 p. (2) 1 an. 4 p. (3) 5 as. 4 p.

EXAMPLES. I.

Find by Practice the value of the following number of articles :—

1. 256 at 10 p. each.
2. 131 at 11*d*. each.
3. 371 at 18*s*. 8*d*.
4. 1572 at 16*s*. 4*d*.
5. 1025 at 25 as. 10½ p.
6. 2013 at 14 as. 7½ p.
7. 1224 at 17*s*. 8¾*d*.
8. 2030 at 18*s*. 9½*d*.

EXAMPLE 2. Find the cost of 527 things at £3-14-8$\frac{3}{4}$ each.

	£	s.	d.	
	527	0	0	= cost of 527 things at £1 each.
			3	
	1581	0	0	= .... £3 ,,
10s. = $\frac{1}{2}$ of £1	263	10	0	= .... 10s. ,,
4s. = $\frac{1}{5}$ of £1	105	8	0	= .... 4s. ,,
8d. = $\frac{1}{6}$ of 4s.	17	11	4	= .... 8d. ,,
$\frac{1}{2}$d. = $\frac{1}{16}$ of 8d.	1	1	11$\frac{1}{2}$	= .... $\frac{1}{2}$d. ,,
$\frac{1}{4}$d. = $\frac{1}{2}$ of $\frac{1}{2}$d.	0	10	11$\frac{3}{4}$	= .... $\frac{1}{4}$d. ,,
	1969	2	3$\frac{1}{4}$	= cost of 527 things at £3-14-8$\frac{3}{4}$ each.

EXAMPLES. II.

Find the cost of the following number of articles :—

1. 885 at £1-6-5 each.
2. 987 at £1-13-4 each.
3. 2564 at Rs. 7-15-4.
4. 475 at £2-16-8.
5. 1031 at £7-11-8.
6. 18426 at Rs. 2-8-9$\frac{1}{2}$.
7. 2204 at Rs. 4-11-7$\frac{1}{4}$.
8. 4284 at £5-16-4$\frac{3}{4}$.

EXAMPLE 3. Required the cost of constructing 1234·5 miles of railway at £485-5-6 per mile.

The cost of 1234·5 miles at £1 per mile = £1234·5.

	£	
	1234·5	= cost at £1 per mile.
	485	
	49380	= .... £400 ,,
	98760	= .... £80 ,,
	6172·5	= .... £5 ,,
5s. = $\frac{1}{4}$ of £1.	308·625	= .... 5s. ,,
6d. = $\frac{1}{10}$ of 5s.	30·8625	= .... 6d. ,,
	599071·9875	= .... £599071-19-9.

EXAMPLES. III.

Find by Practice the cost of constructing roads whose lengths are given in miles—

1. 874·5 at Rs. 784-9-9.
2. 2006·6 at £1891-11-8.
3. 364·4 at £119-15-8.
4. 709·25 at £1729-3-4.

———

263. When the given unit is a **Mixed Number,** convert the fraction into a decimal and proceed.

EXAMPLE 4. Find the cost of $167\frac{3}{8}$ cwt. at £3-2-6 each.

The cost of $167\frac{3}{8}$ cwt. at £1 each = £167·375.

	£	
	167·375	= cost of $167\frac{3}{8}$ cwt. at £1 each.
	3	
	502·125	=………………………… £3 ,,
2*s*. 6*d*. = $\frac{1}{8}$ of £1	20·921875	=………………………… 2*s*. 6*d*. ,,
Adding	523·046875	= £523-0-$11\frac{1}{4}$ Ans.

EXAMPLES. IV.

Find the cost of—

1. $784\frac{1}{2}$ yds. at Rs. 13-9-$9\frac{1}{2}$.
2. $237\frac{1}{4}$ cwt. at £3-11-$6\frac{3}{4}$.
3. $702\frac{3}{5}$ cwt. at £2-6-$7\frac{1}{2}$.
4. $1234\frac{5}{8}$ tons at £7-11-8.
5. $999\frac{7}{8}$ ac. at Rs. 999 10 as.
6. $1015\frac{2}{3}$ yds. at 3*s*. $10\frac{1}{2}$*d*.
7. $986\frac{5}{12}$ mds. at Rs. 10-8 as.
8. $905\frac{2}{3}$ mds. at Rs. 14-12-6.

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264. In working questions in Practice, we may sometimes with advantage employ subtraction of Aliquot parts, instead of addition; thus, 13 as. 4 p. = Re. 1 — $\frac{1}{6}$ of Re. 1.

EXAMPLE 5. Find by Practice the value of 125 things at 13 as. 4 p. each.

13 as. 4 p. = 1 rupee — 2 as. 8 p.

	RS.	A.	P.	
	125	0	0	= cost of 125 things at Re. 1 each.
2 as. 8 p. = $\frac{1}{6}$ of Re. 1	20	13	4	=……………………2 as. 8 p. ,,
Subtracting	104	2	8	=……………………13 as. 4 p. ,,

EXAMPLES. V.

Find the value, by using subtraction of Aliquot parts, of the following number of articles:—

1. 199 at 13 as. 4 p.
2. 420 at 17*s*. 6*d*.
3. 1000 at 18*s*. 4*d*.
4. 370 at 18*s*. 8*d*.
5. 3456 at £ 2 19*s*.
6. 4840 at Rs. 3 15 as.
7. 960 at Rs. 2-14-8.
8. 999 at £5-18-4.

265. Questions such as the following bearing on Exchange, Bankruptcy, etc., may be easily worked by Practice.

EXAMPLE 6. Find the value of Rs. 23,450 in English money at 1*s.* 4*d.* a rupee.

	£	*s.*	*d.*	
	23450	0	0	= the value when Re. 1 = £1.
1*s.* = $\frac{1}{20}$ of £1	1172	10	0	= .... Re. 1 = 1*s.*
4*d.* = $\frac{1}{3}$ of 1*s.*	390	16	8	= .... Re. 1 = 4*d.*
	£1563	6	8	= the value of Rs. 23,450 when Re. 1 is worth 1*s.* 4*d.*

EXAMPLES. VI.

Find by Practice the value in English money of—

1. Rs. 12,340 at 1*s.* 4*d.* a rupee.
2. Rs. 9,999 at 1*s.* 4*d.* a rupee.
3. Rs. 3,45,678 at 1*s.* $4\frac{1}{16}$*d.* a rupee.
4. Rs. $2{,}379\frac{3}{4}$ at 1*s.* $3\frac{3}{4}$*d.* a rupee.

EXAMPLE 7. A bankrupt has debts amounting to Rs. 4,800; but he can pay only 10 as. 10 p. in the rupee. Find by Practice what his assets are.

	Rs.	
	4800	= his effects if he pays the whole debt.
8 as. = $\frac{1}{2}$ of Re. 1	2400	= ....8 as. in the Re.
2 as. = $\frac{1}{4}$ of 8 as.	600	= ....2 as..............
6 p. = $\frac{1}{4}$ of 2 as.	150	= ....6 p...............
4 p. = $\frac{1}{6}$ of 2 as.	100	= ....4 p...............
	Rs. 3250	= ....10 as. 10 p................

EXAMPLES. VII.

1. A bankrupt's debts amount to Rs. 15,640; and he can pay 13 as. 11 p. in the rupee; what are his assets?

2. A bankrupt has debts amounting to Rs. 5,440; but he can pay only 12 as. 8 p. in the rupee. What is he worth?

3. A man has a debt of £1,878-17-6; and he can pay 13*s.* $8\frac{3}{4}$*d.* in the £. Find how much he is worth.

4. A man has a debt of £15,756-13-4; and he can pay only 15*s.* $11\frac{1}{2}$*d.* in the £. What are his assets?

BILLS AND INVOICES.

266. When a person completes his purchases at a shop or a business house, the seller of the goods delivers to the buyer a list of the articles bought with their prices specified and the total made up. Such a list is called a **Bill**.

267. An Invoice is a statement in detail, of the quantity, description and price of the goods, sent by a seller to a buyer to his residence along with the goods.

Each separate entry in a bill or an invoice is called an **item**.

If the cost of the goods is paid for at the time of purchase, the invoice is *receipted*; that is the seller or his Cashier writes 'Received' on it, signs it and enters the date of receipt.

If the amount exceeds Rs. 20, one anna stamp must be affixed, which is cancelled by writing the name and the date across it.

In England, if the value of the purchase is £2 or exceeds this sum, a penny stamp is affixed and similarly cancelled.

268. If money is not paid at the time of purchase, when credit is given, a statement is sent to the customer once every month, quarter or at other fixed periods of the year, shewing the amount of the invoices with their dates and total; and this statement is called an **Account**.

Specimen of an Invoice or Bill.

PALAMCOTTAH, *31st Dec., 1898.*

MR. S. M. DANIEL,

Bought of EDWARD & CO.

		Rs.	A.	P.
Dec. 31	11 yds. of flannel at Rs. 1-14-6 ...	20	15	6
	16 yds. of twill at 4 as. 6 p.	4	8	0
	6 dozen handkerchiefs at Rs. 2-4 a dozen.	13	8	0
	12 yds. of silk at Rs. 2-8-6	30	6	0
	Total Rs. ...	69	5	6

EXAMPLES. VIII.

Make out bills for the following :—

1. 100 cocoanuts at ¾ of an anna each ; 75 mangoes at 1½ as. ; 250 oranges at 1¼ as. ; 300 apples at 1½ as.

2. 11 blankets at Rs. 9 a pair ; 11 pairs at Rs. 2 1 an. a single blanket ; 10½ yds. of silk at Rs. 2-3-8 a yd. ; and 50 handkerchiefs at Re. 1 11 as. per dozen.

3. 5 viss of pepper at 6 pies per pal. ; 4½ seers at 3 as. 4 p. a seer ; 3½ viss at 3 pies per pal. ; 1 md. at 2¾ as. a seer.

4. 35¼ lbs. of tea at 2*s*. 6*d*. per lb. ; 18 lbs. at 5*s*. 6½*d*. ; 20 lbs. at 6*s*. 3¼*d*. ; and 10¾ lbs. of coffee at 3*s*. 6*d*. per lb.

5. 8¾ yds. of cloth at 2 as. 6 p. a yd. ; 5 score quill pens at 5 for 2 pies ; 15½ yds. of tape at 3 pies a yd. ; and 13 papers of pins at 2 as. 6½ pies a paper.

6. 77 yds. of silk at 17*s*. 8¾*d*. per yd. ; 72⅓ yds. of velvet at 12*s*. 6*d*. a yd. ; 22½ yds. of flannel at 4*s*. 9¾*d*. a yd.

7. 8½ quires of paper at 3 as. 6 p. a quire ; 24 note-books at Rs. 1-4-6 each ; 120 pen holders at 5½ as. a score ; 3 gross steel nibs at 11 pies a dozen.

8. 1 md. 3 viss of sugar at 1 an. a viss ; 17½ yds. of flannel at Rs. 2¼ a yd. ; 10¾ viss of coffee at Rs. 2½ a viss ; and 12¼ lbs. of tea at 12 as. 8 p. a lb.

COMPOUND PRACTICE.

269. In **Compound Practice,** we find the value of a compound quantity when the value of only one of its denominations is given ; as 2 mds. 3 viss 4 pals. at Rs. 3-8-8 a md.

EXAMPLES.—ORAL.

1. What aliquot parts of a candy are (1) 5 mds. (2) 10 viss ?

2. Express as aliquot parts of 1 viss (1) 8 pals. (2) 5 pals. ?

3. What aliquot parts of 1 cwt. 1 qr. are (1) 35 lbs. (2) 28 lbs. (3) 20 lbs. (4) 14 lbs. ?

4. Express as aliquot parts of 1 fur. (1) 8 poles. (2) 5 poles. (3) 55 yds. (4) 44 yds. (5) 22 yds.

5. What aliquot parts of 1 day are (1) 1½ hrs. (2) 80 min. ?

EXAMPLE. Find the value of 2 mds. 3 viss 4 pals. of sugar at Rs. 3-8-8 a maund.

	Rs.	A.	P.	
	3	8	8	...value of 1 md.
			2	
	7	1	4	= ....2 mds.
2 viss = $\frac{1}{4}$ of 1 md.	0	14	2	= ....2 viss.
1 viss = $\frac{1}{2}$ of 2 viss.	0	7	1	= ....1 viss.
4 pals. = $\frac{1}{10}$ of 1 viss	0	0	8½	= ....4 pals.
	Rs. 8	7	3½	............2 mds. 3 viss 4 pals.

EXAMPLES. IX.

Find by Practice the value of—

1. 5 mds. 3 viss 9 pals. at Rs. 5-10-8 a md.
2. 7 mds. 4 viss 3 srs. 6 pals. at Rs. 5-7-8 a md.
3. 8 cwt. 3 qrs. 12 lbs. at Rs. 27-4-4 per cwt.
4. 9 tons 17 cwt. 2 qrs. 24 lbs. at Rs. 125-6-8 a ton.
5. 7 tons 13 cwt. 24 lbs. at 2½ guineas a ton.
6. 12 reams 12 quires 12 sheets of paper at Rs. 3-14 a ream.

Find the cost of repairing a road—

7. 6 mi. 3 fur. 110 yds. long at Rs. 450-8 as. a mile.
8. 7 mi. 5 fur. 165 yds. long at Rs. 682-7-4 per mile.

EXAMPLE. Find the value of 24 tons 12 cwt. 3 qrs. 21 lbs. at £18-12-6 a ton.

First Method.

Take aliquot parts of ton, cwts., etc., as in the above example and proceed.

Second Method

Or since £18-12-6 = £18·625, we might multiply £18·625 by the given weight, taking aliquot parts as in method 1.

Third Method.

	£	
	24·646875	at £1 per ton.
	18	
	246·46875	at £10 per ton.
	197·175000	at £8 ,,
10*s*. = $\frac{1}{2}$ of £1.	12·3234375	at 10*s*. ,,
2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*	3·080859375	at 2*s*. 6*d*.
	459·048046875	= £459-0-11½ to the nearest penny.

28{ 7 | 21 lbs.
   4 | 3·0
4 | 3·75 qrs.
20 | 12·9375 cwt.
24·646875 tons.

Fourth Method.

Find the product of 24·646875 and 18·625 by contracted multiplication; and find the value to the nearest penny.

$$\begin{array}{r l}
246\cdot46875 & \\
1\cdot8625 & \\
\hline
246\cdot46875 & \\
197\cdot17500 & \\
14\cdot78812 & \\
\cdot49294 & \\
\cdot12323 & \\
\hline
£459\cdot04804 & = £459\text{-}0\text{-}11\tfrac{1}{2}.
\end{array}$$

EXAMPLES. X.

Find by Practice to the nearest pie or penny the cost of—

1. 72 can. 2 mds. 6 viss at Rs. 200-8-4 a candy.
2. 50 reams 14 quires 8 sheets at Rs. 18-12 as. a ream.
3. 21 tons 13 cwt. 3 qrs. 26 lbs. at Rs. 40-13-4 per ton.
4. 55 yds. 2 ft. 9 in. of silk at 3*s*. 9*d*. per yard.
5. 300 tons 18 cwt. 2 qrs. at £240-12-6 a ton.
6. 157 mi. 3 fur. 24 po. of telegraph wire at £11 10*s*. a mile.
7. Constructing a tramway 17 mi. 7 fur. 165 yds. long at Rs. 18,223-5-4 a mile.
8. Making a road 37 mi. 6 fur. 31 po. 3 yds. long at Rs. 1,785-3-4 per mile.

Example. Find the cost of repairing a road 15 mi. 0 fur. $41\frac{1}{4}$ yds. long at Rs. 50-10-10 a furlong.

In this example, (1) we may reduce 15 miles to furlongs and find the cost of 120 fur. $41\frac{1}{4}$ yds. at Rs. 50-10-10 a furlong, or (2) finding the cost of 1 mile to be Rs. 50-10-10 × 8 or Rs. 405-6-8, we may obtain the cost of 15 mi. 0 fur. $41\frac{1}{4}$ yds. at Rs. 405-6-8 a mile, or (3) we may find the value at Rs. 50-10-10 a mile in the usual way and divide it by 8.

This may be easily worked if the yards be expressed in decimals of a mile thus :

220 { 20|41·25 yds.
11|2·0625
8|·1875
Miles 15·0234375

	Rs.
	15·0234375 at Re. 1 a mile.
	405
	6009·37500 at Rs. 400 a mile.
	75·1171875 at Rs. 5 ,,
5 a. 4 p. = $\frac{1}{3}$ of Re. 1.	5·0078125 at 5 as. 4 p. ,,
1 a. 4 p. = $\frac{1}{4}$ of 5 a. 4 p.	1·251953125 at 1 a. 4 p. ,,
	6090·751953125 = Rs. 6090-12-0 to the nearest pie.

EXAMPLES. XI.

Find by Practice to the nearest pie or penny the cost of—

1. 4 mi. 6 fur. 120 yards of wire at £3-12-10 a furlong.
2. 6 can. 18 mds. 7 viss 32 pals. of quicksilver at Rs. 233-5-4 a maund.
3. 2 can. 8 mds. 3 viss 12 pals. at Rs. 2-6-8 a viss.
4. 20 ac. 2 ro. 25 po. at Rs. 450-10-8 per rood.
5. 14 ac. 5 po. $15\frac{1}{8}$ sq. yds. at Rs. 640-8 as. per acre.
6. 25 ac. 6 po. at £82-10-8 an acre.
7. 17 mi. 68 yds. 2 ft. 3 in. at Rs. 3,460-10 as. per mile.
8. 209 tons 2 qrs. 14 lbs. at Rs. 180-5-4 per ton.

EXAMPLES. XII.

MISCELLANEOUS.

Find by Practice—

1. The value of Rs. 720-12-6 at 1*s*. 4*d*. a rupee.
2. The dividend on £720-11-3 at 16*s*. 9*d*. in the £.
3. The value of 99 cwt. 3 qrs. 27 lbs. at £5-2-6 per cwt.
4. The cost of 15 mds. 25 srs. 6 pals. at £12-10-3 a md.
5. The value of 12 can. 1 md. 7 viss 16 pals. at Rs. 840-6-8 a md.
6. The value of 71 yds. 2 ft. 9 in. of cloth at 13 as. 8 p. a yd.
7. The dividend on Rs. 2,400-13-9 at 9 as. 8 p. in the rupee.
8. The weight of 2 mds. 3 viss 4 srs. 4 pals. at 3 lbs 2 oz. per viss.

9. Find the total cost of—

(*a*) A fence 3 fur. 11 po. $3\frac{1}{4}$ yds. long at £183-6-8 per mile.
(*b*) 345 shares at £40-16-9 each.
(*c*) 27 tons 14 cwt. 3 qrs. 14 lbs. of hay at £4 16*s*. per ton.

10. Make out a bill for the following articles, and express the sum in rupees, annas and pies, taking the rupee to be equal to 1*s*. 4*d*.—

(*a*) 1 piece of flannel 40 yds. long, at 2*s*. $10\frac{1}{2}$*d*. a yd.
(*b*) 2 pieces of long cloth, 39 yds. each, at 6 as. 8 p. per yd.
(*c*) 1 cwt. 3 qrs. 1 stone of sugar at £1-15*s*. a cwt.

11. Make out a bill for the following, expressing the total in £ *s*. *d*. taking 1*s*. 4*d*. equivalent to one rupee :—

(*a*) $42\frac{1}{2}$ yds. of twill at 3 as. 9 p. a yd.
(*b*) 10 tons 6 cwt. 3 qrs. 14 lbs. at £3-4*s*. per ton.
(*c*) 4 can. 3 mds. 2 viss at Rs. 61-5-6 per md.

12. Find the cost of constructing a railway consisting of (*a*) 198 mi. 2 fur. 165 yds. at Rs. 39,175-2-8 per mile, (*b*) 2,240 yds. of bridge at Rs. 158-10-10 per yard and (*c*) 675 yds. of tunnel at Rs. 24-9-11 per foot.

CHAPTER XXIV.

Square Measure : Rectangular Areas.

270. The student is already familiar with the meaning of the terms Square and Rectangle.

The **dimensions** of a rectangle, are its length and breadth.

The **area** of a rectangle, square or any other figure is the amount of space occupied within its bounding lines.

If a side of a square is one inch, the area of the square is one square inch ; if a side of a square is one centimetre, the area of the square is one square centimetre ; if a side of a square is one foot, the area is one square foot ; and if a side of a square is one metre, its area is one square metre.

Square inch, square centimetre, square foot and square metre are square units.

We thus see that a square unit is a square whose side is one linear unit.

The area of a figure is measured by the number of square units it contains.

271. To find the area of a rectangle, the length and breadth being given.

Take the rectangle ABCD, whose side AB represents 6 in. length and the side BC represents 5 in. breadth.

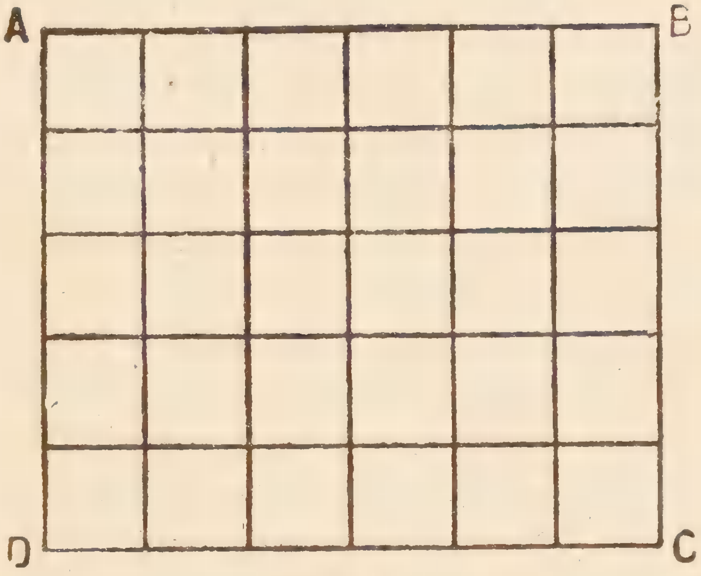

FIG. 35.

Divide AB into 6 equal parts and BC into 5 equal parts. Then each part represents one inch.

Through the points of division draw parallel lines to the sides as shewn in the figure.

The rectangle ABCD is thus divided into squares representing square inches.

ABCD now contains 6 columns of 5 squares; therefore the rectangle contains 6 × 5 squares or 6 × 5 square inches, i.e. 30 sq. in.

∴ *To find the area of a rectangle, multiply the number of linear units in the length by the number of linear units of the same name in the breadth, and the product is the number of corresponding square units in the area.*

A rectangle 3 ft, 6 in. long and 2 feet 3 in. wide is sometimes thus expressed ‘a rectangle 3 ft. 6 in. by 2 ft. 3 in.’ or 3′ 6″ × 2′ 3″.

Practical Exercise.

On squared paper draw the following rectangles (actual size) and find their areas.

1. Length 4 in. breadth 2 in. 2. Length 5 in. breadth 3 in.
3. Length 6 cm. breadth 5 cm. 4. Length 8 cm. breadth 6 cm.
5. 6 in. by 3 in. 6. 6 in. by 4 in. 7. 6″ × 5″.
8. 5″ × 5″. 9. 8 cm. × 8 cm. 10. 12 cm. × 10 cm.
11. 11 cm. × 7 cm. 12. 15 cm. × 12 cm.

EXAMPLES.—ORAL.

Give the areas of the following rectangular objects :—

1. A window 5 ft. by 4 ft. 2. A door 6′ × 5′.
3. A page of a book 7″ × 4″. 4. A frame 16″ by 12″.
5. A piece of cardboard 8 cm. by 7 cm.
6. A post card 14 cm. × 9 cm. 7. A plot of ground 8 yds. × 6 yds.
8. A sheet of paper 21 cm. × 10 cm.
9. A field 13 dm. by 9 dm. 10. A wall 15 ft. long and 14 ft. high.

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272. To construct the Table of Square Measure.

The figure ABCD represents a square each of whose sides is 1 ft.; that is ABCD represents 1 sq. ft. Fig. 36.

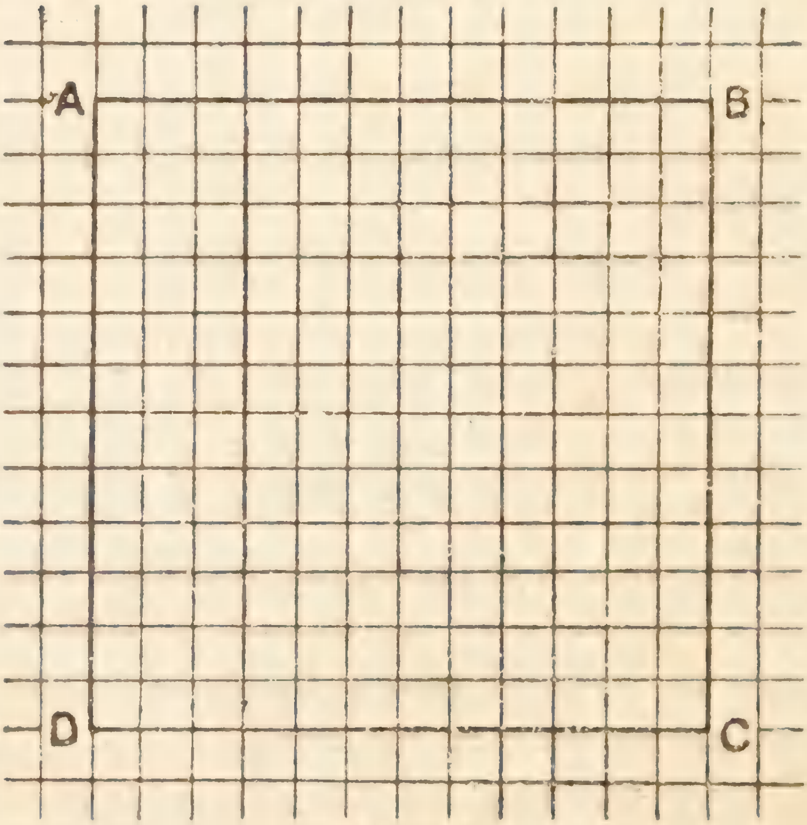

FIG. 36.

Divide AB and BC into 12 equal parts each part representing one inch. Draw parallels to the sides through the points of division. The rectangle is now divided into small squares, each small square representing 1 sq. in.

The whole square contains 12 rows, each row containing 12 small squares.

$\therefore$ ABCD contains 12×12 or 144 small squares ; i.e. 144 sq. in.

We thus see that from 1 ft. or 12 in. 1 sq. ft. 12^2 or 144 sq. in is derived.

We therefore write 12^2 or 144 sq. in. make 1 sq. ft.

Practical Exercise.

Draw diagrams to deduce the following :—

1. From 1 yd. = 3 ft., 1 sq. yd. = 3^2 or 9 sq. ft.
2. From 1 chain = 4 poles, 1 sq. ch. = 16 sq. poles.
3. From 1 chain = 22 yds., 1 sq. ch. = 22^2 or 484 sq. yds.
4. From 1 mile = 80 chains, 1 sq. mile = 80^2 or 6,400 sq. ch.
5. From 1 chain = 100 links, 1 sq. ch. = 100^2 or 10,000 sq. links.
6. From 1 pole = $5\frac{1}{2}$ yds., 1 sq. po. = $(5\frac{1}{2})^2$ or $30\frac{1}{4}$ sq. yds.
7. From 1 cm. = 10 mm., 1 sq. cm. = 10^2 or 100 sq. mm.
8. From 1 dm. = 10 cm., 1 sq. dm. = 10^2 or 100 sq. cm.
9. From 1 m. = 10 dm., 1 sq. m. = 10^2 or 100 sq. dm.

---

273. The student can now draw up for himself the following tables of square measure :—

British.

12^2 or 144 sq. in.	make	1 sq. ft.
3^2 or 9 sq. ft.	,,	1 sq. yd.
22^2 or 484 sq. yds.	,,	1 sq. ch.
$(5\frac{1}{2})^2$ or $30\frac{1}{4}$ sq. yds.	,,	1 sq. po.
4^2 or 16 sq. po.	,,	1 sq. ch.
80^2 or 6,400 sq. ch.	,,	1 sq. mile.

The following also are in use.

40 sq. po.	make	1 rood.
4 roods or 4,840 sq. yds.	,,	1 acre.
100^2 or 10,000 sq. links	,,	1 sq. ch.
10 sq. ch.	,,	1 acre.
640 acres or $(1760)^2$ sq. yds.	,,	1 sq. mile.

Metric Square Measure.

10^2 or 100 sq. mm. = 1 sq. cm.
10^2 or 100 sq. cm. = 1 sq. dm.
10^2 or 100 sq. dm. = 1 sq. m. = 1 centiare.
10^2 or 100 sq. m. = 1 sq. Dm. = 1 are.
10^2 or 100 sq. Dm. = 1 sq. Hm. = 1 hectare.
10^2 or 100 sq. Hm. = 1 sq. Km.

Sq. Dm. and sq. Hm. are seldom used.

Used in Surveying.

100 centiares = 1 are.
100 ares = 1 hectare.
100 hectares = sq. Km.

Table of Square Measure (Madras).

2,400 square feet = 1 manei or ground.
24 grounds = 1 cawnie.
1 cawnie = 6,400 square yards.
160 acres = 121 cawnies
100 cents = 1 acre.

Note.—There is no corresponding linear measure from which an acre is derived; but since an acre contains 4·840 sq. yds., and a square whose side is 70 yds. contains 70^2 or 4,900 sq. yds. or 1·0124 of an acre, an acre may approximately be taken as equal to 70 × 70 sq. yds.

EXAMPLES.—ORAL.

Find approximately the number of acres in the following rectangular plots of land, whose dimensions are given in yards:—

1. 210 × 70. 2. 350 × 70. 3. 210 × 140. 4. 280 × 210.
5. 350 × 280. 6. 210 × 35. 7. 280 × 35. 8. 280 yds. square.

Note.—The student must distinguish between 280 yds. square and 280 sq. yds.

280 yds. square denotes a surface that is a square in shape containing an area of 280 × 280 sq. yds.

280 sq. yds. denotes a surface whose area is 280 sq. yds. without any reference to its shape.

Fig. 36 represents 12 in. square; but one row of it denotes 12 sq. in.

A foot square and a square foot have the same area; the former is a square while the latter is not.

EXAMPLE 1 (*a*). Draw on squared paper a plan of a room 15 ft. by 12 ft., 1 division representing 1 foot, and find the area.

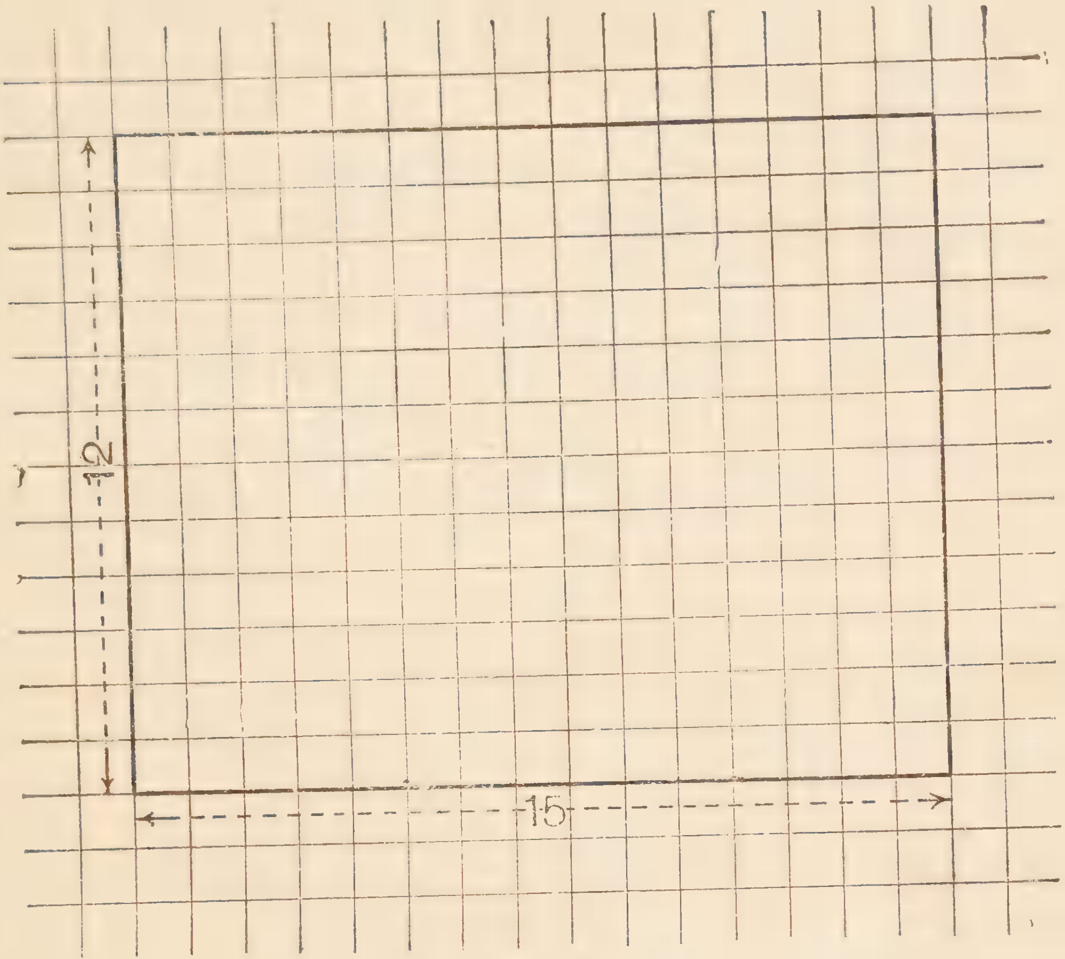

FIG. 37.

Draw the rectangle ABCD, having a length of 15 divisions and breadth of 12 divisions. Since 1 division represents 1 ft., each small square represents 1 sq. ft.

And the rectangle contains 15 × 12 or 180 squares; and therefore the area of the rectangle = 180 sq. ft.

∴ The area of the room = 180 sq. ft. = 15 × 12 sq. ft.

Note.—It is a common error to write 15 ft. × 12 ft. = 180 sq. ft. This is wrong since we cannot multiply foot by foot. It is correct to write 15 sq. ft. × 12 = 180 sq. ft.

EXAMPLE 1 (*b*). Draw on squared paper a rectangle 1·4 in. by 0·8 in. and find its area.

Take one division representing $\frac{1}{10}$ in. as the unit. Then the small square representing $\frac{1}{100}$ sq. in. is the corresponding unit of area.

Draw the rectangle ABCD taking for its length 14 divisions and for its width 8 divisions.

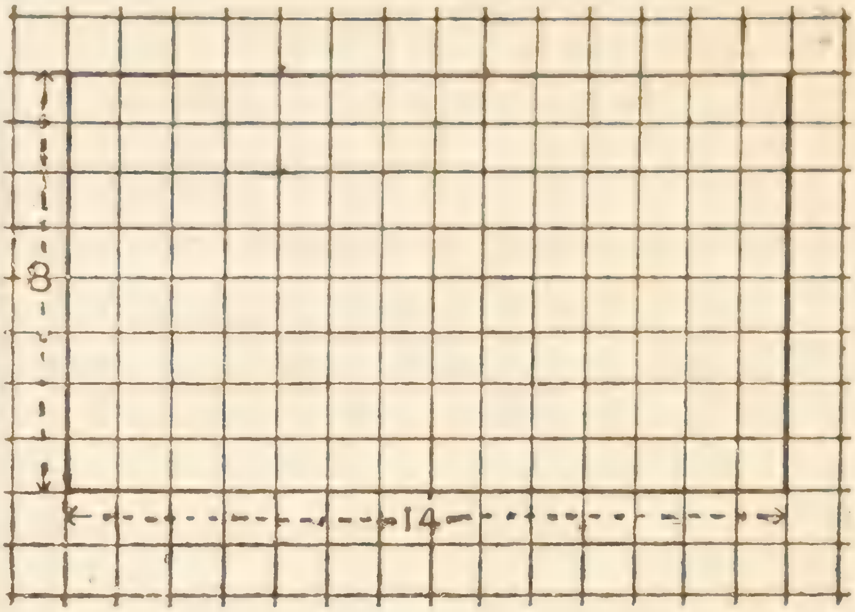

FIG. 38.

The rectangle ABCD contains 14 × 8 small squares or 14 × 8 units of area and therefore represents.

$\frac{14 \times 8}{100}$ sq. in., i.e. 1·12 sq. in.

∴ Area of the rectangle = 1·4 × ·8 sq. in. = 1·12 sq. in.

The rule given under Art. 271 for finding the area of a rectangle, holds good not only when the numbers giving the length and breadth in linear units are integral as in example 1, but also when they are fractional as in this Example.

274. *We may therefore state generally that—*

If L denotes the number of linear units in the length of a rectangle, B the number of linear units of the same name in the breadth, and A the area of the rectangle in corresponding square units, $A = L \times B$.

275. If the numbers denoting length and breadth are given in different denominations, express each in units of any the same denomination; and their product will give the area in corresponding square units.

EXAMPLE 1 (*c*). Find the area of a rectangular room whose length is 17′ 3″ and width 13′ 10″.

$$\text{Area of the room} = 17\tfrac{1}{4} \times 13\tfrac{5}{6}\text{ sq. ft.} = \frac{69}{4} \times \frac{83}{6}\text{ sq. ft.}$$

$$= \frac{1909}{8}\text{ sq. ft.} = 238\text{ sq. ft. } 90\text{ sq. in.}$$

$$= 26\text{ sq. yds. } 4\text{ sq. ft. } 90\text{ sq. in.}$$

Practical Exercise.

On squared paper draw the following rectangles and find their area :—

1. 9 divisions by 8 divisions. 2. 17 divisions by 13 divisions.
3. 25 mm. by 15 mm. 4. 5·5″ by 4·8″.
5. 1·3″ by 0·7″. 6. 4·6 cm. by 3·8 cm.
7. 10″ by 8″, 1 in. to represent 2 in.
8. 4′ 6″ by 5′ 6″, 1 in. to represent 1 ft.
9. 28 cm. by 24 cm., scale 1 mm. to 1 cm.
10. 3 dm. by 25 cm. ,, ,, ,,
11. 16 ft. by 14 ft., scale 0·1 in. to 1 ft.
12. 12 m. by 9 m., scale 1 cm. to 1 m.

EXAMPLES.—ORAL.

Express the areas of the following rectangles :—

In sq. inches—

1. 1′ 6″ by 9′. 2. 2′ 6″ by 10″. 3. 3′ by 8′. 4. 2′ 6″ by 1′.
5. 1′ 10″ by 1′ 8″. 6. 3′ 4″ by 1′ 8″. 7. *a* in. by *b* in. 8. *x* in. by *y* in.

In square feet—

1. $3\frac{1}{2}$′ by $2\frac{1}{2}$′. 2. $3\frac{1}{2}$′ by $3\frac{1}{4}$′. 3. $4\frac{1}{2}$′ by $2\frac{1}{4}$′. 4. 8′ × 2′ 9″.
5. 12′ × 1′ 3″. 6. 2′ 6″ × 1′ 6″. 7. *a* ft. by *b* ft. 8. *x* ft. by *y* in.

In square yards—

1. 1 yd. 1′ by 2′. 2. 2 yds. 2′ by 3′. 3. 2 yds. 1′ by 1 yd. 1′.
4. 3 yds. 2′ by 2 yds. 2′. 5. 4 yds. 2′ by 4 yds. 1′.
6. 5 yds. 1′ by 3 yds. 2′. 7. *a* yds. by *b* ft.
8. *x* yds. by *y* ft. 9. *y* ft. by 3 in.

In sq. cm.—

1. 20 mm. × 15 mm. 2. 15 cm. × 30 mm. 3. 90 mm. × 8 cm.
4. 12 dm. × 8 cm. 5. 7 dm. × 5 cm. 6. 13 m. × 9 m. in centiares.
7. 5 Dm. × 4 m. in centiares. 8. 1 Km. × 16 m. in ares.
9. 2 Km. × 1 Km. in Hectares. 10. 1 Km. × 15 dm. in sq. metres.

EXAMPLES. I.

Find the area of the following rectangles whose lengths and breadths are respectively :—

1. 29 ft. and 27 ft.
2. 35 in. and 32 in.
3. 34 yds. and 17 yds.
4. $26\frac{1}{3}$ yds. and 21 yds.

Find the area of the following rectangular rooms :—

5. 4 yds. 1 ft. 6 in. long and 3 yds. 2 ft. 3 in. wide.
6. 6 yds. 2 ft. 6 in. by 4 yds. 1 ft. 3 in.
7. 30 ft. 10 in. long and 20 ft. 8 in. broad.
8. 16 ft. 4 in. long and 9 ft. 8 in. wide.

Find the area of the following :—

9. A verandah 14 ft. 9 in. long and 6 ft. 6 in. broad.
10. A table cloth 8 ft. 6 in. by 5 ft. 9 in.
11. A mango plank 10 ft. 8 in. long and 1 ft. 6 in. wide.
12. A wall 12 ft. 6 in. long and 8 ft. 9 in. high.

---

276. Since in a square, the length is equal to the width, its area is found by squaring the number of linear units in one of the sides.

Example 2. Find the area of the square field whose side measures 6 chains 4 links.

Area = 6·04 × 6·04 sq. ch.
= 36·4816 sq. ch.
= 3 ac. 6 sq. ch. 4816 sq. links.

$$\begin{array}{r} 6{\cdot}04 \\ 6{\cdot}04 \\ \hline 36{\cdot}24 \\ {\cdot}2416 \\ \hline 36{\cdot}4816 \end{array}$$

Practical Exercise.

Draw on squared paper to the scale given against each of the following squares, and find their area, when their sides respectively are :—

1. 2 ft. 6 in. R.F. $\frac{1}{12}$.
2. 25 cm. R.F. $\frac{1}{10}$.
3 1 metre R.F. $\frac{1}{25}$.
4. 3 ft. 4 in. R.F. $\frac{1}{4}$.
5. 3 yds. 1 ft. R.F. $\frac{1}{20}$.
6. 5 dm. R.F. $\frac{1}{10}$.

EXAMPLES —ORAL.

Find in square inches the area of a square whose side is—

1. 1′ 3″. 2 1′ 8″. 3. 2′ 6″. 4. 3′ 4″. 5. 4′ 2″. 6. 5′ 10″

Find in sq. ft. the area of a square whose side is—

1. $3' \, 4''$. 2. $4' \, 6''$. 3. $3' \, 8''$. 4. 2 yds. $2'$.
5. 3 yds. $1'$. 6. 4 yds. $1'$. 7. $2a$ ft. 8. $3x$ yds.

Find in sq. yds. the area of a square whose side is—

1. $4\frac{1}{2}$ yds. 2. $5\frac{1}{2}$ yds. 3. $3\frac{1}{4}$ yds. 4. 2 yds. 1 ft.
5. 3 yds. 2 ft. 6. 5 yds. 1 ft. 7. $5y$ yds. 8. $4x$ ft.

EXAMPLES. II.

Find the area of the following :—

1. A square room 20 ft. 9 in. long.
2. A courtyard $45' \, 11''$ square.
3. A square field whose side measures 1 Km. 5 m.
4. A playground 20 ch. 48 links square.

Find the area of the following square plots whose sides are respectively :—

5. 550 yds. in acres. 6. 10 ch. 25 links in acres.
7. 29 po. $2\frac{3}{4}$ yds. in acres. 8. 1 Km. 8 m. in Ha.

Express in acres the area of the following rectangular surfaces :—

9. A street 165 chains long and 10 yds. wide.
10. Three paddy fields each measuring 3 ch. 3 links by 2 ch. 44·5 links.
11. Ten plots of ground each 32 yds. by 15 yds. $4\frac{1}{2}$ in.
12. A tank measuring $3\frac{1}{2}$ miles by 4 furlongs.

EXAMPLE 3. On a map drawn to a scale of $\frac{1}{3600}$, the sides of a rectangular field are represented by 4·4 in. and 3·3 in. Find the area of the field in acres.

The length of the field $= 4{\cdot}4 \times 3600$ in. $= 44 \times 10$ yds.
The width of the field $= 3{\cdot}3 \times 3600$ in. $= 33 \times 10$ yds
$\therefore$ the area of the field

$$= \frac{44 \times 10 \times 33 \times 10}{4840} \text{ ac.} = 30 \text{ ac.}$$

EXAMPLES. III.

1. A person desires to form a rough estimate of the acreage of a rectangular field by stepping along two

adjacent sides. If his steps are on the average 33 in. and the two sides are 159 and 57 steps respectively, shew that the field contains about $1\frac{1}{2}$ acres.

2. On a map drawn to a scale of $\frac{1}{2500}$, the sides of a rectangular field are 4·8 cm. and 3·2 cm., respectively. Find the area of the field in *ares*.

3. On a map drawn to a scale of 1 : 4400, an estate measures 12 in. by 9 in. Find in acres, the area it represents.

4. A map drawn to a scale of a centimetre to 10 Km. is 13·5 cm. by 8·4 cm. Find correct to 10 sq. miles the area represented, if 1 Km. = 0·6124 mile.

Carpeting the floor, painting the ceiling, etc.

277. Since the carpet is laid on the floor, the quantity of carpeting required for a room is obtained by finding its area.

The ceiling has the same area as the floor.

EXAMPLE 4. Find the quantity of carpeting required for a room 18 ft. 10 in. long and 15 ft. 8 in. wide, and its cost at Rs. 13-8 as. per sq yard.

The quantity of carpeting required $= 18\frac{5}{6} \times 15\frac{2}{3}$ sq. ft.

$$= \frac{113}{6} \times \frac{47}{3} \text{ sq. ft.} = \frac{5311}{18} \text{ sq. ft.}$$

$= 295\frac{1}{18}$ sq. ft. = 295 sq. ft. 8 sq. in.

= 2 sq. yds. 7 sq. ft. 8 sq. in.

$$\text{Cost at Rs. 13-8 per sq. yard} = \frac{5311}{18 \times 9} \times \frac{27}{2} \text{ Rs.} = \text{Rs. } \frac{5311}{12}$$

= Rs. 442-9-4.

EXAMPLES. IV.

Find the quantity of carpeting required for the following rectangular rooms :—

1. 15 ft. 3 in. by 10 ft. 6 in. 2. 16 ft. 8 in. by 14 ft. 9 in.

3. A carpet 12 ft. by 10 ft. is spread in a room 15 ft. by 12 ft. What area is not covered by the carpet ?

4. A room 12 yds. by 10 yds. is covered with mats to the extent of 35 ft. by 25 ft. What area is left without matting?

Find the cost of carpeting the following rooms :—

5. 25 ft. ~~6 in~~. long and 16 ft. broad at Rs.2-7-11 per sq. yd.

6. 30 ft. 3 in. by 12 ft. 6 in. at 4*s*. 6*d*. a sq. ft.

Find the sum spent on painting the ceiling of the following rooms :—

7. 12·5 m. by 6·8 m. at 8·25 francs per sq. metre.

8. 10·45 m. by 8·54 m. at 5·625 fr. per sq. m.

EXAMPLES. V.

MISCELLANEOUS.

1. A flower garden 35 yds. by 20 yds. contains a lawn 80 ft. by 50 ft. What area is occupied by paths and flower beds?

2. One inch = 2·54 cm.; find to 3 significant figures the number of sq. cm. in a sq. in.

3. The number of pages in the *Madras Mail* issue of August 5, 1909 was 14, each page contained information measuring 23 in. by 15·1 in. How many sq. ft. of information did it contain?

4. Express in acres the area of a street 15 chains by 72 links.

5. A house site is $\frac{3}{4}$ ac. in extent; and a house is built on it with a frontage of 60 ft. and a depth of 80 ft. How much space remains vacant?

6. Postage stamps are 2·2 cm. by 1·8 cm.; find to the nearest sq. cm. the area of Rs. 10 worth of (1) quarter-anna, (2) half-anna, (3) one-anna stamps.

7. A courtyard is paved with Cuddapah slabs 2 ft. 6 in. by 1 ft. 8 in.; and 150 are required. What is the area of the yard?

8. The floor of a room is partly covered with a carpet 15′ 6″ by 12′ 6″, and the remaining area is 4 sq. yds. 6 sq. ft. Find the area of the room.

9. Find the area of the following to 3 significant figures :—

(1) A post card 5·8 in. by 2·9 in.
(2) Another post card 14 cm. by 8·8 cm.
(3) A foolscap sheet of paper 33·6 cm. by 21 cm.
(4) An envelope 4·8 in. by 3·7 in.

10. Find the greatest and least possible value of the area, and hence the possible error in determining the area of the following :—

(1) A rectangular field 325 m. by 236 m. to the nearest metre.
(2) Another measuring 1025 m. by 148 m. to the nearest metre.
(3) A rectangle whose sides are 36·8 cm. by 25·4 cm. to the nearest millimetre.
(4) Another whose sides are 45·5 cm. by 34·4 cm. to the nearest mm.

EXAMPLE 5. A rectangular plot of ground is 32 ft. by 28 ft. and a path 4 ft. wide runs round inside close to the boundary. Find the area of the path.

Let one division represent 2 ft. Then the rectangle ABCD represents the plot.

Leaving out all round a space of two divisions to represent 4 ft., draw the inner rectangle EFGH.

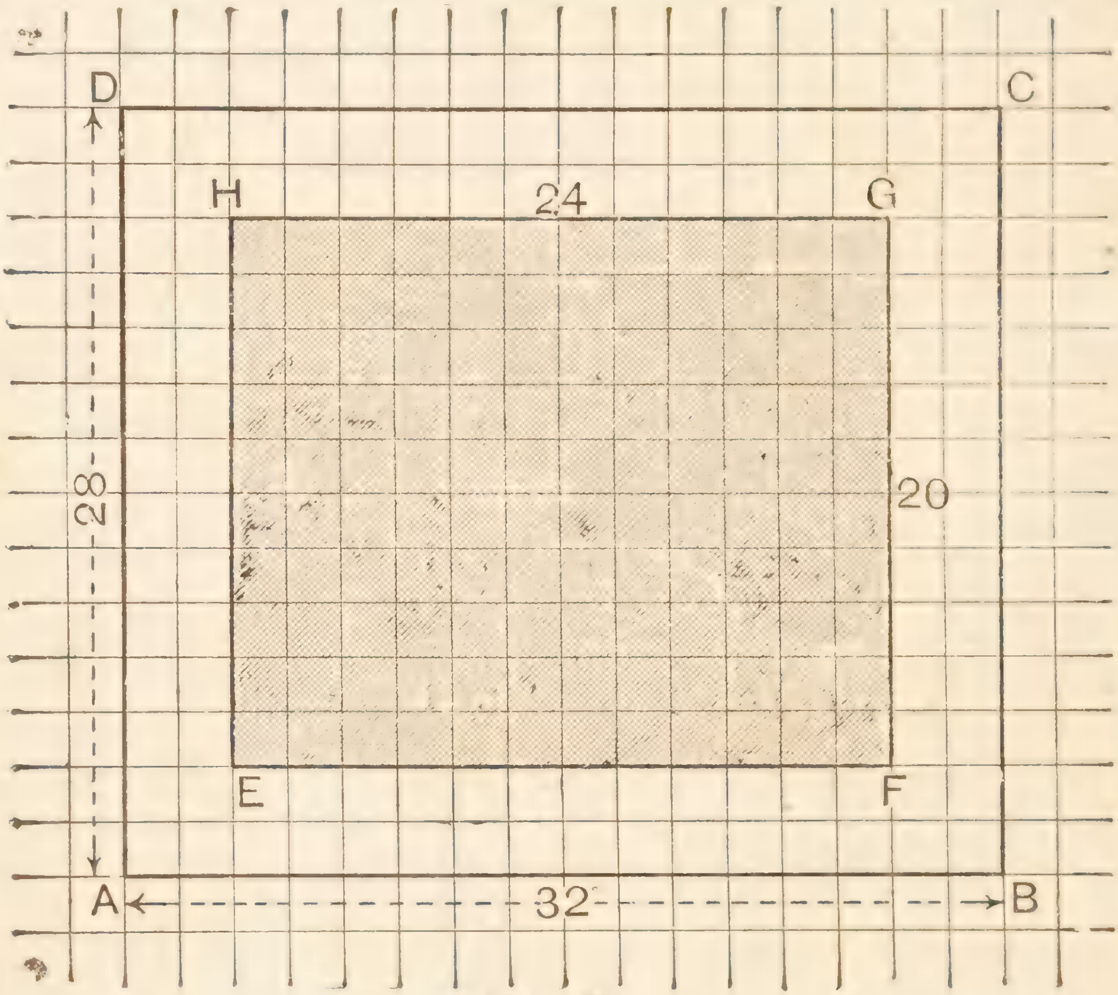

FIG. 39.

Then EF = (32 — 2 × 4) ft. = 24 ft.
And EH = (28 — 2 × 4) ft. = 20 ft.
∴ Area of EFGH = 24 × 20 sq. ft. = 480 sq. ft.
∴ Area of ABCD = 32 × 28 sq. ft. = 896 sq. ft.
∴ Area of the path = (896 — 480) sq. ft. = 416 sq. ft.

EXAMPLES. VI.

Draw a Diagram for each Example.

1. A rectangular plot of ground measures 30 yds. by 25 yds. A path 3 ft. wide runs round inside along the

boundary. Find the space occupied by the path. Scale 0·1 inch to represent 1 yd.

2. A room 21 ft. long and 16 ft. 6 in. wide is carpeted, leaving a space 3 ft. 6 in. all round. Find the cost of the carpet at Rs. 3 per sq. yd. Scale 0·1 inch to 1 foot.

3. A square field whose side is 75 yds. contains a rectangular lawn surrounded by a path 4 ft. wide. Find the cost of gravelling the path at 1 an. 2 p. a sq. yd. Scale 1 cm. to 5 yds.

4. A room is 20 ft. 10 in. long by 16 ft. How much carpet will it require so as to leave a margin 1 foot wide uncarpeted all round the room? Scale 1 cm. to 1 foot.

---

278. In Art. 274, we have seen that $L \times B = A$, where L and B denote the number of linear units in the length and breadth respectively of a rectangle, and A, the area in corresponding square units.

$$\therefore L = \frac{A}{B}; \text{ and } B = \frac{A}{L}.$$

EXAMPLE 6. The area of a school room is 1,083 sq. ft. 15 sq. in.; if the length be 17 yds. 2 ft. $\frac{3}{5}$ in., find the breadth.

$$\text{The area} = 1{,}083 \text{ sq. ft. } 15 \text{ sq. in.} = \frac{51989}{48} \text{ sq. ft.}$$

$$\text{And length} = 17 \text{ yds. } 2 \text{ ft. } \tfrac{3}{5} \text{ in.} = \frac{1061}{20} \text{ ft.}$$

$$\therefore \text{Breadth} = \frac{51989}{48} \text{ sq. ft.} \div \frac{1061}{20} \text{ ft.}$$

$$= \frac{51989}{48} \times \frac{20}{1061} \text{ ft.} = \frac{245}{12} \text{ ft.} = 20 \text{ ft. } 5 \text{ in.}$$

EXAMPLES.—ORAL.

State the number left blank.

1.	30 sq. yds.	8 yds.	——
2.	30 sq. ft.	2 yds.	——
3.	144 sq. yds.	——	36 ft.
4.	8 sq. yds.	16 ft.	——
5.	1 sq. m.	100 cm.	——
6.	50 sq. m.	——	4 m.

Taking 1 ac. approximately = 70 × 70 sq. yds.

7.	2 ac.	——	70 yds.
8.	3 ac.	140 yds.	——
9.	4 ac.	280 yds.	——
10.	6 ac.	——	140 yds.

Taking 1 ac. = 220 × 22 sq. yds.

11.	1 ac.	440 yds.	——
12.	2 ac.	——	44 yds.

EXAMPLES. VII.

1. The width of a door whose area is 3 sq. yds. 3 sq. ft. 96 sq. in. is 5 ft. 4 in. Find its length.

2. What length must be cut off a rectangular play-ground 25 chains wide so that it may contain exactly 1¼ ac.?

3. The area of a tank is 2½ acres, and its length is 268 yds. 2 ft. 8 in. Find its breadth.

4. The length of a rectangular field which contains 8 ac. 1 ro. 15 po. is 20 ch. 62·5 links. Find its breadth.

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279. To find the length of carpet, mat, etc., required.

EXAMPLE. How many yards of carpet 30 in. wide will be required for a room 30 ft. long and 15 ft. broad and what will it cost at Rs. 3-5-4 a yard?

Draw on squared paper a plan of the room taking 0·1 inch (1 division) to represent 1 foot.

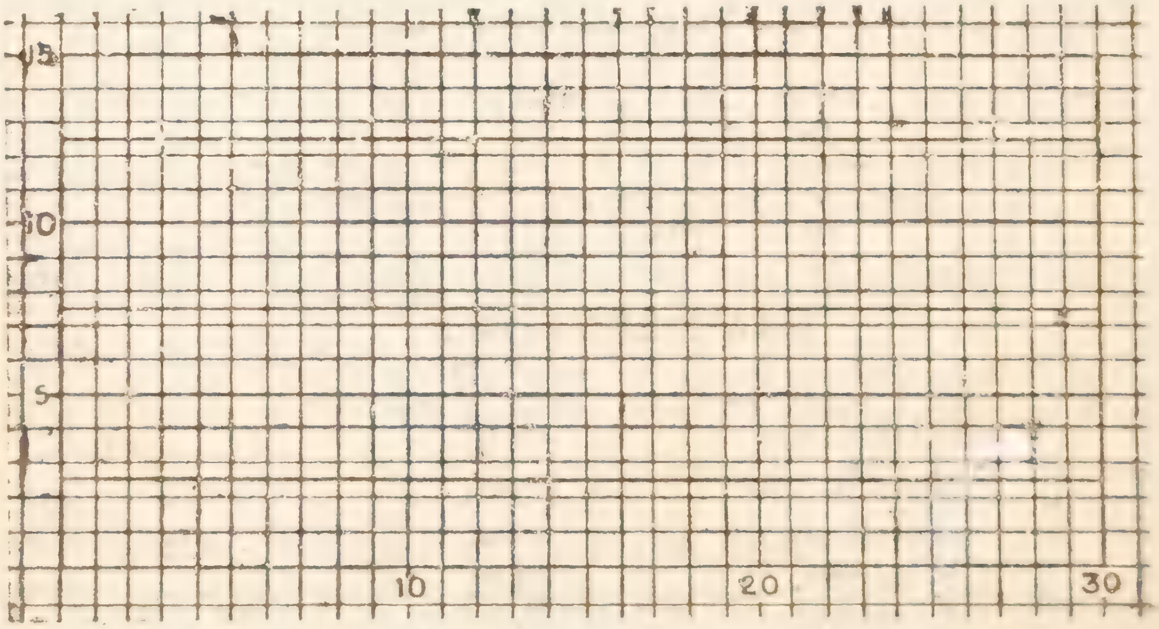

FIG. 40.

Divide the rectangle into strips of 30 in. or $2\frac{1}{2}$ ft. wide as shewn in the figure ; the number of strips is 6 and the length of each strip is 30 ft.

$\therefore$ The total length of the strips $= 30$ ft. $\times\ 6 = 60$ yds.

The working is as follows :—

Area of the carpet required $= 30 \times 15$ sq. ft.
Width of carpet$=30$ in. $= \frac{5}{2}$ ft.
$\therefore$ Length of carpet required $= 30 \times 15$ sq. ft. $\div \frac{5}{2}$ ft.
$= 30 \times 15 \times \frac{2}{5}$ ft. $= 30 \times 6$ ft.
$= 60$ yds.
$\therefore$ The cost at Rs. 3-5-4 per yd. $= 60 \times$ Rs. $3\frac{1}{3} =$ Rs. 200.

Practical Exercise.

On squared paper, draw plans of the following rectangular rooms ; and find the number of yards of carpet, etc., required :—

1. 20 ft. long and 10 ft. wide ; carpet 2 ft. wide. Scale 0·1 inch to 1 foot.

2. 20 ft. 9 in. by 16 ft. 6 in. ; carpet 2 ft. 9 in. wide ; use the scale of example 1.

3. $5\frac{1}{2}$ yds. by 3 yds. ; carpet 2 ft. 3 in. wide ; scale $\frac{1}{36}$.

4. 15 m. by 12 m. ; carpet 5 dm. wide ; scale 1 cm. to 1 m.

5. 13 ft. 6 in. by 8 ft.; carpet 1 ft. 6 in. wide ; scale 0·1 inch to 1 ft.

To prevent wastage, place the width of the carpet towards the length of the room.

6. 475 cm. by 264 cm. ; planks 25 cm. wide ; scale 1 inch to 100 cm. Arrange as in example 5.

EXAMPLES. VIII.

1. Find the cost of carpeting a room 20 ft. long and 12 ft. broad with carpet 25 inches wide at Rs. 5 a yard.

2. Required the cost of covering with boards, $\frac{3}{4}$ yd. wide, a hall 100 ft. 9 in. by 70 ft. 8 in. wide at 8 as. a yard.

3. Find the cost of paving a floor 18 ft. 8 in. long and 10 ft. $1\frac{1}{2}$ in. wide with slabs 1 foot 6 in. wide at 2 as. 6 p. a yard.

4. A reception pandal 220 yds. long and 100 yds. wide is to be covered with mats, each 2 yds. long ; if the cost of mat is 2 pies a foot, find the cost.

Example. If 63 yds. of carpet $1\frac{1}{3}$ yds. wide cover a room which is 31 ft. 6 in. long, find the breadth of the room.

Area of carpet $= 63 \times \frac{4}{3}$ sq. yds. $= 84$ sq. yds.

$\therefore$ Area of the room $= 84$ sq. yds.

And length of the room $= 31$ ft. 6 in. $= 10\frac{1}{2}$ yds.

$\therefore$ Breadth of the room $= 84$ sq. yds. $\div \frac{21}{2}$ yds.

$= 84 \times \frac{2}{21}$ yds. $= 8$ yds.

EXAMPLES. IX.

1. Required the length of a room 23 ft. 4 in. wide, which takes 140 yds. of carpet, 5 ft. wide.

2. If 172 yds. 1 foot 6 in. of carpet, 4 ft. 6 in. wide are required for a hall, 21 ft. $6\frac{3}{4}$ in. wide, what is its length?

3. The cost of repairing the floor of a room 24 ft. wide, at 2 as. 8 p. a sq. yd. is Rs. 12; find the length of the room.

4. A hall is 64 ft. long, and the cost of paving the floor at 2*s*. 8*d*. a sq. yd. amounts to £16. What is its width?

Example. Find the number of slabs 3 ft. by 2 ft. that would be required to pave the floor of a room 24 ft. long and 10 ft. wide.

Draw a plan of the room on a scale of $\frac{1}{8}$ in. to 1 foot.

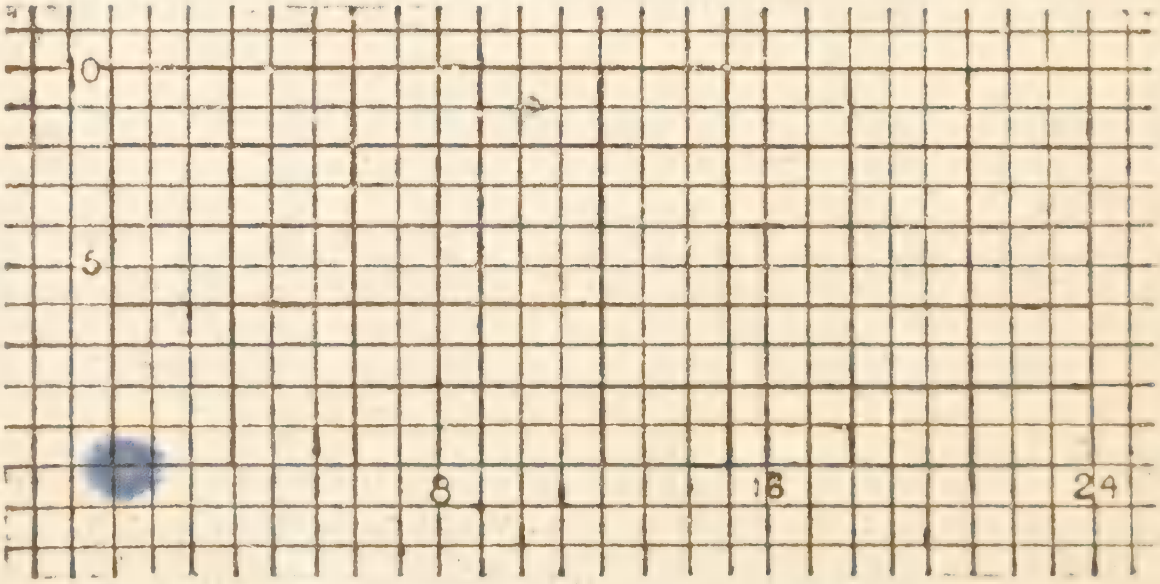

FIG. 41.

Arrange so that the length of the slab is in the direction of the length of the room.

Each slab occupies 3 ft. in the length and 2 ft. in the breadth of the room. We divide the whole rectangle into smaller rectangles 3 ft. by 2 ft.; and there are 40 slabs.

Note.—Give a reason for the arrangement of the slabs.

$$\text{Area of the room} = 24 \times 10 \text{ sq. ft.}$$

$$\text{Area of a slab} = 3 \times 2 \text{ sq. ft.}$$

$$\text{No. of slabs required} = \frac{24 \times 10}{3 \times 2} = 40.$$

Practical Exercise.

Draw on squared paper a plan on a convenient scale for each of the following :—

1. A courtyard 27 ft. by 20 ft. is paved with stones 3 ft. by 1½ ft. How many are there?

2. Find the number of bricks each 8 in. long and 6 in. wide that would be required for the floor of a room 15 ft. long and 12 ft. wide.

3. Find the number of marble slabs each 1 foot 6 in. by 1 foot 3 in. that will pave the floor of a hall 50 ft. long and 30 ft. wide.

4. How many bricks, each 8 in. by 6 in., will be required for the floor of a room 20 ft. long and 12 ft. wide? What will be their cost at Rs. 25 per 1,000?

5. How many mats 5 ft. 6 in. long and 2 ft. 3 in. wide will be required for matting a hall 44 yds. long and 9 yds. broad? Find the cost at 3 as. 4 p. a mat.

6. Find the number of square bricks each 9 in. wide required for the floor of a room whose length is 29 ft. 3 in. and breadth 19 ft. 6 in.

EXAMPLES. X.

1. Find the number of bricks each 6 in. by 3 in. that would be required to pave the floor of a room 18 ft. 6 in. long and half as broad as long.

2. Find the number of bricks 9 in. by 6 in. required for the floor of a room 15 yds. 1 foot long and 24 ft. wide.

3. The breadth of a room is 30½ ft. and its area 1,281 sq. ft. Find its length, and the number of bricks each 9 in. long and 8 in. broad required for the floor.

4. If 69 yds. of carpet, ¾ yd. wide, cover a room which is 10½ yds. long, find its width and the number of square stones each 1½ ft. long required to pave the floor.

Area of Walls, Papering, etc.

280. The area of a single wall is found by multiplying the number of linear units in the length by the number of linear units in the height.

A rectangular room has four walls; and if it is a square, since the areas of the walls are equal to one another, the area of the walls may be obtained by multiplying the area of one wall by four.

When it is not a square, two of the walls are long, each having the same length and two are short, each with the same length.

The four walls of a room may be represented by the four sides of an open box; or a sheet of paper may be folded so as to shew the four walls of a room.

Take, for example, an empty cardboard box whose length is 8 in., width 6 in. and height 4 in.; cut through the side edges and lay out the sides and ends so that they lie in a plane with the bottom as in Fig. 42. (Scale 0·1 in. to 1 in.)

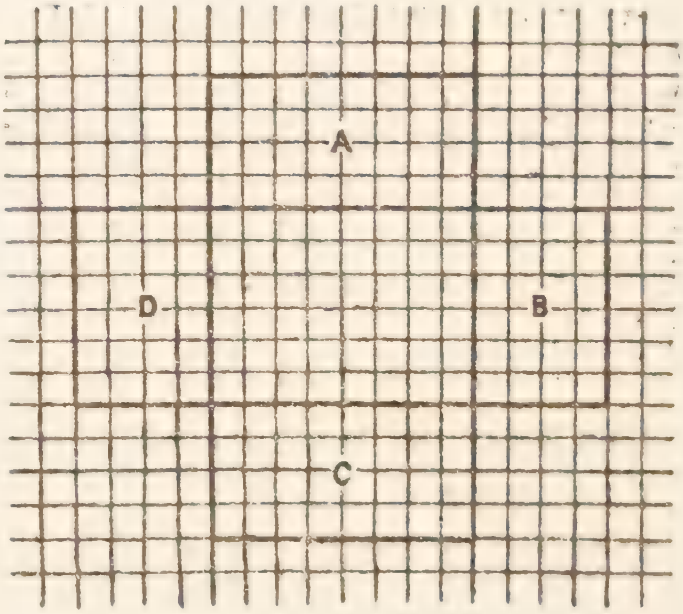

Fig. 42.

The total area of the four sides, A, B, C and D = 8 × 4 sq. in. + 6 × 4 sq. in. + 8 × 4 sq. in. + 6 × 4 sq. in. = (32 + 24 + 32 + 24) sq. in. = 112 sq. in.

If the sides are cut out and placed alongside of each other as in Fig. 43, we get one rectangle whose length is 28 in., and breadth 4 in.; and its area = 28 × 4 sq. in. = 112 sq. in.

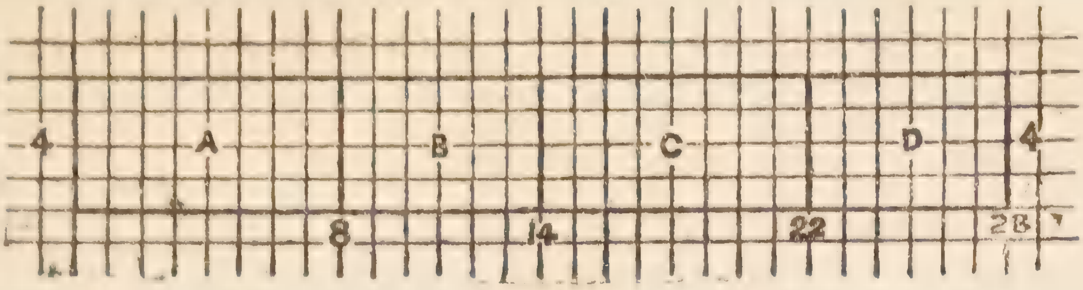

FIG. 43

The total length of 28 in. is made up of twice the length + twice the breadth ; i.e. 2 × (8 in. + 6 in.).

But twice the length + twice the breadth = the Perimeter of the rectangle ; and 4 in. is the height.

∴ The area of the sides representing the 4 walls may be obtained in one step by multiplying the perimeter by the height.

∴ *The area of the walls of a rectangular room = Perimeter × Height = 2 × (Length + Breadth) × Height.*

EXAMPLE. Draw a figure representing the four walls of a room 30 ft. long 15 ft. wide and 21 ft. high ; find their area and the cost of colour-washing them at Rs. 3-2 as. per 100 sq. ft. Scale 1 in. to 30 feet. (1 division to 3 ft.)

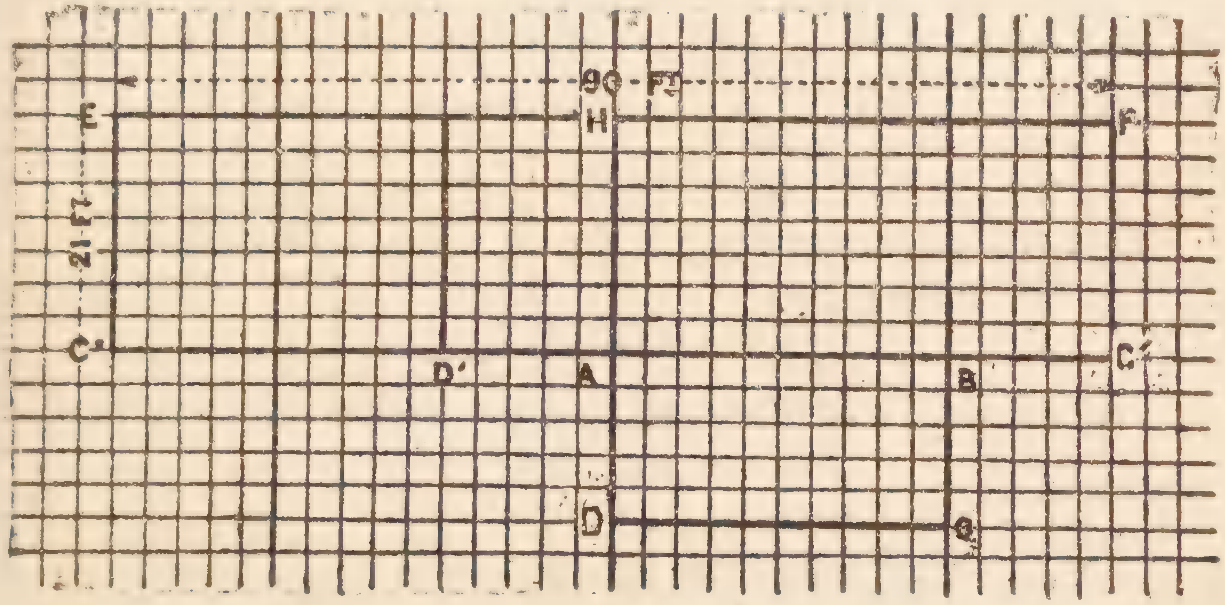

FIG. 44.

In the figure the rectangle ABCD represents the floor of the room, and AH, the height 21 ft.

EF the perimeter = 2 × (30 + 15) ft. = 90 ft.

The rectangle EFC′C″ represents the area of the four walls.

Area of EFC′C″ = EF × FC′ = 90 × 21 sq. ft. = 1890 sq. ft.

Cost at Rs. 3 2 as. per 100 sq. ft. = Rs. $3\frac{1}{8} \times \frac{1890}{100}$ = Rs. 59-1 an.

Practical Exercise.

1. Take several soap boxes, cigar boxes, biscuit boxes, envelope boxes and cigarette boxes, all open, cut through the side edges and lay out the sides and ends as in Fig. 42 and find the total area of the bottom and sides in some ; and in others cut out the sides and arrange them as in Fig. 43 and find the area of the four sides in each.

2. Draw a rectangle, on plain or squared paper, cut out a piece and fold it so as to represent the four walls or sides of each of the following :—

   (1) A room 15 ft. long, 10 ft. wide and 8 ft. high ; scale 1 cm. to 1 foot ; and find the area of the walls.

   (2) A tank 20 ft. by 15 ft. and 12 ft. high ; scale 0·1 in. to 1 foot ; and find the total area of the sides and the bottom.

   (3) A cistern 30′ by 16′ and 4′ deep ; scale 1 cm. to 2 ft. ; and find the cost of lining the inside with lead at 10 as. per sq. foot.

   (4) A box 3 ft. long, 2 ft. wide and 1 foot deep ; scale 1 mm. to 1 in. Find the cost of painting the inside surface at 4 as. a sq. foot.

EXAMPLES.—ORAL.

The following are respectively the length, breadth and height of rectangular rooms ; find the perimeter and the area of the walls :—

1. 4 yds., 3 yds., 2 yds.
2. 10 m., 6 m., 4 m.
3. 12 ft., 10 ft., 6 ft.
4. 15 yds., 12 yds., 8 yds.
5. 20 ft., 15 ft., 12 ft.
6. 12·5 m., 12 m., 8 m.
7. x ft., y ft., z ft.
8. a yds., b yds., c yds.

EXAMPLES. XI.

Find the areas of the walls of the following rectangular rooms :—

1. 90 ft. long, 20 ft. broad, 25 ft. high.

2. 33 ft. long, 12 ft. broad, 12 ft. 6 in. high.

3. Find the cost of painting the walls of a room 15 yds. by 10 yds., and height 8 yds., at 1*s.* 8*d.* a sq. foot.

4. Find the cost of papering a room 18 ft. 10 in. × 15 ft. 2 in. and 10 ft. high with paper at 2 as. a sq. yd.

EXAMPLE. A room 24 ft. 7 in. long, 20 ft. 5 in. broad, 15 ft. high is to be painted; there is a door 6 ft. 6 in. by 3 ft., and there are 3 windows each 4 ft. 9 in. by 2 ft. 10 in. Required the cost of painting the walls at 2*s.* 4*d.* a sq. yd.

Perimeter $= 2 \times (24$ ft. 7 in. $+ 20$ ft. 5 in.$) = 2 \times 45$ ft.

Area of the walls $= 2 \times 45 \times 15$ sq. ft. or 1350 sq. ft.

Area of the door $= 6\frac{1}{2} \times 3$ sq. ft. or $19\frac{1}{2}$ sq. ft.

Area of the three windows $= 3 \times 4\frac{3}{4} \times 2\frac{5}{6}$ sq. ft. or $80\frac{3}{4}$ sq. ft.

The space which does not require painting, that is, the area of the door and of the three windows $= (19\frac{1}{2} + 80\frac{3}{4})$ sq. ft. or $100\frac{1}{4}$ sq. ft.

$\therefore$ The area requiring painting $= (1350 - 100\frac{1}{4})$ sq. ft. or $1249\frac{3}{4}$ sq. ft.

$$= \frac{4999}{4} \times \frac{1}{9} \text{ sq. yds.}$$

$$\text{Cost at } 2s.\ 4d. \text{ a sq. yd.} = \frac{4999}{4} \times \frac{1}{9} \times \frac{7}{3}s. = \frac{34993}{108}s.$$

i.e. £16-4-0¼ to the nearest penny.

EXAMPLES. XII.

1. A room 30 ft. long, 20 ft. broad and 12 ft. high has two windows, each 5 ft. by 4½ ft. and one doorway 6 ft. by 5 ft. Find the cost of painting the walls at 8 as. a sq. yd.

2. A schoolroom 35 ft. 7 in. long, 21 ft. 5 in. broad and 18 ft. high has two doors each 6½ ft. by 5½ ft. and 4 windows each 4¼ ft. by 4 ft. Find the cost of papering the walls at 2 as. a sq. yd.

3. A room 22 ft. 6 in. long and 12 ft. 6 in. wide and 15 ft. high has 4 windows each 4 ft. by 3½ ft. and 2 doors each measuring 6 ft. by 4½ ft. Find the cost of painting the walls and the ceiling at Rs. 2-8 as. a sq. yd.

4. A room 25 ft. 4 in. × 20 ft. 8 in. and 15 ft. 3 in. high has two doors each 10 ft. by 6 ft. and 3 windows, each 5 ft. 6 in. by 4 ft. 9 in. Find the cost of plastering and white-washing the walls of the room inside at one anna a sq. foot.

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281. Wall paper is sold by the piece; each piece is 12 yds. long and generally 21 in. wide. As no fraction of a piece is sold, a fraction in the answer must be reckoned as a whole piece.

Example. Find the length of paper required for the walls of a room 19 ft. 2 in. long, 12 ft. 4 in. wide and 10 ft. high, paper being 1 foot 9 in. wide.

Perimeter = 2 × (19 ft. 2 in. + 12 ft. 4 in.) = 63 ft.
Area of the walls = 63 × 10 sq. ft. = 630 sq. ft.
∴ Area of the paper required = 630 sq. ft.
But width of paper = 1 foot 9 in. = $\frac{7}{4}$ ft.
∴ Length of paper required = 630 sq. ft. ÷ $\frac{7}{4}$ ft.
= 630 × $\frac{4}{7}$ ft. = 360 ft. = 120 yds.
No. of pieces = 10.

EXAMPLES. XIII.

1. Find the length of paper required for the walls of a room 45 ft. 6 in. by $10\frac{1}{2}$ ft. $12\frac{3}{4}$ ft. high; paper 1 ft. 9 in. wide.

2. How many pieces of wall paper, 21 in. wide, would be required for a room whose length is 21 ft. 10 in., breadth 17 ft. 2 in., and height 12 ft. 3 in.

3. A room is 18 ft. long, $13\frac{1}{2}$ ft. broad and $12\frac{1}{2}$ ft. high; what will be the cost of papering it, if each piece of paper, 21 in. wide, cost Re. 1 8 as.?

4. A room 20 ft. long, 11 ft. 6 in. wide and 18 ft. high is to be papered with rolls of paper each 2 ft. 3 in. square. How many such rolls are required, the area occupied by the doors and windows being one-fourth of the whole area?

Example. The area of the walls of a room whose length is twice its breadth is 1224 sq. ft. Find how much carpet it will require, if the height is 12 ft.

Area of the 4 walls = 2 × (L + B) × H.
∴ 2 × (L + B) × H = 1224 sq. ft.
But the length is twice the breadth.
∴ 2 × (2B + B) × 12 ft. = 1224 sq. ft.
∴ 2 × 3B × 12 ft. = 1224 sq. ft.

$$\therefore \text{Breadth} = \frac{1224}{2 \times 3 \times 12} \text{ ft.} = 17 \text{ ft. and Length} = 34 \text{ ft.}$$

∴ Area of carpet required = 34 × 17 sq. ft. = 578 sq. ft

EXAMPLES. XIV.

1. The area of the walls of a room whose length is $1\frac{1}{2}$ times its breadth is 1120 sq. ft. Find the length and breadth, if its height is 14 ft.

2. The height of a room is 12 ft.; and the area of the walls is 80 sq. yds. If the length is $1\frac{2}{3}$ times the breadth, required the quantity of carpeting.

3. The length of a room is $32\frac{1}{2}$ ft.; its area, $601\frac{1}{4}$ sq. ft. and the area of the walls, $164\frac{1}{3}$ sq. yds. Find the height.

4. The length of a room is 64 ft. The cost of painting the walls at Re. 1 8 as. a sq. yd. is Rs. 504; and the cost of paving the floor at 1 an. 6 p. a sq. foot is Rs. 120. Find the breadth and height of the room.

EXAMPLES. XV.

MISCELLANEOUS.

1. A rectangular floor is 12·5 m. by 10 m.; find its area in sq. ft. and sq. in., if 1 m. = 39·375 in.

2. A strip of land is 44 yds. wide; what length must be bought to make an acre. Find the cost of fencing round it at 2*s*. 6*d*. per chain.

3. A plot of ground is 4 ch. 72 links by 3 ch. 4 links. Find its area in acres, roods, poles.

4. Find in yards the width of a street, if its area is 75 acres per kilometre of length. 1 Ha. = $2\frac{1}{2}$ ac., 1 Km. = $\frac{5}{8}$ mile.

5. A roll of oil cloth contains 40 sq. yds. and its width is $2\frac{1}{2}$ ft. If it costs Rs. 22 8 as., find the price per yard.

6. Find the total area of a brick 9 in. long, 6 in. wide and 4 in. thick.

7. A corridor 30 ft. long and 5 ft. 4 in. wide is covered with planks 10 ft. by 6 in. laid parallel to the sides of the corridor. How many are required and what area is wasted?

8. A platform is 100 ft. long and 70 ft. wide; how should boards 14 ft. by 9 in. be arranged so that there might be the least possible wastage. Draw a diagram and find the number of boards required.

9. A page of squared paper notebook is 8·3″ × 6·6″, the portion ruled to the tenths of an inch is 8″ × 6″. Find the number of squares in the page and the ratio of the unruled part to the ruled portion.

10. A district map is drawn to a scale of one inch to the mile. If it is 24 in. by 18 in., what is the area it represents?

11. If a map 18 in. by 15 in. represents an area of 300 ac., what is the scale in inches to the mile?

12. A map 20 in. by 12 in. is on a scale of $\frac{1}{2500}$; find in acres the area represented.

13. A map is drawn to a scale of 1·5 cm. to the mile; what area on the map will represent 1750 Ha.? 1 sq. m. = 1550 sq. in. and 1 Ha. = $2\frac{1}{2}$ acres.

14. The length of a square room is $18\frac{3}{4}$ ft.; painting the walls to a certain height costs £2-3-9 at $4\frac{2}{3}$*d.* per sq. yd. Find it.

15. A piece of a tennis net is 5′ 10″ by 4′ 1″, and each mesh is a square of side $\frac{7}{8}$ in. How many meshes are there? What is the length of string used excluding that used for knots?

16. A piece of flannel is 20 yds. by $1\frac{1}{2}$ yds. and when soaked in water loses $\frac{1}{16}$ of its length and $\frac{1}{18}$ of its breadth. How much is lost by shrinkage?

17. If Rs. 169 be spent in buying carpet $\frac{3}{4}$ yd. wide at Rs. 3 4 as. a yd. for a room 24 ft. long and 18 ft. 4 in. wide, how much of the floor will remain uncovered?

18. In a football ground, 120 yds. by 80 yds. is the portion in which the game is played; and a space of 5 yds. is roped off all round. What is the area of the enclosure?

19. A rectangular grass plot is surrounded by a path 4 ft. wide. The outside measurements are 40 yds. by 30 yds. Find the area of the path.

20. A room is 6 yds. long, 5 yds. broad and 3 yds. 1 foot high. The cost of the paper used, 2 ft. 3 in. wide, is Rs. 45-13-4. Find the price of the paper per piece of 12 yds.

21. To calculate the number of pieces of paper 12 yds. by 21 in. required to paper a ceiling, the following rule is sometimes given:—'Multiply the length in feet by the breadth in feet and divide by 50.' What does this allow for waste for a room $22\frac{1}{2}$ ft. long and $17\frac{1}{2}$ ft. broad?

22. A picture frame is 18 in. by 16 in.; if the frame is $1\frac{1}{2}$ in. wide, find the area of the front of the frame.

23. A dealwood box without a lid is lined with tin weighing $\frac{1}{2}$ lb per sq. foot. If the inside measurements are 5 ft. 6 in. long, 4 ft. wide and 3 ft. 9 in. high, find the weight of lead used.

24. The cost of carpeting a room 21 ft. long at 5*s.* a sq. yd. is £6-2-6; and the painting of the walls at 9*d.* a sq yd. costs £2-12-6. Find the height of the room.

CHAPTER XXV.

CUBIC MEASURE: VOLUMES.

282. A **solid** is a body which has three dimensions length, breadth and thickness (height or depth).

A **rectangular solid** is a body bounded by six rectangular faces.

EXAMPLES. A wooden box with a plane lid, a biscuit tin with the cover on, a brick well smoothed to form plane faces. In all these notice that opposite pair of faces are equal.

283. A **Cubic** is a body bounded by six *equal square* faces; so that the length, breadth and thickness are all equal.

A cube is therefore a particular kind of rectangular solid. When the length, breadth and thickness of a rectangular solid are not all equal, it is called a CUBOID.

Any one of the sides bounding any face of a cube is called an **edge** of the cube.

284. The **Volume,** capacity or cubic content of a body is the amount of space included within its bounding faces.

A cubic unit is a cube whose edge is one unit.

A cubic inch, a cubic foot, a cubic centimetre, a cubic decimetre, etc., are cubic units.

A cubic inch is the Volume of a solid each of whose edges is one inch; a cubic decimetre is the Volume of a solid each of whose edges is one decimetre.

The volume of a body is measured by the number of cubic units contained in it; i.e. by the number of cubic inches, cubic centimetres, etc.

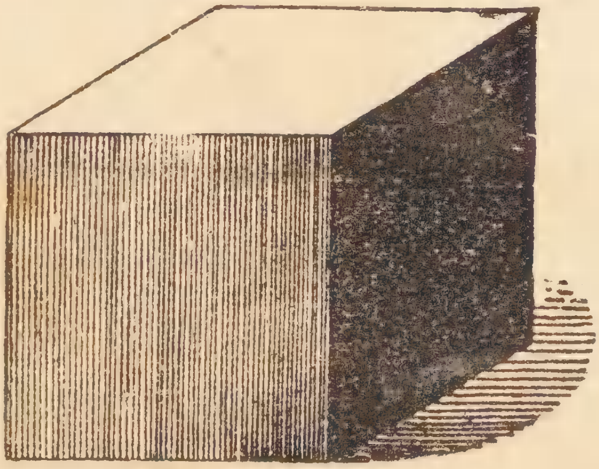

FIG. 45.

An inch cube is a cube whose length, breadth and thickness are each one inch.

FIG. 45 represents one cubic inch; also one inch cube

Practical Exercise.

1. Take a cube from your box of cubes or one from a box of picture building bricks.

How many faces or sides has it? How many edges? How many corners? What is the area of each face in sq. inches and in sq. cm.? What is the height in inches and millimetres?

2. Place 6 one-inch cubes one over the other so as to form a pile. What is the height? How many cubic inches are there in the pile?

3. Place 3 cubes in a row. How many rows form a square? How many cubic inches does the solid so formed contain?

4. Place a cube upon each one of these cubes. How many cubes are there altogether? How many cubic inches in all? What is the height of the solid now formed? What is the area of the top surface and of the bottom?

5. Place one more cube upon each one of the cubes. How many layers of cubes have you now? What is the height; and what is the area of the top surface?

6. How many cubes are there in all and how many cubic inches?

7. Take 8 one-inch cubes, and arrange them in the form of a square. Measure the length, breadth and height of the solid so formed. How many cubic inches are there? What relation does the number of cubic inches bear to any one of the dimensions?

CONSTRUCTION OF TABLE OF CUBIC MEASURE.

285. British Table.

Place 12 inch cubes in a row and arrange 12 rows and you have a solid containing 144 inch cubes or 144 cubic inches, that is a solid, whose height is 1 inch and the area of whose top surface is 1 square foot. If then you form 12 such solids and place them one over another, you build up a cube whose length, breadth and thickness are each 1 foot; and you have now a cubic foot. Since 144 × 12 inch cubes or cubic inches have been used in making it, it is evident that 1,728 cubic inches make one cubic foot.

FIG. 46 represents one cubic foot.

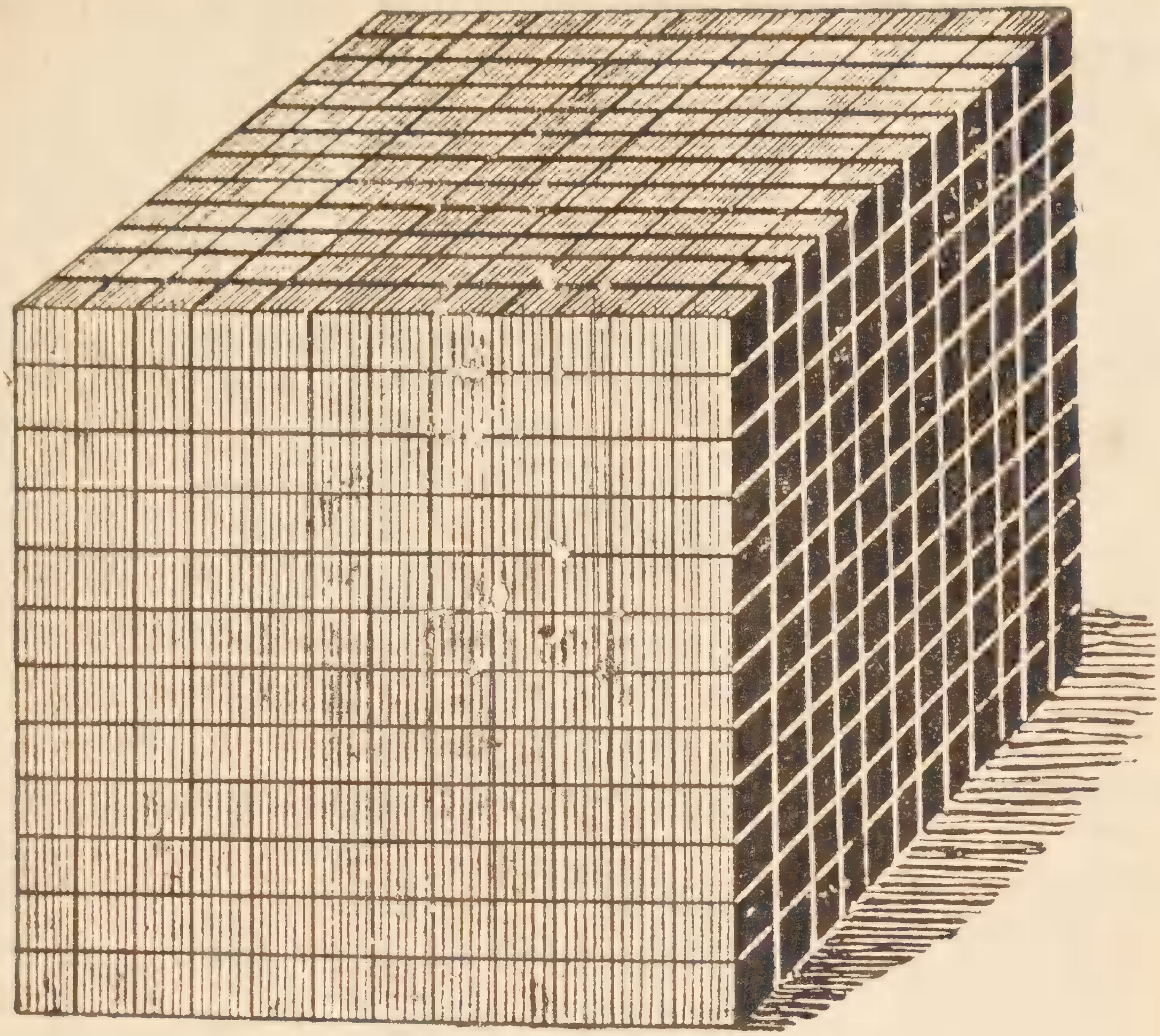

FIG. 46.

Similarly, it may be shewn that 3^3 or 27 cubic feet make one cubic yard.

286. Metric Table.

Take centimetre cubes and place 10 of them in a row, and arrange 10 such rows. You have now a solid containing 100 cubic centimetres. Now form 10 such solids and place them one over another and you are building up a solid whose length, breadth and height are each 10 cm., i.e. a cubic decimetre which contains 100×10 or 1000 cubic centimetres.

∴ 10^3 or 1000 cubic centimetres make 1 cubic decimetre. Similarly 10^3 or 1000 cubic millimetres maybe shewn to make 1 cubic centimetre and so on.

Practical Exercise.

1. Get a cubical block, one cubic foot in size.

What is the area of each of the faces ?

Divide the top and the other face of the cube into square inches.

How many sq. in. are there in one face? If a slice one inch thick be cut out of the cube, how many cubic inches can be cut out of the slice? How many slices each one inch thick can be cut out of the cube? How many cubic inches are there altogether in the cube? What do you infer from this?

2. Get a block one cubic decimetre in size, and divide the top and other faces of the cube into square centimetres. How many sq. cm. are there in each face? How many cubic centimetres in a slice one centimetre thick? How many such slices can be made out of the cube? How many cubic centimetres go to make one cubic decimetre?

3. Arrange cubes or draw diagrams to show that—

(1) 3^3 or 27 cubic feet make one cubic yard.
(2) 10^3 or 1000 cubic millimetres make one cubic centimetre
(3) 10^3 or 1000 cubic decimetres make one cubic metre.

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287. The student can now construct for himself the following tables of cubic measure British and Metric:—

BRITISH MEASURE OF VOLUME.

12^3 or 1728 c. in. make 1 cubic foot (c. ft.)
3^3 or 27 c. ft. make 1 cubic yard (c. yd.)

METRIC MEASURE OF VOLUME.

10^3 or 1000 cubic millimetres (c. mm.) make 1 cubic centimetre (c. cm.)
10^3 or 1000 c. cm. make 1 cubic decimetre (c. dm.)
10^3 or 1000 cubic decimetres make 1 cubic metre (c. m.)

288. We have already noticed British Measures of capacity both Dry Measure (gallons, bushels, etc.), and Liquid Measure (pints, gallons, etc.).

In the Metric Measure of capacity, the principal unit is the **Litre.**

The litre is the volume of one cubic decimetre or 1000 cubic centimetres.

METRIC MEASURE OF CAPACITY.

10 centilitres (cl.)	make	1 decilitre (dl.)
10 decilitres	,,	1 **litre** (l.)
10 litres	,,	1 decalitre (Dl.)
10 decalitres	,,	1 hectolitre (Hl.)
10 hectolitres	,,	1 Kilolitre (Kl.)

289. One gram is the weight of 1 cubic centimetre of distilled water at 4° C.

One litre of water weighs 1000 grams or 1 kilogram.

Therefore the litre is the volume of one kilogram of water at 4° C.

A litre = $1\frac{3}{4}$ pints nearly.

One cubic inch of distilled water at 4° C. weighs about $252\frac{1}{4}$ grains.

Therefore one cubic foot of water weighs approximately 62·264 lbs. or 62·3 lbs. or roughly $62\frac{1}{2}$ lbs. or 1000 oz.

'A pint of pure water weighs a pound and a quarter.'

∴ A gallon weighs 10 lbs. av.

1 gal. = 277·463 c. in. or 277·5 c. in. approximately; and 6·23 gals. are almost exactly equal to one cubic foot.

∴ 1 c. foot of water weighs 6·23 × 10 lbs. or 62·3 lbs.

Example 1. Express 4 c. m. 567 c. dm. in cubic cm.

4 c.m. 567 c. dm. = 4567 c. dm. = 4567000 c. cm., the multiplier in each step being 1000.

Example 2. How many kilograms of water are there in a tank containing 6789 cubic centimetres.

6789 c. cm. = 6·789 c. dm. = 6·789 litres.
= 6·789 kilograms.

EXAMPLES.—ORAL.

1. How many cubic feet are there in $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{4}$ c. yd. and in 3, 4, 5, 6 c. yds.

2. Express in c. cm.—(1) 54 c. dm. 326 c. cm. (2) 26 c. dm. 95 c. c.m. (3) 6 c.m. 8 c. dm. (4) 2 c. m. 34 c. dm. 56 c. cm.

3. Express in c. m. and decimals of a c. m.—(1) 345678 c. cm. (2) 1234567 c. cm. (3) 5678912 c. dm. (4) 9101112 c. dm.

4. Express in litres and decimals of a litre—(1) 3 Dl. 3 l. 3 dl. (2) 6 l. 7 dl. 8 cl. (3) 5 Hl. 4 l. 6 dl. (4) 10 l. 9 dl. 8 cl.

5. Express the capacity of the examples in 2 above as litres.

6. Express the cubic content of the examples in 4 above in kilograms of water.

Practical Exercise.

1. Take a bar of soap and cut out 1-inch cubes, as many as you can.

2. Use soap, cheese, clay or some plastic substance and cut out centimetre cubes.

3. With similar material cut out 2-inch cubes, 3-inch cubes and decimetre cubes.

290. Volume of Cubes.

The volume of a 2-inch cube is 2^3 or 8 c. in.; of a 3-inch cube is 3^3 or 27 c. in.

The volume of a cube whose edge is 2 ft. 6 in. is $2\frac{1}{2}\times 2\frac{1}{2}\times 2\frac{1}{2}$ c. ft or $\frac{125}{8}$ c. ft., i.e. $15\frac{5}{8}$ c. ft.

The volume of a cube is found by cubing the number of units of length in any one of its edges.

291. To find the volume of a cuboid.

Example. Find the volume of a cuboid 4 inches long, 3 inches wide and 2 inches high.

Take a number of inch cubes of wood, or of soap if you have cut out a sufficient number; and place 4 in a row, and arrange 3 such rows. This solid consists of 4 × 3 or 12 cubes, and contains 12 cubic inches.

Form another exactly similar solid and place it over the first; and you have now a rectangular solid consisting of 4 × 3 × 2 or 24 cubes and containing 24 cubic inches.

And this solid measures 4 in. long, 3 in. wide and 2 in. high, and is exactly the same as the solid whose volume is required.

∴ The volume of the cuboid is 24 c. in.

Or we may proceed thus:

Since the solid is 2 inches thick, it can be divided into two halves, each of which is one inch thick.

Since the length is 4 in. and width 3 in., each half can be subdivided into smaller blocks each of which is one cubic inch, the number of such blocks being 4 × 3 or 12, and the number of cubic inches in them being 4 × 3 or 12.

Therefore the number of cubic inches in the solid, i.e. in the two halves $= 4 \times 3 \times 2 = 24$.

Hence the volume of a rectangular solid or a cuboid can be found by expressing the length, breadth and height in terms of a common linear unit and multiplying the three numbers together; the product is the volume of the solid in corresponding cubic units.

If L, B and H denote respectively the number of linear unit in the length, breadth and height of a cuboid.

$L \times B \times H = V$ *(volume in corresponding cubic units).*

Practical Exercise.

1. Build up cuboids with inch cubes, whose length, breadth and height are—

(1) 2″, 2″, 1″. (2) 3″, 2″, 2″. (3) 3″, 3″, 2″.
(4) 4″, 2″, 3″. (5) 4″, 3″, 3″. (6) 6″, 3″, 2″.

How many cubic inches does each contain?

2. Use centimetre cubes to build up the following cuboids :—

(1) 3 cm. × 2 cm. × 3 cm. (2) 4 cm. × 3 cm. × 2 cm
(3) 5 cm. × 4 cm. × 3 cm. (4) 6 cm. × 3 cm. × 5 cm.

How many cubic centimetres are there in each?

EXAMPLE. Find the cubic content of a wall 18 ft. long, 13 ft. 6 in. high and 1 foot 8 in. thick.

$$\text{Cubic content} = 18 \times 13\tfrac{1}{2} \times 1\tfrac{2}{3} \text{ c. ft.}$$
$$= \frac{18 \times 27 \times 5}{2 \times 3 \times 27} \text{ c. yds.}$$
$$= 15 \text{ c. yds.}$$

EXAMPLES.—ORAL.

Find the volumes of the cubes whose edges are—5 in. ; 8 in. ; 3 ft. 6 in. ; 1 foot 3 in. ; 2 ft. 8 in. ; 2 ft. 9 in.

Required the volumes of the following cuboids, the length breadth and height being respectively—

1. 6 in., 5 in., 4 in. 2. 12 ft., 8 ft., 5 ft.
3. 15 cm., 12 cm., 8 cm. 4. 20 ft., 15 ft., 12 ft.
5. 25 cm., 16 cm., 10 cm. 6. 7·5 in., 6 in., 2·5 in.
7. a ft., b ft., c ft. 8. x dm. y dm., z dm.

How many inch cubes can be put into boxes whose inside measurements are—

1. 6″ × 6″ × 5″. 2. 8″ × 6″ × 5′. 3. 1′ × 8″ × 4″.
4. 1′ × 10″ × 8″. 5. $7\frac{1}{2}$″ × 8″ × 6″. 6. $12\frac{1}{3}$″ × 6″ × 4″.
7. x in. × y in. × z in. 8. a in. × b in. × c in.

EXAMPLES. I.

1. Find the cubic content of a wall 30 ft. long, 10 ft. high and 2 ft. 6 in. thick.

2. Find the volume of air in a room 21 ft. long, 15 ft. wide and 12 ft. 6 in. high.

3. What is the cubic content of a box 6′ 9″ long, 2′ 6″ wide and 1′ 4″ deep.

4. Find the volume of water in a tank 15 metres long, 5 metres wide and 80 cm. deep.

5. Find the cost of digging a trench 5′6″ long, 4′ 3″ wide and 2 yds. deep at 5 as. 4 p. a cubic foot.

6. What is the cost of a beam 14 ft. long, 16 in. wide and 9 in. thick at Rs. 1-0-8 per cubic foot ?

7. A cistern is 1·5 metres long, 80 cm. wide and 60 cm. deep. How many kilograms of water can it hold ?

8. A bar of iron is 20 ft. long, 3 in. wide and 0·5 in. thick. Find its weight, if a cubic foot of iron weighs 480 lbs.

9. Find the volumes of the cubes whose edges are—
(1) 2′ 6″, (2) 3′ 4″, (3) 5·4 cm., (4) 4·5 cm.

292. Since L × B each expressed in linear units = A. Area in corresponding square units, we have A × H = V.

Also from the formula L × B × H = V, we have

$$(1)\ L=\frac{V}{B\times H}, \quad (2)\ B=\frac{V}{L\times H}, \quad (3)\ H=\frac{V}{L\times B} \text{ or } \frac{V}{A}.$$

EXAMPLE. A sheet of iron 4·25 metres long and 2·68 metres wide has a volume of 2733·6 c.cm. Find its thickness.

Volume = 2733·6 c. cm.

Area = 425 × 268 sq. cm.

$$\therefore \text{ The Thickness } = \frac{2733\cdot6}{425\times268} \text{ cm.}$$

$$\begin{array}{r} \cdot024 \\ 1\cdot139)\overline{\cdot027336} \\ 2278 \\ \hline 4556 \\ 4556 \\ \hline \end{array}$$

$$=\frac{2733\cdot6}{113900}\text{ cm.} = \frac{27\cdot336}{1139}\text{ cm.} = \frac{\cdot027336}{1\cdot139} = \cdot024 \text{ cm.}$$

EXAMPLES.—ORAL.

	Length.	Breadth.	Thickness.	Volume.
1.	15 in.	4 in.	?	180 c. in.
2.	12 cm.	8 cm.	?	192 c. cm.
3.	?	12 ft.	3 ft.	540 c. ft.
4.	15 in.	?	4 in.	720 c. in.
5.	20 cm.	?	5 cm.	1200 c. cm.
6.	?	12 ft.	6ft.	1800 c. ft.

EXAMPLES. II.

1. The cubic content of a room is 7,500 c. ft. Its height being 10 ft. and breadth 25 ft., find its length.

2. The cross section of a beam is 144 sq. in.; and its volume is twenty-one cubic yards. Find its length.

3. What length must be cut off a straight plank 1·2 ft. wide and 9 in. thick that it may contain 11·25 c. ft.?

4. The thousandth part of a cubic foot of gold is beaten into leaves whose area is 40·5 sq. in. What is the thickness of a leaf?

5. A tank contains 17,600 litres of water; find the depth, if it is 3·2 metres by 2·75 metres.

6. If a mile of wire can be made out of one cubic foot of copper, find the area of the cross section in decimals of a sq. inch to two significant figures.

7. It is estimated that 30,250 gallons of rain water fell over an acre of ground. If one cubic foot of water contains $6\frac{1}{4}$ gals., find in inches the amount of rainfall.

8. A cistern holds 375 gals. of water and its depth is 3 ft. Find the area, given that a gallon of water weighs 10 lbs. and 1,000 oz. measure 1 cubic foot.

Example. Find the number of bricks required for building a wall 20 ft. long, 15 ft. high and $2\frac{1}{2}$ ft. thick, each brick with its share of mortar measuring 9 in. × 6 in. × 4 in.

The cubic content of the wall $= 20 \times 15 \times \frac{5}{2}$ c. foot $= 750$ c. ft.
The cubic content of each brick $= \frac{3}{4} \times \frac{1}{2} \times \frac{1}{3}$ c. foot $= \frac{1}{8}$ c. foot.
∴ The No. of bricks required $= 750$ c. ft. $\div \frac{1}{8}$ c. foot.
$= 750 \times 8 = 6{,}000$.

EXAMPLES. III.

Find the number of bricks required for building the following walls:—

1. 100 ft. long, 2 ft. 6 in. thick and 15 ft. high, each brick measuring 9 in. × 5 in. × 4 in.

2. 80 ft. 6 in. long, 9 ft. high and 2 ft. 9 in. thick, each brick being 9 in. × $4\frac{1}{2}$ in. × 3 in.

3. 15 ft. long, $1\frac{3}{4}$ ft. wide and 12 ft. high, each brick measuring 8 in. × 7 in. × 3 in.

4. How many bricks are there in a stack 36 ft. by 10 ft. by 12 ft., each brick measuring 9 in. × $4\frac{1}{2}$ in. × 3 in. ?

5. How many bars each 10 ft. long, 1·5 in. broad and 0·375 in. thick can be made from 10 c. yds. of iron ?

6. Find the number of planks 6′ 9″ long, 1′ 6″ wide and 1″ thick that can be got from 27 c. yds. of timber.

7. If, in a deal wood box 3′ 6″ long, 1′ 4″ deep and 1′ 8″ wide a bookseller packs 144 copies of a book 9 in. × 5 in. × $1\frac{1}{2}$ in., find how many more of a size $4\frac{1}{2}$ in. × 4 in. × $1\frac{1}{3}$ in. are required to fill it.

8. How many cubes each side being 3 in. long can be cast from a block of metal 14 in. × 10 in. × 6 in. How many cubes each side being 3 in. long can be sawn out of a block of wood of the same dimensions ?

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293. To find the volume of material used in the construction of rectangular boxes.

Practical Exercise.

Procure deal wood cases, some open and others closed.

1. Take one of the open boxes (without a lid) ; measure the external and internal dimensions, also the thickness of the wood ; shew that internal length = external length *minus* twice the thickness, internal breadth = external breadth *minus* twice the thickness and internal height = external height *minus* the thickness. Find the external volume and the internal volume, and subtract the latter from the former. What is the difference of the volume ?

2. Take one of the closed boxes and take the external measurements. Remove the cover ; find the thickness of the wood and take the internal measurements. Shew that the inside measurements are in each case the external measurements *minus* twice the thickness of the wood. Find the external volume of the box and the internal capacity ; subtract the latter from the former and thus find the volume of the wood used in the construction of the box.

3. Take one or two more open and closed boxes and proceed as shewn above.

Example. Find the volume of dealwood, one inch thick required for making a packing case whose external measurements are length 3′ 6″, breadth 2′ 5″, and height 1′ 10″.

External measurement.		Internal measurement.
Length	3′ 6″	3′ 6″ − 2″ = 3′ 4″
Breadth	2′ 5″	2′ 5″ − 2″ = 2′ 3″
Height	1′ 10″	1′ 10″ − 2″ = 1′ 8″

From each external measurement twice the thickness of the wood is subtracted.

External volume.	Internal volume.
$3\frac{1}{2} \times 2\frac{5}{12} \times 1\frac{5}{6}$ c. ft.	$3\frac{1}{3} \times 2\frac{1}{4} \times 1\frac{2}{3}$ c. ft.
$= \frac{7}{2} \times \frac{29}{12} \times \frac{11}{6}$ c. ft.	$= \frac{10}{3} \times \frac{9}{4} \times \frac{5}{3}$ c. ft.
$= \frac{2233}{144}$ c. ft $= 15\frac{73}{144}$ c. ft.	$= \frac{25}{2}$ c. ft. $= 12\frac{1}{2}$ c. ft.

Volume of the material $= 15\frac{73}{144}$ c. ft. $- 12\frac{1}{2}$ c. ft.
$= 3\frac{1}{144}$ c. ft. $= 3$ c. ft. 12 c. in.

We thus see that a closed box whose length in linear units is l, breadth is b, and height is h, and of uniform thickness t, has

(1) the external volume in cubic units $= lbh$

(2) the internal dimensions $= l - 2t,\ b - 2t,\ h - 2t$.

(3) the internal volume in cubic units $= (l - 2t)\ (b - 2t).\ (h - 2t)$.

(4) the volume of the material $= lbh - (l - 2t)\ (b - 2t)\ (h - 2t)$.

Note.—When the box is open, that is, when it has no lid, the internal height is external height *minus* thickness of the material.

EXAMPLES. IV.

1. Find the capacity (internal volume) of an open box whose outside measurements are length 15 in., breadth 12 in. and height 8 in., the wood of the box being $\frac{1}{2}$ an inch thick.

2. An open box of wood 2 cm. thick measures externally 26 cm. × 20 cm. × 16 cm. Find the volume of wood used and the number of litres of water it can hold.

3. A closed box measures externally 3′ 8″ long, 2′ 3″ wide, 2′ 4″ high and is made of wood one inch thick. What volume of wood has been used?

4. A closed vessel formed of metal one inch thick, whose external dimensions are 8′ 6″, 7′ 6″ and 4′ 3″, weighs 8 cwt. 1 qr. 8 oz. What would be its weight, if it were all solid?

EXAMPLES. V.

MISCELLANEOUS.

1. One inch of rain falls on an acre of ground. How much will it weigh, reckoning the weight of one cubic foot as 1,000 oz.?

2. If a cubic foot of marble weighs 2·716 times as much as a cubic foot of water, find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad, and 2 ft. thick. A cubic foot of water = 1,000 oz.

3. Find the expense of building a wall 108 yds. long, 3 ft. 8 in. high, and 13½ in. thick at £14-13-4 per rod; a rod of brick work consisting of 272¼ sq. ft., the thickness being 13½ in.

4. A cubic foot of water weighs 62½ lb., and a room 18 ft. 9 in. by 13 ft. 4 in. is flooded to the depth of 2 in., what is the weight of the water in the room?

5. Supposing a cubic foot of water weighs 1,000 oz., find how many tons of water will a cistern hold if the length, breadth and depth of the cistern is 18 ft. 8 in., 18 ft. 4 in., and 6 ft. 9 in. respectively.

6. Supposing a gallon of water to contain 277¼ c. in., find approximately the number of gallons of water which would cover a square mile of ground to the depth of an inch.

7. A cistern 12 ft. long, 2 ft. wide and 6 in. deep contains pulp for making paper If half the volume of the pulp is lost in drying, how many sheets of paper 8 in. by 6 in. will be obtained, if 300 sheets in thickness go to an inch?

8. The water in a rectangular cistern 8 feet by 7 feet weighs 4 tons 13 cwt. 3 qr. Find the depth of the cistern, if 1 c. foot of water weighs, 1,000 oz.

9. The cubic content of a room 25 ft. long and 12 ft. 6 in. high is 3,750 c. ft. Find the cost of painting its walls at 12 as. a sq. yd.

10. A room 15 ft. 6 in. long and 12 ft. 4 in. wide contains 2,150 c. ft. 1,080 c. in. Find the cost of papering the room with paper 2 ft. 6 in. wide at 6 as. per piece of 12 yds.

11. If 82 c. ft. 864 c. in. be the capacity of an open cistern 5′ 6″ long and 3′ 4″ wide, what will be the cost of lining it with lead at 12 as. a sq. yd.

12. An open cistern made of wood 2 in. thick measures externally 3′ 10″ by 2′ 10″ by 2′ 4″. Find the cost of lining it with tin at 9*d*. per sq. foot.

13. A beam of teak is 5·4 metres long, 25 cm. wide and 20 cm. thick. Find its weight in kilograms, if teak is 0·75 as heavy as water.

14. Given that mercury is 13½ times as heavy as water, find the volume of 27 Kg. of mercury. To what depth will 405 Kg. of mercury fill a rectangular trough 1·5 metres by 15 cm.?

15. A wall 24 ft. long, 2¼ ft. thick and 12 ft. high has a window 3 ft. by 2 ft. and a door 5½ ft. by 4½ ft. Find the cost of building it at 3 as. a c. foot.

16. The area of a lake is 0·65 sq. mile; and the volume of water is estimated to be 572 millions cubic feet. Find its depth to the nearest tenth of a foot.

17. A cistern 6′ 4″ long, 4′ 6″ broad and 5′ deep is filled with water and leaks till the level of water sinks 9 in. How many gallons of water are left? 1 c. foot of water = 6½ gals.

18. A side of a square cistern is 12′ 6″. What weight of water should be drawn off that the level of water might sink 3′ 9″? 1 c. foot of water = 62½ lbs.

19. A brick measures 8″ × 5″ × 4″. Find to the nearest rupee, the cost of building a wall 17′ 6″ × 7′ 4″, × 12″, if 1,000 bricks cost Rs. 27-8 as. and the workmen are paid Rs. 3 per 10 c. ft.

20. A certain vessel contains 100 c. ft. of water and the depth is 2′ 6″. A flat piece of stone 2′ 8″ by 1′ 6″ is dropped into the water; and this raises the level of the water by half an inch. Find the thickness of the stone.

21. The external measurements of a wooden cistern are 6′ by 5′ by 4′. If the thickness of the wood is 1 inch, find the weight of wood used, if a cubic inch weighs 5 oz.

22. The internal dimensions of a tank are 3 m. by 2 m. by 1 m., and its weight when empty is 450 Kg. Find its weight when it is full of water?

23. The outside measurements of a closed box are 2′ 7½″ length, 2′ 5½″ breadth, and 2′ 1¾″ height; the top and sides are ¾ in. thick, and the bottom 1 in. thick. Find the number of gallons of water it can hold. 1 c. foot of water = 6¼ gals.

24. How long would it take to fill with water a canal, that is, 26 miles 1,357 ft. long, 12 ft. broad and 4 ft. deep through a channel that discharges 1,000 gals. per minute, if a gallon contains 277·274 c. in.?

CHAPTER XXVI.

ELEMENTARY MENSURATION.

294. Areas of Rectangular surfaces and Volumes of Rectangular solids have been dealt with in Chapter XXIV and Chapter XXV respectively. We shall now treat of Mensuration of other plane and solid figures.

295. Area of a Parallelogram.

A parallelogram is a four-sided figure having its opposite sides parallel and equal.

A rectangle is a parallelogram; but a parallelogram is not necessarily a rectangle.

In figures 47 and 48, ABCD and EFGH are parallelograms but not rectangles, because none of the angles is a right angle.

ABCD is a **Rhomboid** and EFGH a **Rhombus.** In a rhombus you see that all the sides are equal to one another.

Any side of a parallelogram may be regarded as its **Base,** and the perpendicular to the base from any point in the opposite side is called the **Altitude.**

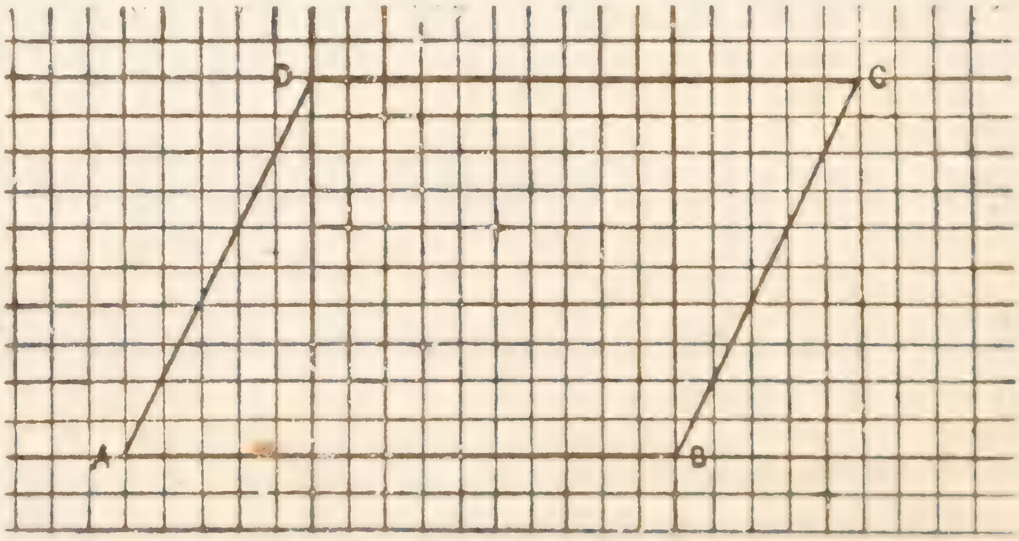

FIG. 47.

296. Area of a Rhomboid. Fig. 47.

(1) Count the number of squares in ABCD, and calculate its area thus:

Number of sqq. = ...
Area of each sq. = ...sq. in.
Area of ABCD = ...sq. in.

(2) Drop a perpendicular to the base AB from any point on the opposite side. Measure the base and the altitude.

Base = ... in.
Altitude = ... in.
Base × altitude =...×...sq. in.
=........sq. in.

Do the two results agree?

Note.—In counting squares, a portion of a square, that is, greater than a half is taken as one and a portion less than a half is neglected.

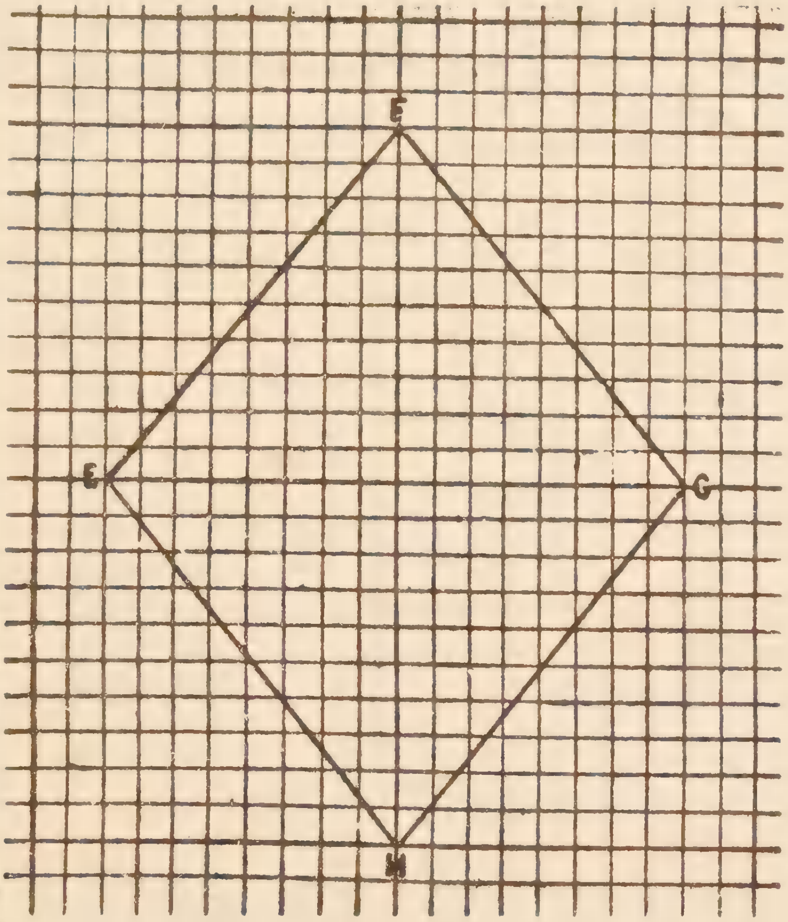

FIG. 48.

297. Area of a Rhombus. Fig. 48.

(1) Proceed as in (1), Art. 296. | (2) Proceed as in (2), Art. 296.

See if the two results are the same.

Area of a parallelogram = base × altitude.

298. Area of a Triangle.

The altitude of a triangle is the perpendicular drawn to the base from the opposite angle.

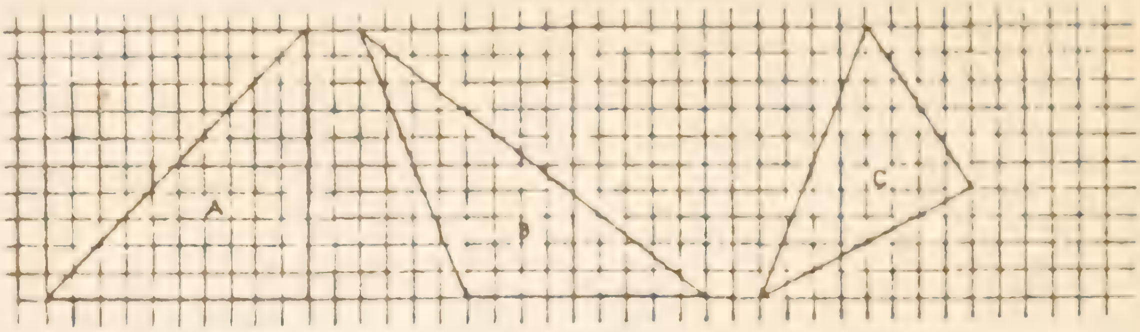

FIG. 49.

(1) Count the number of sqq. in A and calculate its area.

Number of sqq. = ...
Area of each sq. = ...sq. in.
Area of A = ...sq. in.

(2) Drop a perpendicular to the base from the opposite angle. Measure the base and the altitude.

$\frac{1}{2}$ the base = ... in.
Altitude = ... in.
Product = ... sq. in.

See if the two results are equal.

Do the same with Fig. 49 B and C; and see if the two results in each case agree.

Area of a triangle = $\frac{1}{2}$ the base × altitude.

Practical Exercise.

Draw the different parallelograms and three different triangles on (*a*) squared paper; (*b*) plain paper, and use the area obtained from (*a*) to check the correctness of the area got, from (*b*) by measurement.

EXAMPLES. I.

1. A parallelogram-shaped playground has its base 5 furlongs and altitude $\frac{1}{2}$ a furlong. What is its area in acres?

2. A field in the form of a parallelogram has one of its sides 27 chains long and the perpendicular distance 32 rods. Find its yield at 5 kalams per acre.

3. The base of a triangular field is 40 yds. and its altitude 30 ft. What is the area?

4. Find the rental of a triangular estate whose base is 660 yds. and altitude 55 yds. at Rs. 40 per acre.

299. Area of a Quadrilateral.

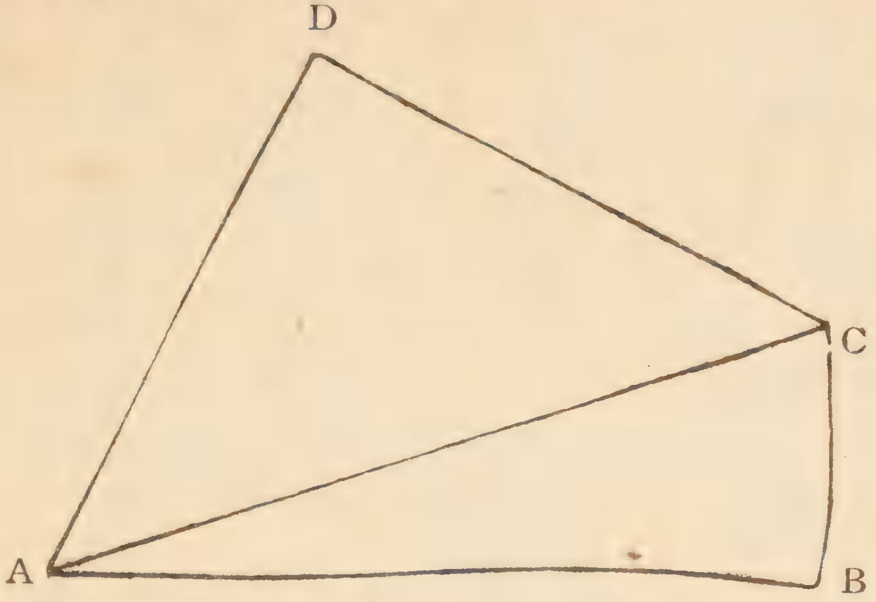

FIG. 50.

Let ABCD be any quadrilateral figure.

Divide the figure into two triangles by joining two of the opposite angles by the straight line AC.

Then area = sum of the areas of the two triangles.

300. Area of a Polygon.—A polygon is a figure of many sides.

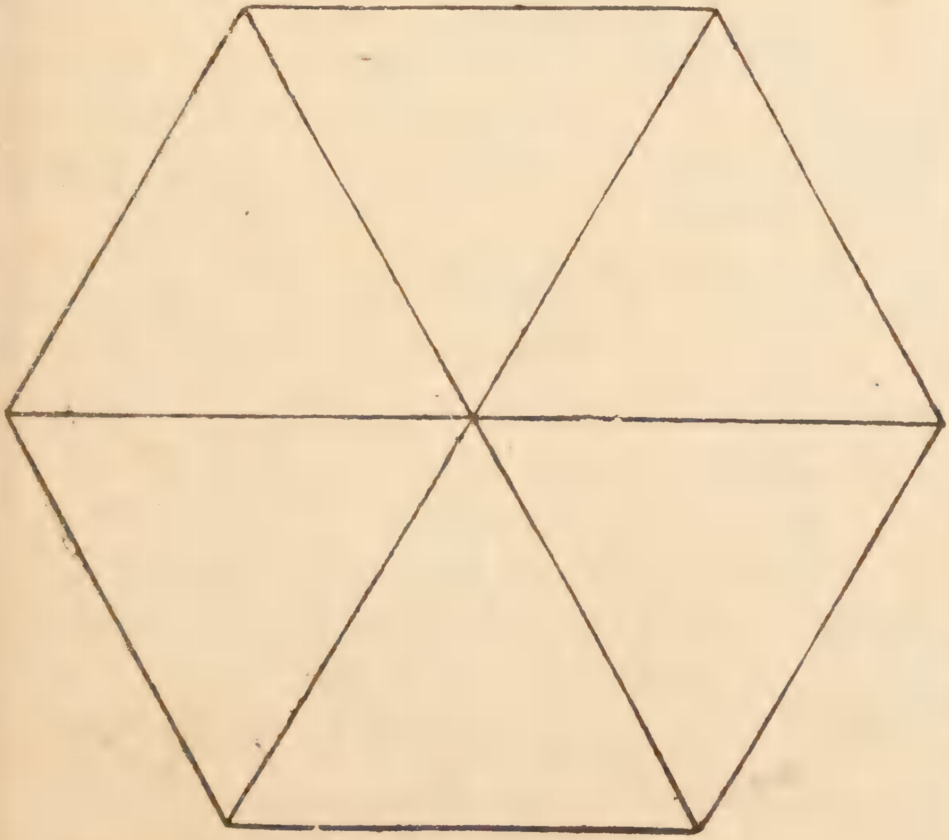

FIG 51.

The area is found by the method of triangulation, i.e. by dividing the polygon into triangles and taking the sum of the areas of the triangles.

301. Area of a Trapezoid.—A trapezoid is a four-sided figure having two of its opposite sides parallel. Fig. 52.

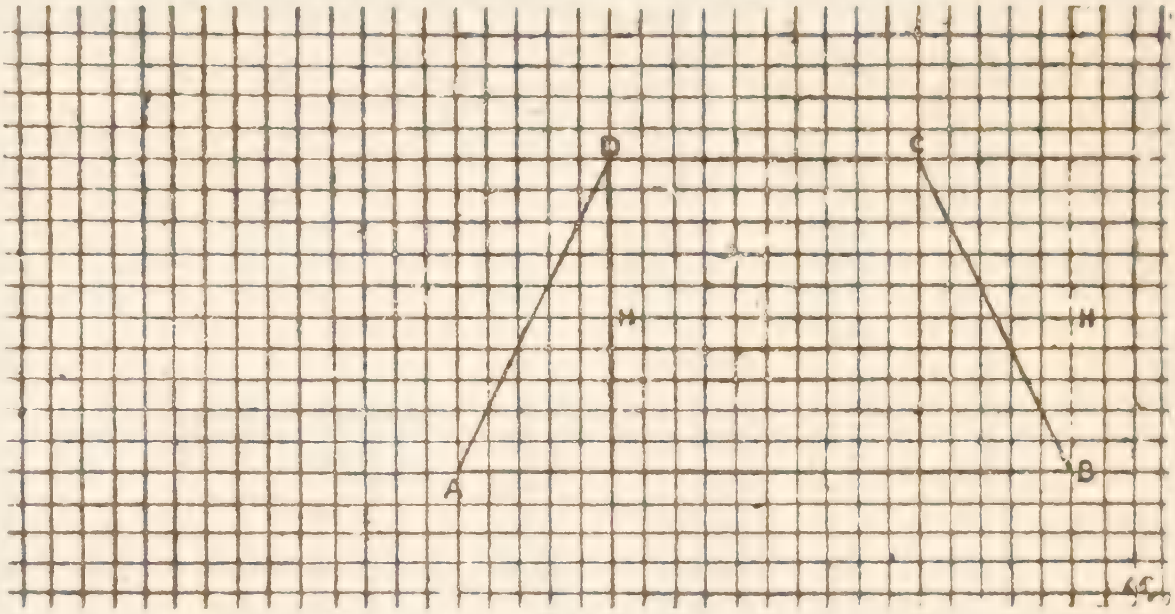

FIG. 52.

Draw AC.

Area of ABCD = Area of ADC + area of ACB.
$= \frac{1}{2}$ DC × Altitude × $\frac{1}{2}$ AB × Altitude.
= Altitude × $\frac{1}{2}$ (CD + AB).

Area of a Trapezoid = Altitude × $\frac{1}{2}$ the sum of the parallel sides.

Practical Exercise.

Draw (1) Trapezoids, (2) Quadrilaterals that are not parallelograms and (3) Polygons on (*a*) squared paper, (*b*) plain paper, and use the area calculated from (*a*) to verify the area obtained from (*b*) by measurement.

EXAMPLE. A stream of water 8 yds. broad at the surface and 6 yds. at the bottom and 2 yds. deep flows at the rate of $1\frac{1}{3}$ miles an hour into a tank. How many tons of water flow into the tank per hour. A cubic foot of water = $62\frac{1}{2}$ lbs.

The transverse section of the stream of water is a trapezoid whose area $= \frac{1}{2}(8 + 6) \times 2$ sq. yds. = 14 sq. yds.

The volume of water that flows into the tank per hour
$= 14 \times \frac{4}{3} \times 1760$ c. yds. $= 14 \times \frac{4}{3} \times 1760 \times 27$ c. ft.

Since 1 c. foot of water $= 62\frac{1}{2}$ lbs.,

$$\text{No. of tons of water} = \frac{14 \times 4 \times 1760 \times 27 \times 125}{3 \times 2 \times 112 \times 20}$$
$$= 22 \times 9 \times 125 = 24{,}750.$$

EXAMPLES. II.

1. Find the rent of a field, trapezoid in shape, whose parallel sides are 58 yds. and 38 yds. and the altitude 30 yds. at Rs. 30 4 as. an acre.

2. Find the cost, at Rs. 2 4 as. a cubic yard, of raising an embankment 1,500 yds. long, 10 ft. wide at the bottom and 8 ft. at the top, the vertical height being 5 ft.

3. Find the volume of water that passes every minute into a tank through a canal 10 ft. deep, 18 ft. broad at the surface and 14 ft. broad at the bottom, flowing at the rate of 3 miles an hour.

4. How many tons of water pass from a river every hour into a canal 12 ft. deep, 30 ft. broad at the surface and 20 ft. wide at the bottom, flowing at the rate of $2\frac{1}{2}$ miles an hour? 1 cubic foot of water = 1,000 oz.

---

302. To measure the circumference of a circle.

(1) With dividers and scale.

Draw a circle of 3 cm radius. Make a mark anywhere on the circumference with a fine pointed pencil. Open the dividers to a convenient distance, say 5 mm. Fix one end of the dividers firmly on the mark and bring the other end till it is exactly on the circumference. Keeping this end firm, move the first leg so that it may rest on the circumference. Go on measuring thus and counting till the mark is reached. The number of times the dividers were moved multiplied by 5 mm. gives the entire length of the circumference. This will be found to be nearly 18·85 cm.

(2) With thread and scale.

Stick a number of pins close to one another all along the circumference of the circle described, pass a piece of fine thread round the pins and cut it off where it overlaps. Measure the length cut off. See if you get nearly 18·85 cm. as a result of this measurement.

303. To find the relation between circumference and diameter.

The diameter of a circle of 3 cm. radius is 6 cm. The circumference of the circle has been measured and found to be *approximately*

18·85 cm. Now divide 18·85 cm. by 6 cm., and the quotient is 3·14 correct to 2 decimal places.

Draw circles of different radii and measure both the circumference and the diameter and divide the former by the latter in every case; the quotient correct to 2 places of decimals will be found to be 3·14.

∴ The ratio of the circumference to the diameter is 3·14 : 1.

This ratio is expressed by the Greek letter π (pronounced *pie*). It is 3·14159 correct to 5 decimal places, and its equivalent is $3\frac{1}{7}$ or $\frac{355}{113}$ approximately.

Practical Exercise.

Draw circles of different radii and measure their circumferences and diameters. Take at least three measurements of each and take the average; and note down thus:

Diameter in in. or cm.	1	2	3	4	5	6
Circumference in do.	..	..	..	..	..	..

Are the ratios of any pairs of corresponding numbers equal?

Any slight differences are due to the fact that the circumferences cannot be measured exactly.

304. Area of a Circle.

(i) Draw a circle of radius AB 2 cm. long.

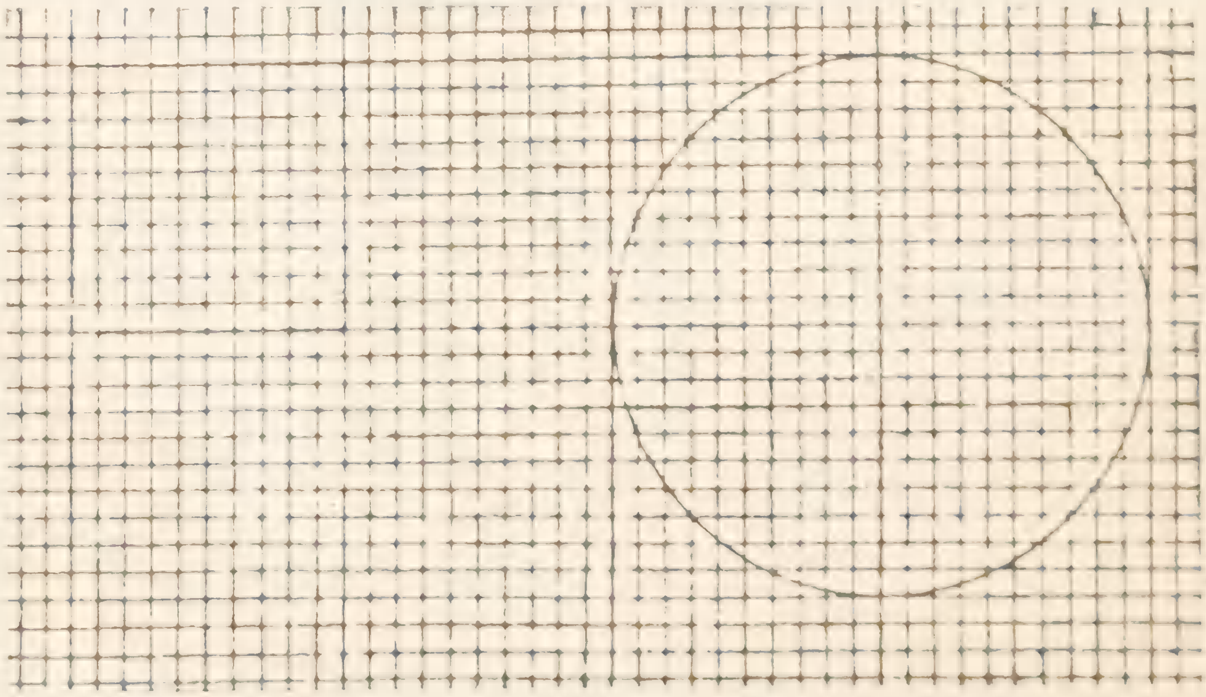

FIG. 53.

Count the number of small squares enclosed within the circumference and calculate the area of the circle.

No. of squares in the circle = ... (1)

Area of the circle = ... $\times$ ...... sq. cm. = ... sq. cm. ...... (2)

Draw a square ABCD, whose side is 2 cm., the radius of the Circle.

No. of small squares in the square = ... (3)
The square of radius = ... sq. cm. .... (4)
Divide (1) by (3) ; the quotient = ... (5)
Divide (2) by (4) ; the quotient = ... (6)
The quotient in each case = 3·14 correct to 2 places.
∴ Area of the circle = 3·14......... $\times$ (radius)^2

But 3·14 .... is expressed by π. Art. 303.

∴ *Area of a circle* $= \pi r^2$.

(ii) Measure the circumference of the circle and half the radius.

Circumference = ... cm.
Half the radius = ... cm.
∴ Circumference $\times \frac{1}{2}$ radius = ... sq. cm.

Is the result the same as (5) or (6) above ?

Area of a circle = circumference $\times$ half the radius

$$= 2\,\pi\, r \times \frac{r}{2}.$$

(iii) **By the Method of Weighing.**—Draw on a piece of cardboard a circle of one inch radius. On the same cardboard draw four squares each with a side 1 inch, i.e. the length of the radius.

Now cut out the circle and the squares neatly. Place the circle in one of the scale pans of a balance, and balance it with the squares placed in the other scale pan.

The squares will weigh more ; remove one of the squares, and the circle will weigh more.

Cut off about a seventh of the square that was taken out and place the part cut off along with the other squares.

The circle will now nearly counterpoise the squares ; and the balance will almost be in equilibrium. We thus see that the area of a circle approximately $= 3\frac{1}{7} \times (\text{radius})^2$.

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Practical Exercise.

1. Draw circles with the following radii :—

(1) 3 cm. (2) 2·5 in (3) 1·5 cm. (4) 3 in.

(*a*) on squared paper, (*b*) on plain paper, and use the area calculated from (*a*) to verify the area obtained from (*b*) by measurement.

2. Try the ‘method of weighing’ with the circles in Example 1.

EXAMPLE. The radius of a circular lawn is 42 yds. long. How many acres of grass are there ?

$$\text{Area of the lawn} = 3\tfrac{1}{7} \times 42 \times 42 \text{ sq. yds.}$$
$$= \frac{22}{7} \times \frac{42 \times 42}{4840} \text{ acres}$$
$$\text{No. of acres of grass} = \frac{63}{55} \text{ ac.} = 1\tfrac{8}{55} \text{ ac.}$$

EXAMPLES. III.

1. The diameter of a circular garden measures 56 yds. Find the circumference and the area.

2. Given $\pi = 3{\cdot}1416$ correct to 4 decimal places, find the length of the diameter of a circle whose circumference is 24,900 miles correct to the nearest hundred.

3. How many cubic feet of earth should be dug out to sink a well 35 ft. deep and 5 ft. in diameter ?

4. Inside a circular garden 63 yds. in diameter, there is a circular path 5 ft. wide. Find the space occupied by the path.

5. In the centre of a circular field 120 ft. in diameter, a circular well is dug, 14 ft. in diameter. How much space remains for cultivation ?

6. The radius of a circular room is 7½ ft. and a verandah 6 ft. wide runs round it. Find the area of the verandah.

7. Find the cost of painting at 3 as. a sq. foot the ceiling and walls of a circular room 15 ft. high, the radius being 7 ft.

8. A circular room is 20 ft. in diameter and 21 ft. high. Find the cost of painting the ceiling and walls at 4*s*. a sq. yd.

Mensuration of Solids.

305. Cylinder.—EXAMPLES: A ruler, lead pencils, etc.

You can easily make a hollow cylinder by taking a rectangular sheet of paper and rolling it round till two opposite sides meet edge to edge.

The open ends form the circumferences of two equal circles. If you take away the hand, the paper unrolls itself and you get a rectangle, the circumferences of the open ends now becoming straight lines.

The ends open or closed form the bases of the cylinder, each base being a circle. Except at the two ends, the surface of a cylinder is curved.

The area of the two bases or of the circular ends $= 2\pi r^2$; and the area of the curved surface of the cylinder = circumference of the base × height $= 2\pi rh$, where r is the radius of the circular ends or *of the cross section*, and h is the height of the cylinder.

The area of a cylinder $= 2\pi rh + 2\pi r^2$.

The volume of a cylinder = area of base × height

$$= \pi r^2 \times h.$$

EXAMPLE. Find the volume of iron in a cylindrical pipe 3 ft. 6 in. long, the material of which is 1 inch thick, having an external diameter of 10 in.

The external radius is 5 in., and internal radius 4 in.

External volume $= \frac{22}{7} \times 5^2 \times 42$ c. in.

Internal volume $= \frac{22}{7} \times 4^2 \times 42$ c. in.

Volume of iron reqd. = the difference between the external and internal volume.

$= \frac{22}{7} \times 42 \times (5^2 - 4^2)$ c. in. $= \frac{22}{7} \times 42 \times 9$ c. in.

$= 1{,}188$ c. in.

306. Sphere.—EXAMPLES: Cricket balls, marbles, etc.

The earth is not an exact sphere, as it is somewhat flattened at the Poles.

The area of the surface of a sphere will be found on measurement to be equal to the area of four circles having the same radius as that of the sphere.

∴ *The area of the surface of a sphere* $= 4\pi r^2$, *where* r *is the radius.*

The volume of a sphere $=\frac{4\pi}{3}\times r^3$ *or* $\frac{4\pi}{3}\left(\frac{d}{2}\right)^3=\frac{\pi}{6}d^3$, *where d is the diameter.*

EXAMPLE. Taking the earth to be a sphere of 3,960 miles radius, find the area of its surface to the nearest million square miles.

Taking $\pi = 3·14$.

$$\begin{aligned} \text{Area} &= 4 \times 3·14 \times 3{,}960 \times 3960 \text{ sq. mi.} \\ &= 3·14 \times 4 \times 39{,}600 \times 396 \text{ sq. mi.} \\ &= 3·14 \times 158400 \times 396 \text{ sq. mi.} \\ &= 3·14 \times 1584 \times 396 \text{ million sq. mi.} \\ &= 3·14 \times 62·7264 \text{ million sq. mi.} \\ &= 197 \text{ million sq. mi. to the nearest million.} \end{aligned}$$

Using contracted multiplication.

$$\begin{array}{r} 62·7264 \\ 3·14 \\ \hline 188·1792 \\ 6·2726 \\ 2·5090 \\ \hline 196·9608 \end{array}$$

EXAMPLES. IV.

1. Taking the earth to be a sphere of 3,960 miles radius, find the number of cubic miles it contains to the nearest million.

2. If π were taken as $3\frac{1}{7}$ instead of 3·1416, what difference would this make in the calculation of the area of the surface of the earth? Radius = 3,960 miles.

3. The diameter of the moon = 0·273 times the earth's diameter. Shew that the volume of the moon is approximately $\frac{1}{49}$ that of the earth.

4. The external diameter of a flat ring is 1 foot 6 in. and the internal radius $7\frac{1}{2}$ in., the height being 1·5 in.; find its volume.

5. The outer diameter of a water pipe is 2 ft., the inner diameter 1 ft. 8 in., and the length of the pipe 40 ft. How many cubic inches of metal are used in making it?

6. A cylindrical tank 14 ft. in diameter contains 25,000 gals. of water. What is the depth, if 11 gals. of water have a volume of 3049·75 c. in.

7. A hollow spherical shell is 1 in. thick, and its outer diameter is 6 in. What is the total outside and inside surface?

What would be the difference in the total area, if the shell was cut into two halves?

8. One quarter of the volume of a cylindrical boiler 12 ft. long and 5 ft. in diameter internally is fitted with cylindrical heating tubes fitted longitudinally, and $7\frac{1}{2}$ in. in diameter. Find the number of tubes.

CHAPTER XXVII.

EASY GRAPHS.

307. We have already seen in the examples of Chapter XX how two quantities are related to one another.

For example, we know that if 1 man earns daily half a rupee, 2 men of equal ability earn daily twice as much or Re. 1, 3 men earn thrice as much, that is, Rs. $1\frac{1}{2}$, 4 men earn four times as much, namely, Rs. 2, and 10 men earn ten times as much or Rs. 5.

Here the number of men and the sum earned are so related that if the number of men increases or decreases the sum earned also increases or decreases. This may be expressed by saying that the sum earned is directly proportional to the number of men.

Again, we know that if 1 man can do a piece of work in 20 days, 2 men of equal ability can do it in 10 days, 4 men can do it in 5 days and 10 men can do it in 2 days.

Here the number of men and the number of days are related; but their relation is not one such as is seen to exist in the previous example; for as the number of men *increases*, the number of days *decreases*. Anyhow, when there is a change in one quantity, there is a corresponding change in the other.

We thus see that two quantities may be so related that if a change is made in the value of the one, there is a corresponding change in the value of the other.

308. If 1 article costs 4 as., 2 cost 8 as., 3 cost 12 as. and 5 cost 20 as. and 12, 48 as. Here again the cost is directly proportional to the number of articles.

If x denote the number of articles, and y the number of annas, for 5 different values of x, 5 corresponding values of y are given below:

x	1	5	12	22	35
y	4	20	48	88	140

A quantity which has not always the same value, but may have a series of different values, is called a **variable.**

In the above example, x the number of articles is a variable and y the number of annas is also a variable.

The population of a town, the price of rice per measure, the readings of the barometer and the thermometer are other examples of variables.

When variables are thus related, their relation may be clearly represented by diagrams drawn on squared paper.

309. Method of Representation. On squared paper two intersecting lines are chosen and are slightly thickened to help the eye.

The horizontal line is named XOX′ and the vertical line YOY′. The point of intersection of these two lines, O, is called the origin. These lines are taken as the lines of reference ; XOX′ is known as the X axis and YOY′ as the Y axis.

The position of a point is determined by its perpendicular distances from the two axes ; that is, if we know the perpendicular distances of the point from the axes, the position of the point can be fixed.

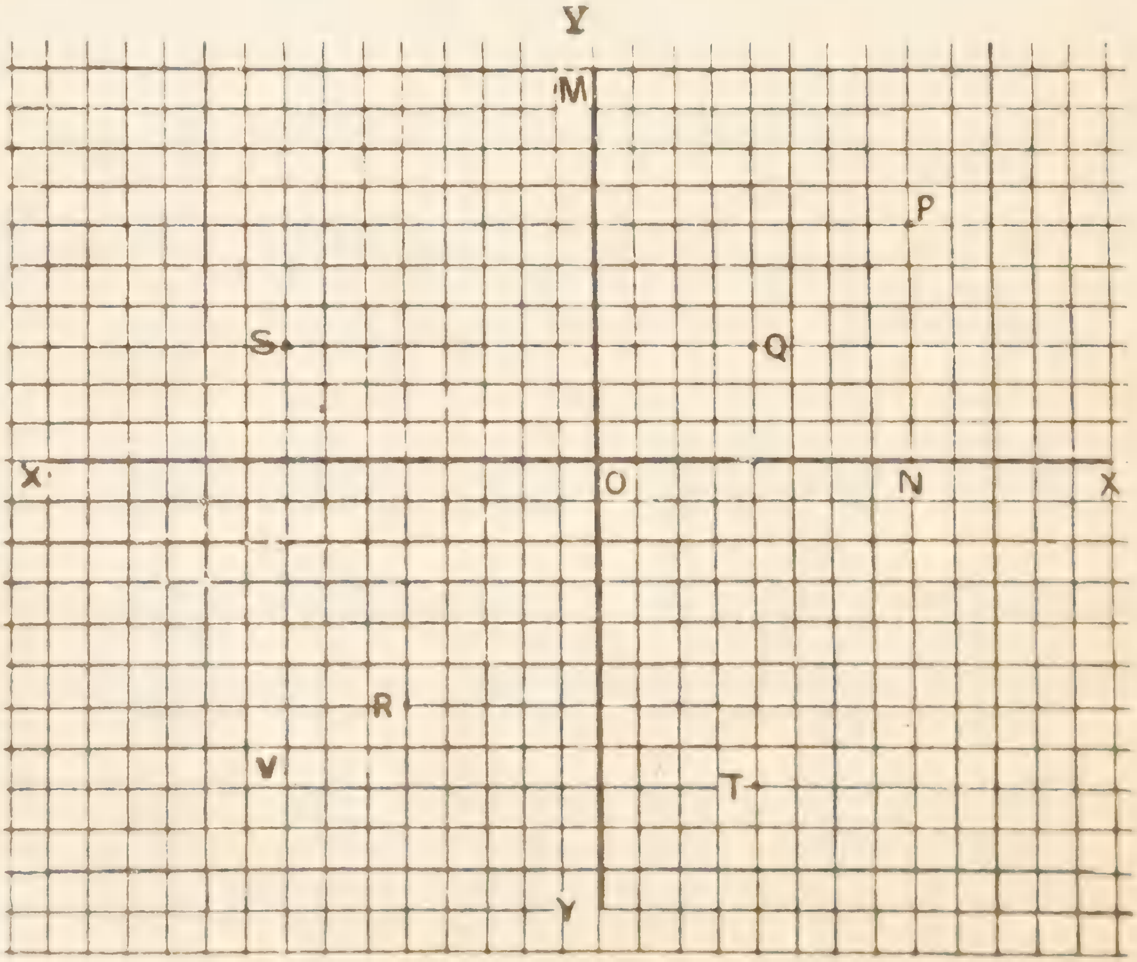


FIG. 54.

Thus the position of any point P is fixed by taking 8 divisions along OX, and then 6 divisions vertically above it, i.e. 8 divisions to the right of O and then 6 divisions up.

The distances 8 and 6 are known as the **Co-ordinates** of the point P; the distance ON being known as the **abscissa** of the point P, and the distance NP the **ordinate** of P.

The position of any point is defined by its co-ordinates. A point whose co-ordinates are 4 and 3 is named *the point* (4, 3), the abscissa of the point being always mentioned first. Such a point is Q in the figure. Generally, when we speak of any point (x, y), we mean the abscissa of that point is x and the ordinate of the point is y.

The process of marking a point by means of its co-ordinates is called **plotting the point.**

310. The two straight lines XOX′ and YOY′ divide the plane (surface) of the paper into four spaces XOY, YOX′, X′OY′ and Y′OX, which are known as the first, second, third and fourth **quadrants** respectively.

Distances taken *along* the X axis to the *right* of O, the origin, are considered *positive* and those taken *along* the X axis to the *left* of the origin are considered *negative* (see Art. 32). Distances measured *vertically above* the X axis are taken as *positive*, and those measured *vertically below* the X axis are taken as *negative*.

It will thus be seen that the abscissa of a point is positive in the 1st and 4th quadrants, and negative in the 2nd and 3rd quadrants; and the ordinate of a point is positive in the 1st and 2nd quadrants, and negative in the 3rd and 4th quadrants.

In figure 54, the co-ordinates of the point P are (8, 6); of S, (— 8, 3); of R, (— 5, — 6).

EXAMPLE 1. (*a*) Plot the points (8, 0) and (0, 10) and measure the distance between them. Fig. 54.

(*b*) Plot the points (4, 3), (— 8, 3), (—8, — 8), (4, — 8) and calculate the area of the rectangle formed by joining them.

(*a*) Take 8 divisions to the right of O, and no divisions *up* or *down*. The resulting point N is on the X axis.

Since the abscissa is zero, and the ordinate 10, take 10 divisions vertically above the origin. The resulting point M is on the Y axis.

Measuring the distance between M and N, we find it to be 1·5 in.

Note.—The point whose co-ordinates are (0, 0) is clearly the origin itself; in the other words, the co-ordinates of the origin are (0, 0).

(*b*) First plot the point (4, 3) thus :

Since the abscissa and ordinate are both positive, take 4 divisions along OX, i.e. to the right of O, and then 3 divisions vertically above it. The resulting point Q is in the first quadrant.

Next plot the point (— 8, 3) thus :

Since the abscissa is negative, take 8 divisions to the left of O; and since the ordinate is positive, take 3 divisions up. We thus get the point S in the second quadrant.

Then plot the point (— 8, — 8).

Since both the abscissa and the ordinate are negative, take 8 divisions to the left of O and 8 divisions *below* it along the Y axis. The resulting point V is in the third quadrant.

Finally plot (4, — 8).

Take 4 divisions to the right of O, and then 8 divisions below it; and the resulting point T is in the fourth quadrant.

Join the plotted points and form the rectangle SQTV.

The rectangle measures 1·4 in. by 1·3 in.

Its area = 1·82 sq. in.

Practical Exercise.

1. Plot the following pairs of points on squared paper, and draw the line which joins each pair :—

(1) (5, 5), (8, 4). (2) (5, 10), (10, 5). (3) (0, 6), (2, 4).
(4) (6, 8), (0, 0). (5) (2, —2), (10, —17). (6) (—6, 2), (12, —5).
(7) (—2, 6), (1, —3). (8) (—4, 0), (0, —5).

2. Find the lengths of the lines joining the following pairs of points :—

(1) (5, 12), (— 15, — 8). (2) (— 5, 3), (7, — 8).
(3) (0, 0), (5, 6). (4) (7, 8), (— 3, — 4).
(5) (15, — 2), (— 3, — 8). (6) (0, 0), (12, — 5).
(7) (2·5, 1·5), (2·7, 1·8). (8) (4, 5·4), (— 2·5, — 3·7).

3. Plot the following points and find their distances from the origin :—

(1) (12, 9). (2) (8, 16). (3) (—8, — 10).
(4) (— 9, 6). (5) (1·8″, ·6″). (6) (1·2″, 1·6″).
(7) (— 1·4″, 0·8″). (8) (3·9″, — 2·5″).

4. (*a*) Shew that the straight lines joining the points (7, 3) and (− 7, − 3) and the points (− 5, 6) and (6, − 5) are equidistant from the origin.

(*b*) Shew that the distance between the first pairs of points = the distance between the second pairs.

(1) (10, 0), (0, 3) ; (3, 0), (0, 10).
(2) (7, − 5), (− 5, 4) ; (4, − 5), (− 5, 7).

5. Find the co-ordinates of the points of intersection of the straight lines joining :—

(1) (5, 5), (− 7, − 7) and (8, 3), (− 8, − 3).
(2) (6, 9), (− 4, 9) and (0, 0), (0, 10).
(3) (− 3, 7), (− 3, − 6) and (0, 0), (− 9, 0).
(4) (9, 9), (− 9, − 3) and (− 3, 8), (9, − 2).

6. Find the perimeter of the rectangles formed by the lines joining the following points :—

(1) (5, 5), (− 5, 5), (− 5, − 5). (5, − 5).
(2) (3, 7), (− 5, 7), (− 5, − 2), (3, − 2).
(3) (8, 4), (− 8, 4), (− 8, − 4), (8, − 4).
(4) (8, 0), (8, 10), (− 4, 10), (− 4, 0).

Find the area of the rectangles, if each division represents one foot.

7. Plot the following points and shew that they lie in a straight line :—

(1) (5, − 4), (0, 0), (− 5, 4).
(2) (8, − 7), (0, 0), (− 8, 7).
(3) (2, 4), (4, 8), (0, 0), (− 2, − 4).
(4) (− 10, − 5), (0, 0), (4, 2), (10, 5).

8. A person goes 12 miles to the East and then 16 miles to the North. How far is he from the starting point ?

Take one division to represent one mile.

---

311. Statistical Graphs.

EXAMPLE 1. If the value of y in Art. 308 be expressed in rupees, we have—

No. of articles	0	1	5	12	22	35
Cost in rupees	0	$\frac{1}{4}$	$1\frac{1}{4}$	3	$5\frac{1}{2}$	$8\frac{3}{4}$

This relation between the two variables can be represented graphically on squared paper.

Take one small division horizontally to represent 1 article and 1 division vertically to denote $\frac{1}{4}$ rupee.

Plot the points $(1, \frac{1}{4})$, $(5, 1\frac{1}{4})$, $(12, 3)$, $(22, 5\frac{1}{2})$, $(35, 8\frac{3}{4})$. Joining these by a straight line from the origin, we get the graph OP. Fig. 55.

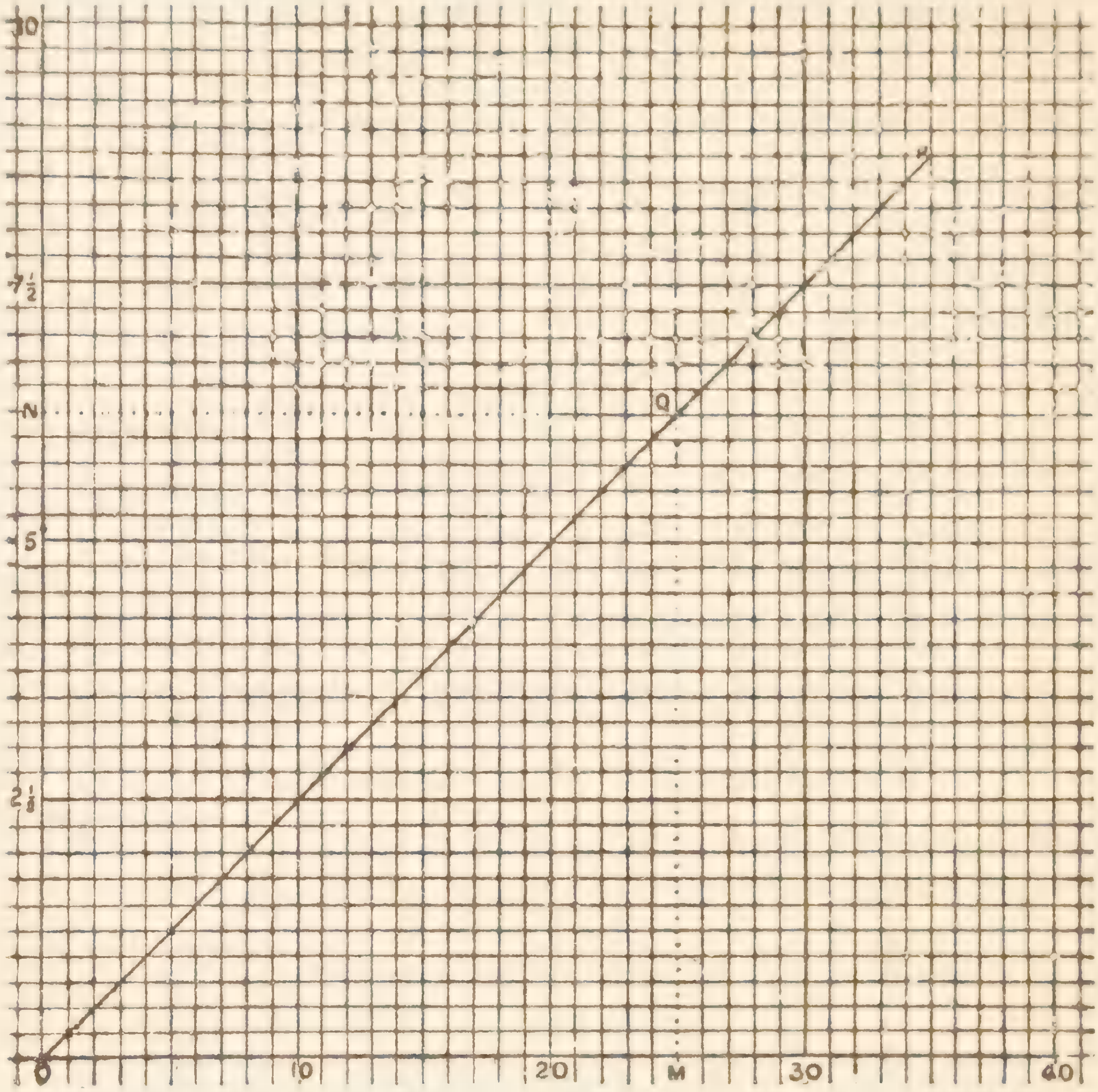


FIG. 55

312. A graph is a line, straight or curved, drawn on a diagram shewing the relation that exists between two connected variables.

In Fig. 55 the graph is a straight line.

When the variables, as in the above example, are directly proportional, the graph is a straight line.

313. We can use the graph to find the price of any number of articles.

Suppose it is required to find the price of 25 articles.

The abscissa being 25, draw from M a perpendicular to meet the graph at Q. From Q, draw a perpendicular to meet the Y axis at N. The co-ordinates of Q are (25, $6\frac{1}{4}$). Therefore the cost of 25 articles is Rs. $6\frac{1}{4}$.

In the same way, the number of articles that can be bought for a given price can be ascertained by using the graph.

The garph in Fig. 55 can be used to find the cost of number of articles up to 40.

The value of any number above 40 can be ascertained by drawing the graph on a larger sheet of paper and producing the graph.

314. To construct a straight line graph, we have only to join two points whose co-ordinates are given, or to join the origin with another given point.

If 50 articles cost Rs. 10, a graph may be constructed to find the value of any number of articles.

We have simply to plot the point (50, 10) and to join it by a straight line from the origin.

For examples see under Conversion Graphs.

315. Choice of scale units.

In the construction of the graph of Fig. 55, one small division horizontally, i. e. one-twelfth of an inch denoted one article, and 1 division vertically or $\frac{1}{12}$ of an inch represented one quarter-rupee.

If we had taken one small division vertically to represent one rupee, the plotted points could not have been apart by a convenient distance and the diagram would heve been only about one inch in width.

Whenever it is convenient to do so, the same scale unit may be taken for both the variables; but it is not necessary that this should be done in every case.

When, for instance, we have to represent the distances covered in given times by a train going at a uniform rate of 30 miles an hour, if we take 1 division horizontally to denote 1 hour, and 1 division vertically to represent 1 mile, for every horizontal division, 30 vertical divisions have to be taken, and the diagram will become inconveniently large.

Of two connected variables, if one increases much more rapidly than the other, it is best to take a small unit for the rapidly increasing variable and a large one for the other.

Practical Exercise.

1. If a mango costs 2 as., plot the values for 4, 9, 15, 25 and 36; and construct a graph for reading off the price of any number up to 100. Take one small division horizontally for 1 mango and vertically for 2 as.

2. If the duty on tobacco is 2*s*. 6*d*. per lb., draw a graph for calculating the duty on any weight up to 1 cwt. Read off the duty on 80 lbs. and 109 lbs

3. If 40 pencils cost 10 as., draw a graph from which the cost of any number of pencils can be found.

4. A train travels uniformly at the rate of 30 miles an hour for 12 hours. Draw a graph to find the distance covered in any time within the 12 hours.

Let 5 horizontal divisions denote 1 hr. and 1 vertical division represent 5 miles.

5. Draw a graph shewing the monthly and weekly wages at £25 a year. For what period is £7 due as wages?

6. Is the graph of the following a straight line?—

x	1	$1\frac{1}{2}$	3	7	10
y	3·75	5·25	10·5	12·25	17·5

7. The following table gives the extension of a spiral spring when loaded with different weights:—

Extension in cm.	0·5	1·0	1·5	2·0	3·0	4·5	6·0
Weight in grams	5	10	15	20	30	45	60

Find the extension for a weight of 50·5 grams, and the weight that would cause an extension of 9·25 cm.

8. The following are the results of an experiment on the bending of a lath:—

Weight in grams	10	15	30	45	70	100
Bending in mm.	1	1·5	3	4·5	7	10

Find the amount of bending for 52·25 grams and for 125·5 grams.

---

316. The corresponding values of two variables are sometimes obtained by observation or experiment. In such cases the data cannot always be regarded as absolutely correct, for there may be errors; and therefore the positions of the plotted points cannot be considered as exactly determined.

If the height of the barometer is read off by observation at equal intervals of time in a given period, by plotting the points, a graph may be drawn shewing how the height of the barometer varied during the given period. The variations of temperature may similarly be shown by a graph. In these cases, the successive points are usually connected by straight lines and the graph consists of an irregular broken line.

Note.—Such a graph can give us no accurate information about intermediate points as is given by a straight line graph, but only a general idea of the total change that has taken place between the plotted points.

EXAMPLE 2. The following barometric readings in inches were recorded in a station for the first ten days of September, 1909. Shew these variations by a graph.

Day	1	2	3	4	5	6	7	8	9	10
Reading	29·66	29·59	29·60	29·67	29·69	29·72	29·74	29·76	29·72	29·69

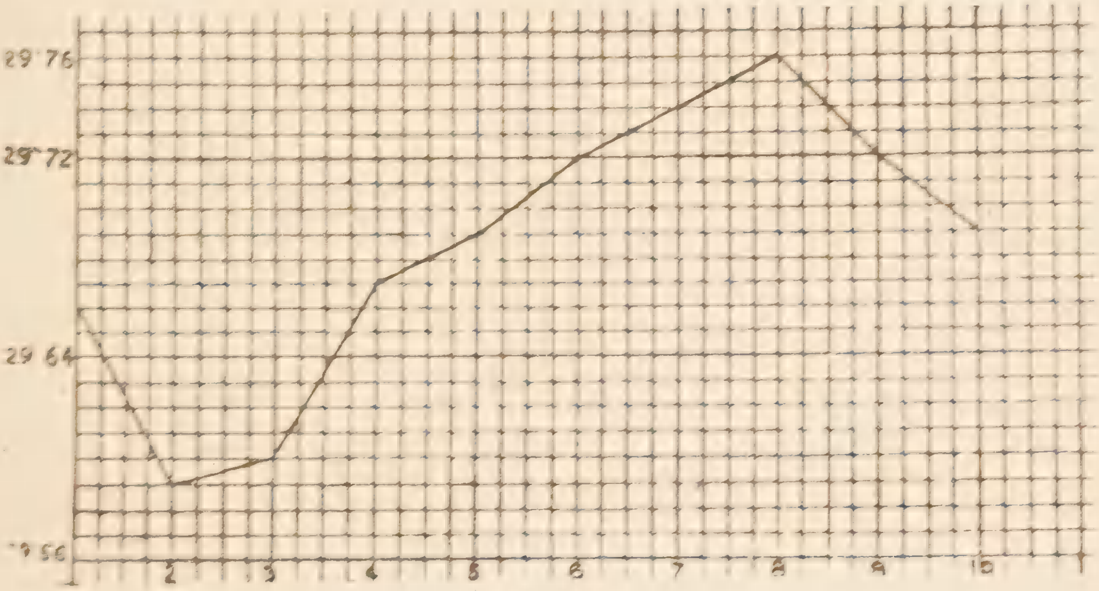


FIG. 56.

Take 4 horizontal divisions to represent 1 day, and one vertical division to represent ·01 of an inch.

Plot the points and join them successively by straight lines.

The lowest record being 29·59, it will be convenient if we begin measuring ordinates at 29·56.

Graphic Exercise.

Join the plotted points in the following by straight lines, and draw the graph shewing the variations :—

1. The barometric readings in inches for the first 10 days in a certain month were—

Day	1	2	3	4	5	6	7	8	9	10
Reading	28·7	28·6	28·8	29·4	29·2	29·3	29·2	29·0	29·1	29·2

2. The mean temperature on the first day of each month calculated on the average of 50 years is as follows :—

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep	Oct.	Nov.	Dec.
37	38	40	45	56	48	46	45	49	45	40	35

3. The following are the amounts of rainfall in inches recorded in a station for the 12 months of the year 1908 :—

Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep
9·5	10·2	8·6	5·4	2·1	0·8	0·0	0·0	2·4	2·5	4·6	2·5

4. The price in pence of copper on the 1st day of each of the years given below was as follows :—

1887	'88	'89	'90	'91	'92	'93	'94	'95	'96	'97	'98	'99	1900
45	43	43	48	45	40	36	29	30	31	28	27	27	28

5. The average price in pence of silver per ounce for the years 1893, '94, '95, '96, '97, '98, '99, and 1900 was 39, 32, 31½, 31, 30, 29, 29, and 31, respectively.

6. The average yearly price of paddy per *kota* (21 markals) was—

Year	1900	'01	'02	'03	'04	'05	'06	'07	'08	'09
Price in Rs.	8	7·5	8	9	9·5	10·5	9·25	10·75	10	10·5

7. In a term of 12 weeks, a boy's places in his form were 7, 10, 6, 11, 9, 8, 6, 4, 2, 1, 1, 3.

8. The number of thousands of people who emigrated from a certain country between 1886 and 1895 was as follows :—

Year	1886	'87	'88	'89	'90	'91	'92	'93	'94	'95
No.	37·5	38·5	41·1	47	75·5	78·4	98·1	92·3	85	80·5

EXAMPLE 3. The population in millions of a certain country at the beginning of the years given below was as follows :—

Year	1821	1831	1841	1851	1861	1871	1881	1891	1901
Popn.	12	14	16	18	20	22·5	26	29	32·5

Draw the graph and find the population at the beginning of 1875 and 1895.

Take one horizontal division to denote 2 years and one vertical division to represent one million of population. Having plotted the points, draw a continuous curved line passing through as many as possible of the plotted points; and you get the graph required. Fig. 57.

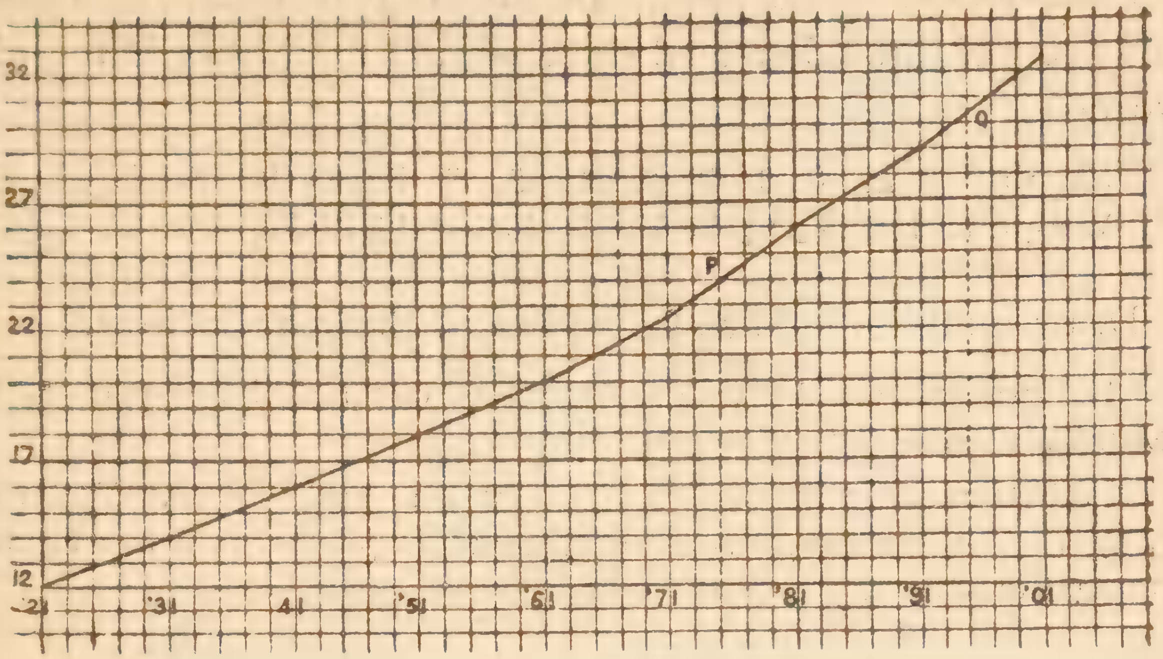


FIG. 57.

From abscissa 1875 draw a perpendicular to meet the graph at P. The perpendicular drawn from P meets the Y axis at 24. The population in 1875 was 24 millions. Similarly the population in 1895 is found to have been 30·5 millions.

These results cannot be exact, but only approximate. The graph in Fig. 57 is said to be a smooth curve, that is, a curve in which the bending is gradual, without there being any angular points.

Note.—To draw a curve, a piece of flexible material may be bent so as to lie along the plotted points and held in position while the line is being drawn.

317. As was observed in Art. 316, the values of two variables obtained by experiment cannot always be regarded as absolutely correct or free from errors.

The position of the plotted points cannot, therefore, be considered as having been exactly determined.

When a limited number of points have thus been plotted, it will be seen that a number of curves can be drawn through them ; and it may be that no curve passes through *all* of them.

The best plan in such cases is to draw a free-hand continuous curve, lying as evenly as possible among the plotted points, passing through some of the points and having the remaining points, some on the one side and others on the other side of it. The simplest curve thus obtained is the graph showing the relation between the two variables.

EXAMPLE 4. The weight of a child at its birth was 7½ lbs. and the weights taken at the end of each month for 12 months were—

Month ...	0	1	2	3	4	5	6	7	8	9	10	11	12
Weight in lbs.	7½	9½	11	13	15½	16	16¼	18	19	19¼	20	21	21½

Draw a smooth curve to represent these variations.

Let two horizontal divisions represent one month and each vertical division one pound weight.

Plot the several points and draw a smooth curve. You will notice it passes through some of the points plotted and lies evenly among the others.

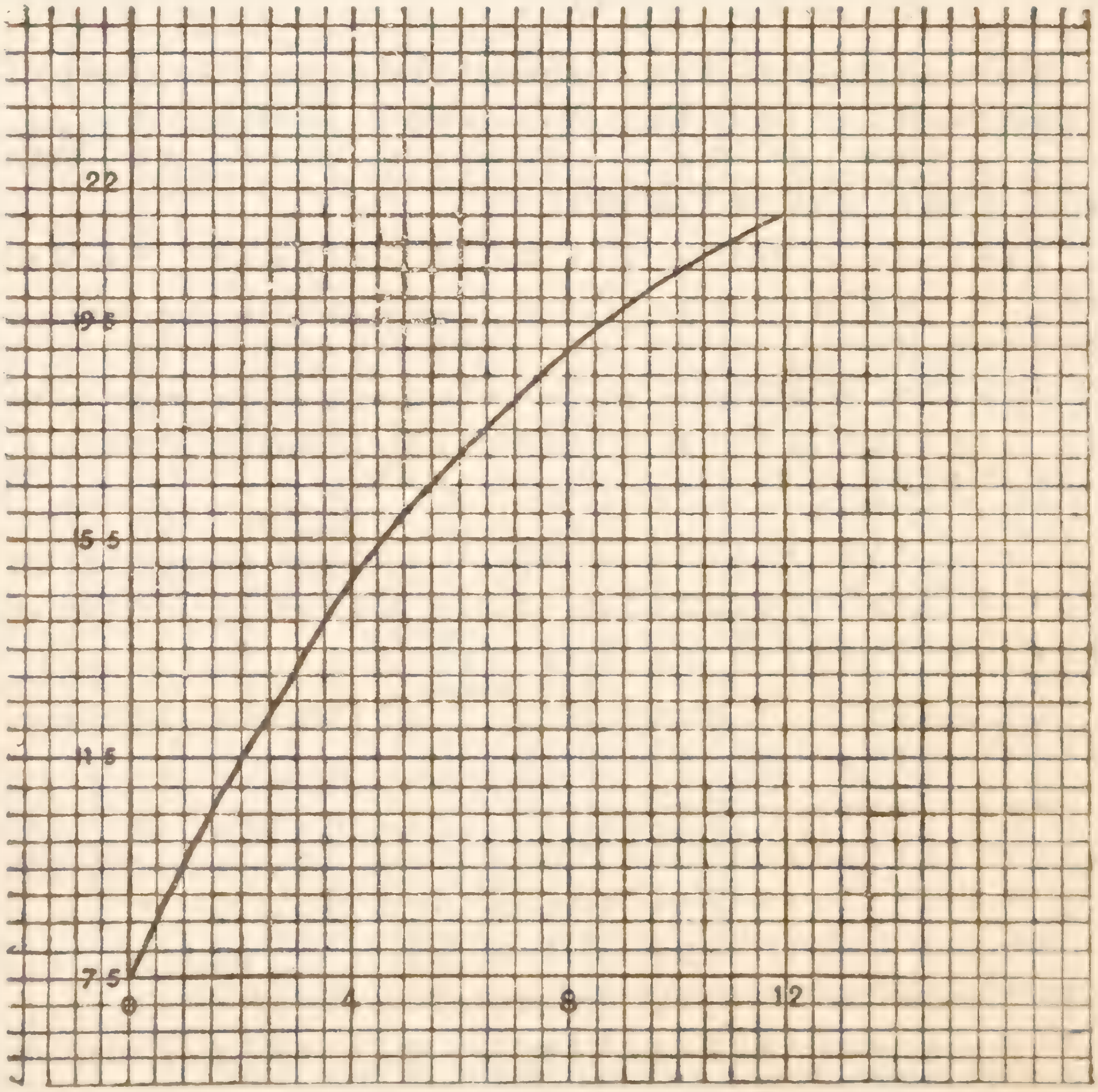


FIG. 58.

Practical Exercise.

Draw smooth curves in the following examples :—

1. The population in millions of a certain country was as follows according to the census taken in the following years :—

Year	1851	1861	1871	1881	1891	1901
Population	20	24	29	34·2	41	49·4

Find the population in 1876 and 1897. *Take one division horizontally for one year, and two divisions vertically for one million.*

2. Premiums limited to 15 years payable to secure Rs. 1,000 are given in the following table. Draw a graph connecting the age and the premium ; and from it find (1) the premium payable at age 33 ; (2) at what age the premium payable is Rs. 34¾. *Take two horizontal divisions to represent one year of the age and two vertical divisions to denote Re. I :—*

Age next birthday	20	25	30	35	40
Premium payable in Rs.	30½	33	36	39¾	44½

3. To shew the variations in the height of the barometer in inches—

Time	6 a.m.	9 a.m.	12 noon	3 p. m.	6 p.m.	9 p.m.	Midnight.
Height	29·90	30·01	29·96	29·91	29·94	29·98	29·94

4. The maximum and minimum shade temperatures in degrees F. for the first 7 days in a certain month are given below—

Day	1	2	3	4	5	6	7
Maximum	59	58	66	68	70	75	69
Minimum	49	43	44	47	52	52	53

Draw the two curves on the same sheet.

5. The heights of a boy taken at different ages are given below—

Birthday	12th	13th	14th	15th	16th	17th	18th
Height	4′4″	4′6½″	4′8¾″	4′11¼″	5′1¼″	5′3″	5′5″

6. The expectation of life for males at different ages are given below—

Age	20	25	30	35	40	45	50
Expectation	40·7	35·8	32·3	28·8	25·4	22·1	18·8

Find the expectation of the life of a man aged (1) 32 yrs. (2) 47·5 yrs., also at what age the expectation of life is (3) 38 yrs (4) 27·7 yrs.

7. A beaker of water is heated over a steady flame ; and the following observations made :—

Time in min.	0	5	10	15	20	30	35	40
Temp. in C°	15	20	24·5	28·6	32	38·4	41	43·8

Find the temperature reached in 25 min.

8. The following table shows the rise of wages per day of a carpenter during the years 1850 to 1900 :—

Year	1850	1860	1870	1880	1890	1900
Wages	6 as. 6 p.	8 as. 3 p.	10 as.	11 as.	12 as. 6 p.	13 as. 8 p.

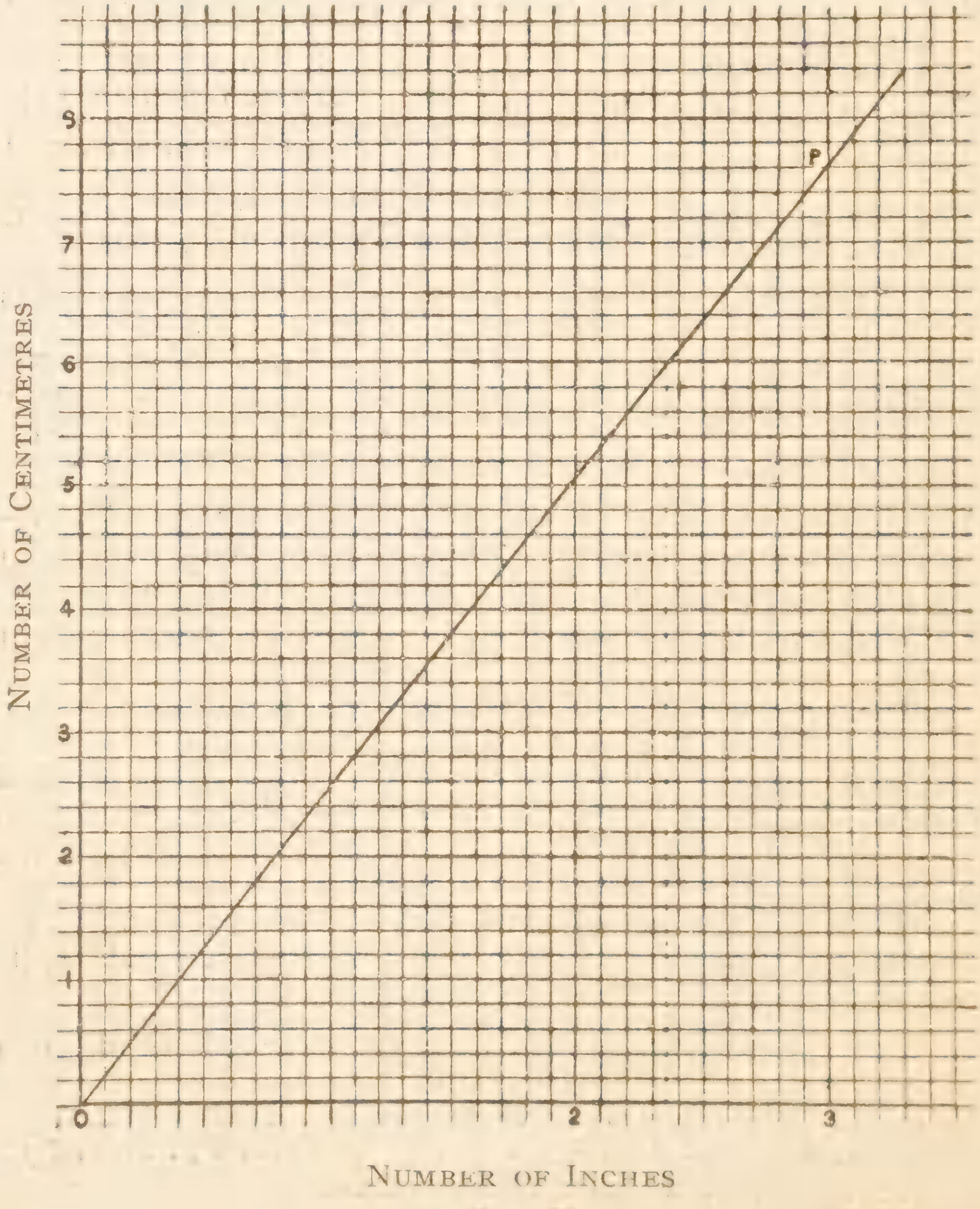
Find the probable wages in 1875 and 1894.

318. Conversion Graphs. To convert inches into centimetres and *vice versâ*.

EXAMPLE 1. Given that 3 inches is equivalent to 7·62 centimetres, draw a graph by means of which any number of inches can be converted into centimetres.

Let 10 horizontal divisions represent 1 in., and 5 vertical divisions 1 cm.

Plot P whose co-ordinates are (3, 7·62); and join P with the origin by the straight line OP. Then OP is the graph required.



NUMBER OF INCHES

FIG. 59.

By drawing the graph on a large sheet of paper the number of centimetres equal to a given number of inches and *vice versâ* can be read off.

From the graph, 1·5 inches will be seen equal to a little over 3·8 cm. and 6 cm. is about 2·4 in.

The values obtained can only be approximate.

319. Any number of degrees in the centigrade scale can be converted into the Fahrenheit scale and *vice versâ* by

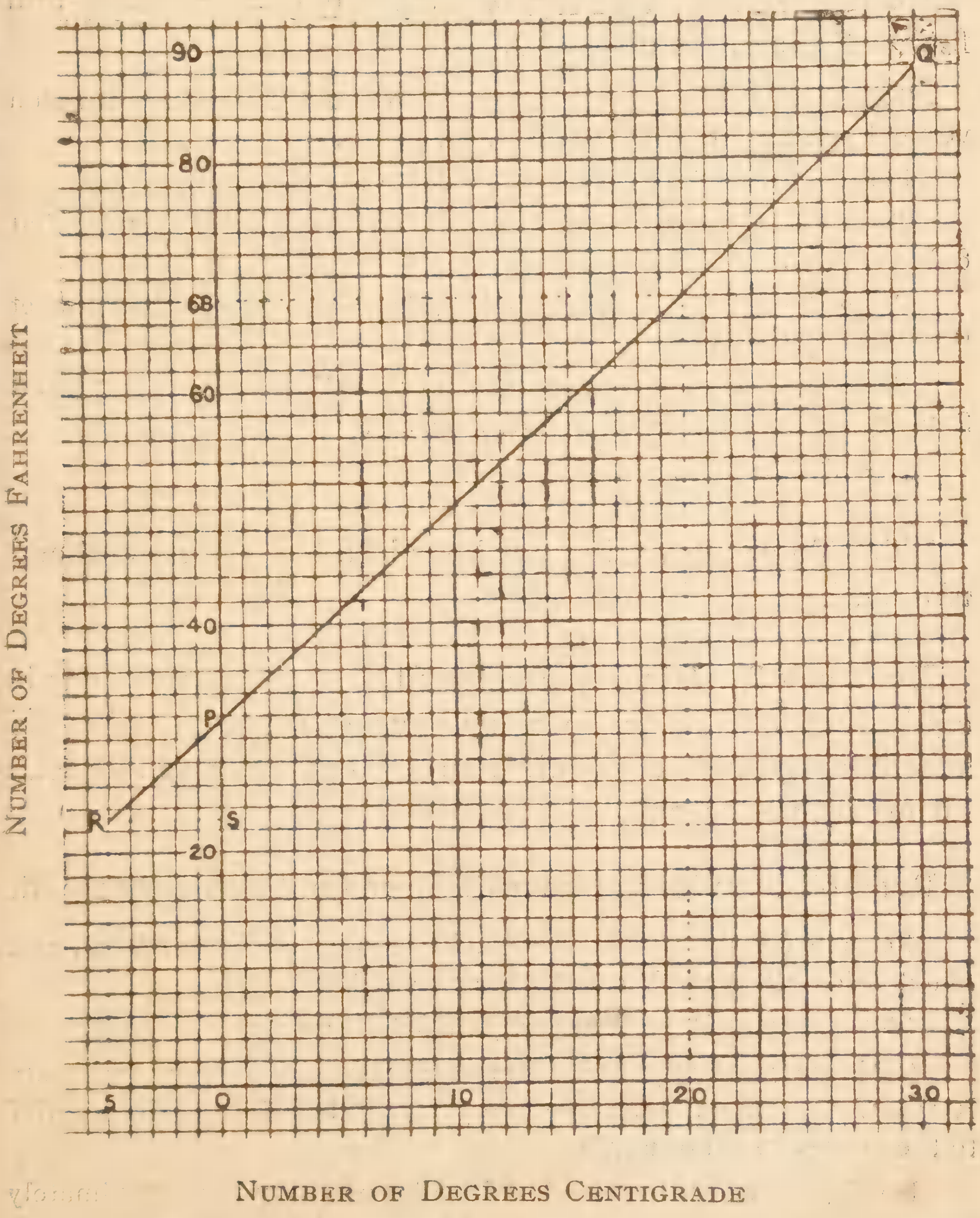


FIG. 60.

means of the graph showing the relation between these two variables.

EXAMPLE 2. It being given 0° C = 32° F and 30° C = 86° F, draw a graph to show that any number of degrees between these two temperatures in the one scale can be expressed in the other scale.

Read off in the Centigrade scale the value of 68° F and find in F the value of — 5° C.

Take one division horizontally to denote 1° C and one division vertically to denote 2° F. See Fig. 60, p. 293.

Plot the points P (0, 32) and Q (30, 86) ; and join PQ.

Then PQ is the graph required. From the graph it is seen that 68° F = 20° C.

From the point representing — 5° C, draw a perpendicular to meet the graph QP produced at R.

From R draw a perpendicular to meet the Y axis at S. The point S is $11\frac{1}{2}$ divisions above 0° and denotes 23°.

$\therefore -5° C = 23° F.$

320. To convert miles per hour into feet per second and *vice versâ*.

EXAMPLE 3. Given that a speed of 60 miles per hour is equivalent to 88 ft. per second, draw a graph from which the speed of any number of miles up to 60 per hour can be read off in feet per second.

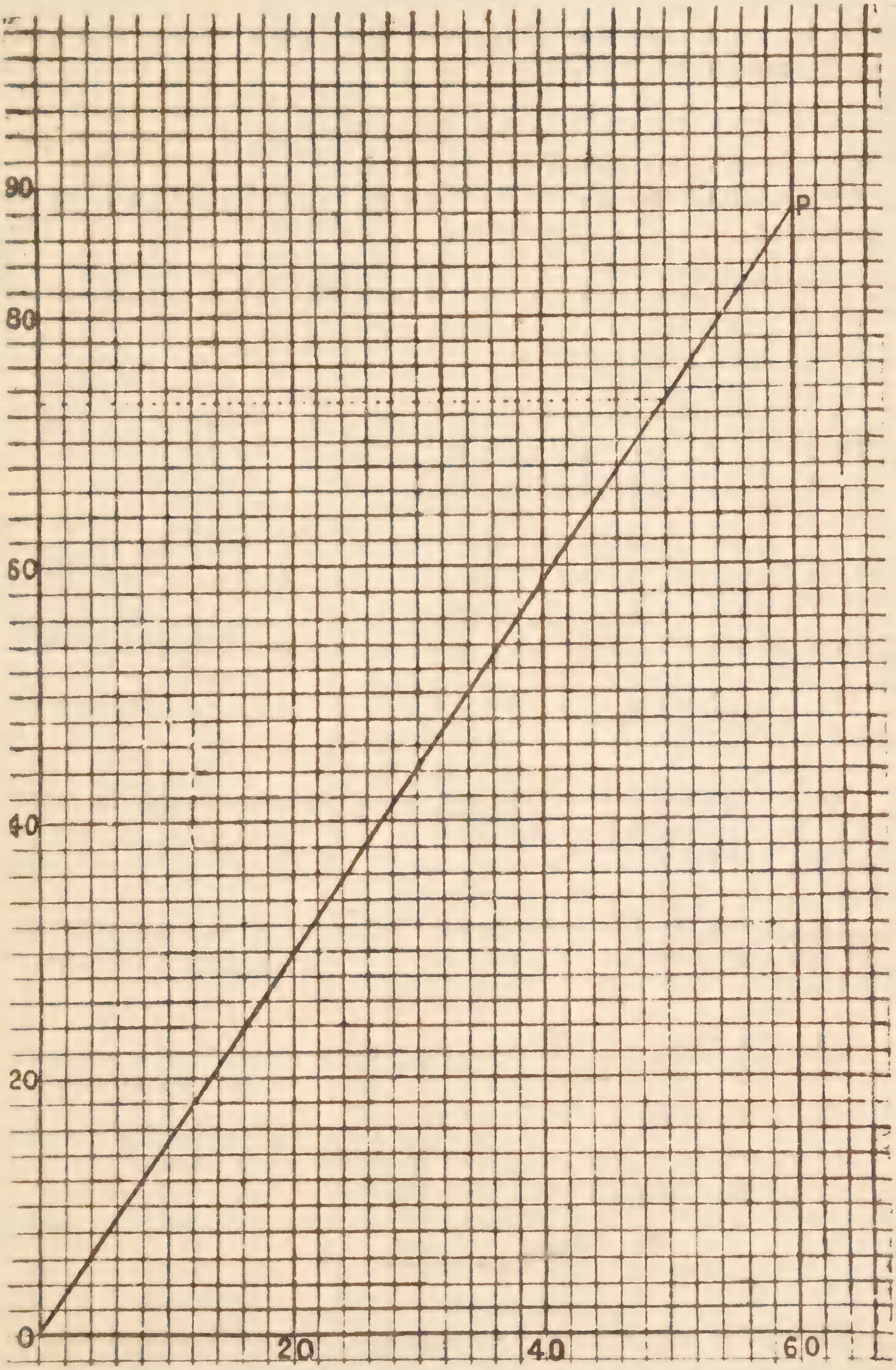
Read off in miles per hour the speed of 70 ft. per second.

Let one division along the X axis denote 2 miles per hour, and one division along the Y axis 2 ft. per second.

Plot the point P, (60, 88) and join OP. See Fig. 61, p. 295.

OP is the graph required. The abscissa corresponding to ordinate 70 is 47·7 approximately. The rate per hour in miles required is 47·7 to the nearest tenth of a mile.

It will be seen that a speed of 50 miles per hour is approximately equal to 73 ft. per second, which is correct to the nearest foot.



Number of Miles per Hour

Fig. 61.

Practical Exercise.

1. Read off from the graph of Fig. 60, 40°C, 55°C, 82°C and −20°C in the Fahrenheit scale; and 30°F, 95°F, 176°F, and 200°F in the Centigrade scale.

2. From the graph of Fig. 61, read off (*a*) in feet per second, the speed of (1) 20 mi. (2) 25 mi. (3) 35 mi. (4) 45 mi. per hour; (*b*) in miles per hour the speed of (1) 60 ft. (2) 80 ft. (3) 45 ft. (4) 50 ft. per second.

3. If £9 = Rs. 135, draw a graph connecting £ and Rs., and from it determine (1) how many rupees there are in £ 5¼; (2) how many pounds = Rs. 100.

4. Given Rs. 75 = 100*s.*, draw a graph connecting rupees and shillings; and find the number of rupees equal to 75*s.* and the number of shillings equal to Rs. 45.

5. An American dollar is equal to 4*s.* 2*d.*, and there are 100 cents in a dollar. Draw a graph showing the relation between English and American money. Find to the nearest penny the value of 3 dollars 40 cents, and of 10*s.* 5*d.* in dollars and cents.

6. A cubic foot of water measures 6·25 gallons. Draw a graph for converting gallons into cubic inches.

7. A rainfall of 0·1 inch represents approximately 10 tons per acre. Draw a graph by which you can find the weight for any rainfall from ·01 to 1 inch.

8. Draw the following conversion graphs for changing British into Metric measures and *vice versâ*, it being given approximately that:—

(1) 100 yds. = 91·5 metres. (2) 100 miles = 161 Km.
(3) 100 litres = 44 gals. (4) 100 lbs. = 45·5 Kg.
(5) 100 sq. yds. = 83·5 metres. (6) 100 ac. = 40·5 hectares.

Find out 35 in each of these in terms of the other; i.e. 35 yds. in metres and 35 metres in yards, and so on for others

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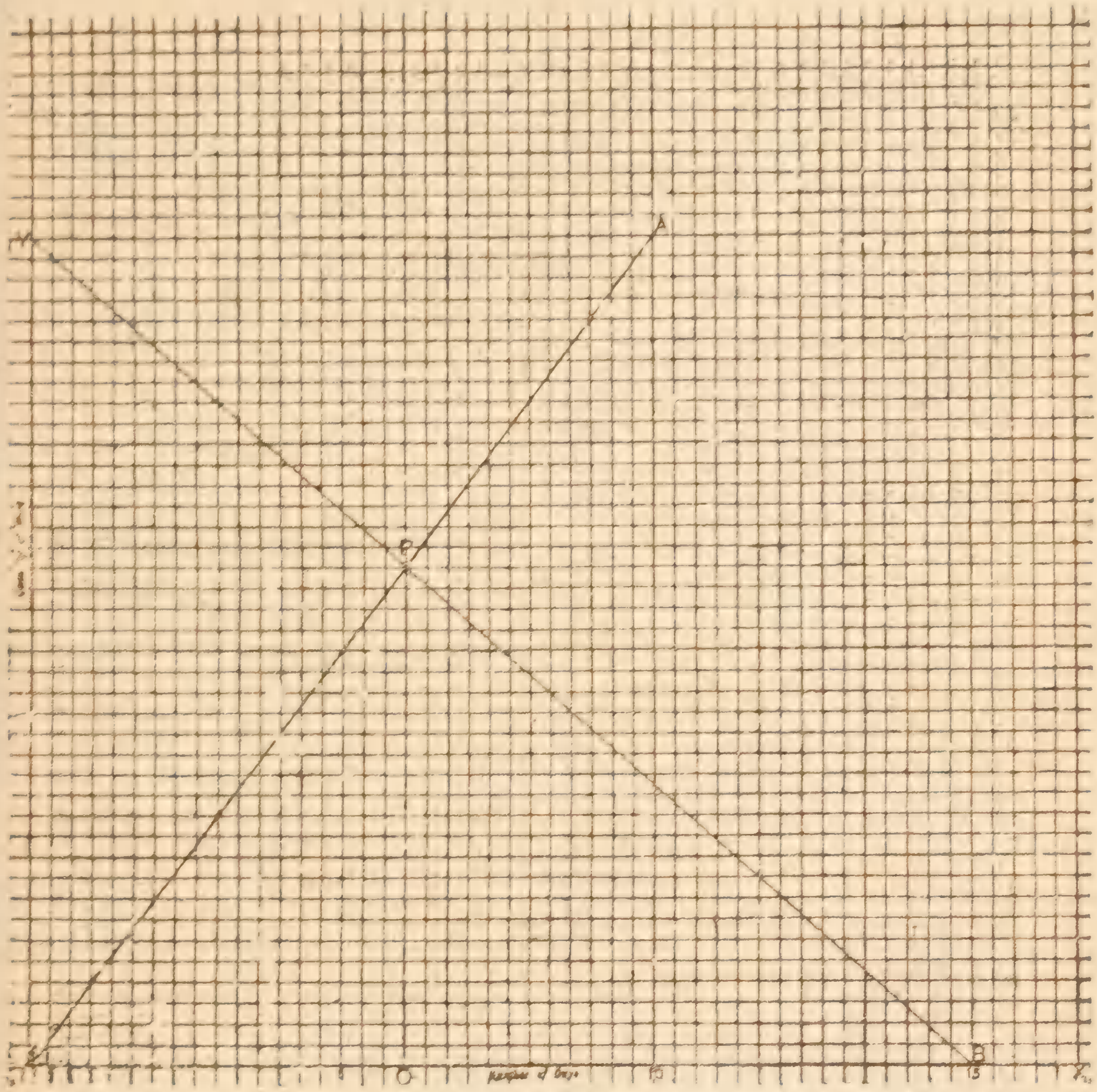
Problems on Work and Pipes: Graphic Solution.

EXAMPLE 1. A can do a piece of work in 10 hrs. and B in 15 hrs. How long will they take to do it together?



NUMBER OF DAYS

FIG. 62.

Let OY denote the whole work and 3 horizontal divisions one day. Then OA is A's graph showing he can do the work in 10 days; and BY is B's graph showing he can finish it in 15 days. From P where the two graphs cut one another drop the perpendicular PQ meeting the abscissa at Q; and Q = 6 days.

∴ A and B finish the work together in 6 days.

EXAMPLE 2. A cistern has a tap which fills it in 6 hrs. and an opening which empties it in 10 hrs. In what time will it be full if both are open?

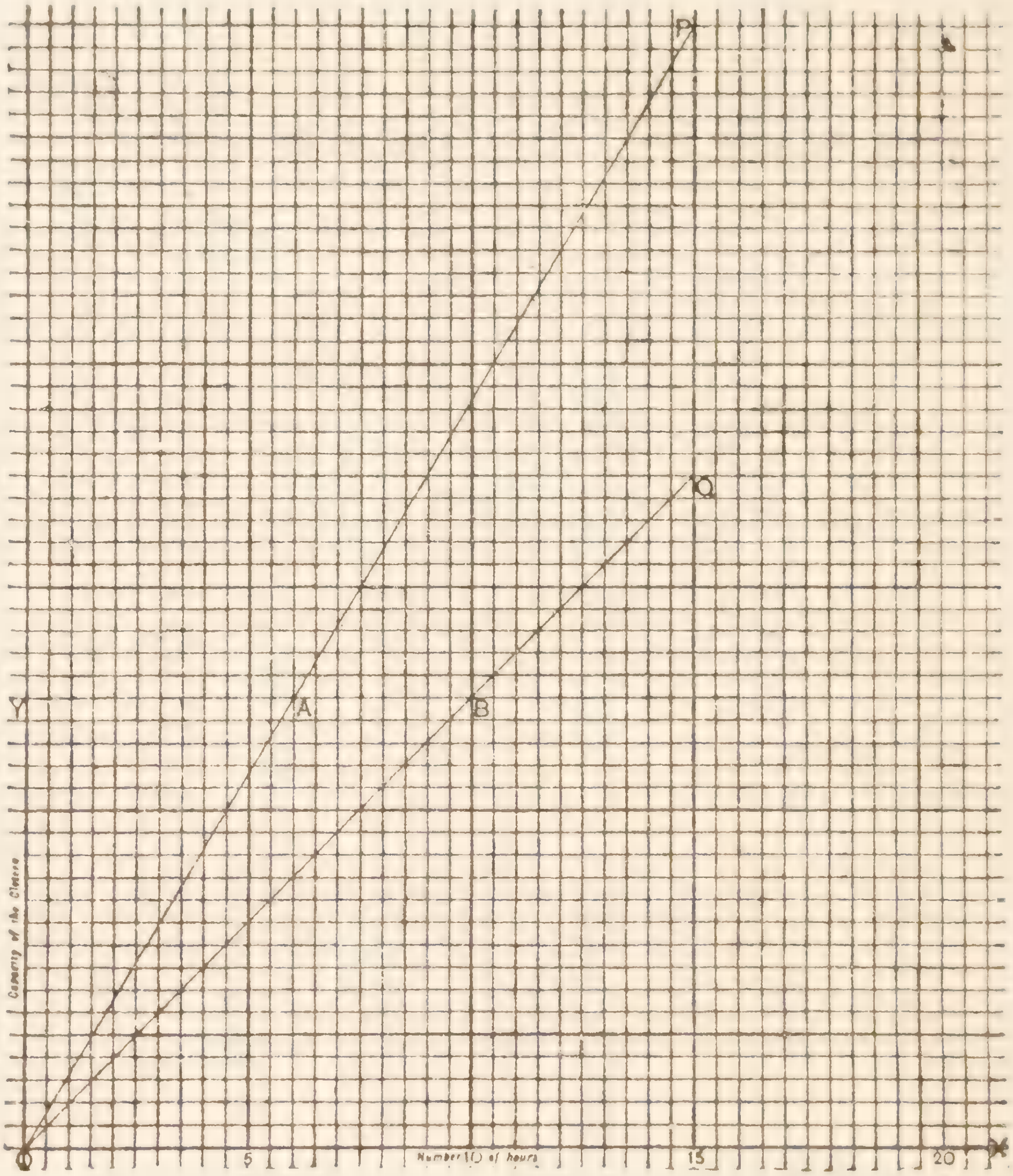


FIG. 63.

Let OY represent the capacity of the cistern and two horizontal divisions 1 hr. Draw the graph OA representing the rate of filling and CB representing the rate of emptying. Produce OA and OB to any length; and with the dividers, find two points P and Q in OA and OB respectively, such that PQ = OY, the capacity of the cistern.

Produce PQ to meet the X axis. PQ meets it at 15.

∴ No. of hours required is 15.

Practical Exercise.

MISCELLANEOUS GRAPHIC EXERCISES.

1. The average yearly prices of wheat per quarter are given below. Draw a graph to show the variation—

Year ...	1892	'93	'94	'95	'96	'97	'98	'99	1900	'01	'02
Price in *s. d.*	30/3	26/4	22/10	23/1	26/2	30/2	34/	25/8	26/11	26/9	28/1

2. Make a graphical record of the following observation of barometric heights :—

Time ...	8 a.m.	10.15 a.m.	2 p.m.	4.30 p.m.	6 p.m.
Reading in inches.	29·1	29·3	29·1	28·95	29·05

3. Draw a graph to shew the relation between the age and weight of a boy—

Age ...	13	14	15	16	17	18
Weight in stone ...	5	5·4	5·7	6	6·5	7·2

4. Rainfall in inches between 1894 and 1902 was recorded in a station as follows :—

Year ...	1894	1895	1896	1897	1898	1899	1900	1901	1902
Rainfall	32·3	26·3	25·8	27·8	22·1	24·8	27·9	23·5	21·9

Show graphically the variations in the rainfall.

5. Imports into Britain from India : cotton, raw material—

Year	1900	1904	1905	1906	1907
Value in million £ ...	0·7	2·2	1·0	1·5	2·3

Imports into India from Britain : cotton, manufactures—

Year	1900	1904	1905	1906	1907
Value in million £ ...	16·1	21·7	22·9	23·0	25·7

Draw the two graphs on the same sheet.

6. Average rainfall in inches both in London and Madras for the 12 months of a certain year is given below—

	Jan.	Feb.	Mar.	Apl.	May.	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
London	2·4	1·7	1·4	1·9	1·7	2·3	2·4	2·3	2·4	2·6	1·9	2·4
Madras	1·0	0·3	0·4	0·6	2·2	2·1	3·8	4·4	4·7	10·8	13·7	5·1

Draw the two graphs on the same sheet.

7. The annual premium payable to a Life Assurance Company to secure £100 at death are given below—

Age ...	20	30	40	50	60
Premium ...	£1-15	£2-6-8	£3-2-6	£4-7-6	£7-2-6

Draw a graph to find the premium payable at age (1) 25, (2) 45.

8. The following table gives the age and 'expectation of life' for males :—

Age	7	8	9	10	20	30	40	50
Expectation in years.	50·8	50·2	49·6	48·8	41·5	34·4	27·6	21·1

Draw a graph to determine the expectation of life of a person aged (1) 18 years, (2) 35 years.

9. Out of a thousand people, the number of persons living up to different ages is given below—

Age ...	5	10	15	20	30	40	45	50
Number ...	877	760	649	547	376	244	191	143

Find graphically the number living up to age (1) 25, (2) 35.

10. Draw a graph showing the variations in the speed of a train between two stations A and B, 12 miles distant.

Miles from station A	0	1	2	3	4	5	6	7	8	9	10	11	12
Speed in miles per hr.	12	21	24	34	35	30	36	34	32	21	18	12	0

11. Draw a graph to show the variation of temperature with time.

Time ...	8 a.m.	9	10	11	noon	1	2	3	4	5	6	7	8 p.m.
Temperature	48	50	58	60	68	75	77	75	67	60	58	50	45

12. Draw a graph to convert bushels to hectolitres given that 100 bushels = 36·5 Hl. Find the number of hectolitres equal to 35 bushels.

13. A can do a piece of work in 12 days which B can do in 20 days. In what time will they both do it together?

14. Two pipes can fill a bath in 4 and 6 hrs. respectively. If both are open, when will the bath be full?

15. A bath has a tap which fills it in 10 hrs., and an opening which empties it in 30 hrs. If when filling it, the latter by mistake is not closed, after how many hours will the bath be full?

16. A can do a piece of work in 320 days; with the assistance of B, the work is done in $7\frac{1}{2}$ days. What time will B take to do it alone?

17. A man starts at 2 p.m. and walks 12 miles at the rate of 4 miles an hour. Shew graphically where he is at 3·15 and find the time when he has walked 10 miles.

18. A and B start at the same time from the same place and travel in the same direction at 4 and 6 miles an hour respectively. How far apart will they be at the end of 5 hrs.? When will there be a distance of $22\frac{1}{2}$ miles between them?

19. Two persons A and B walk in opposite directions along a road 40 miles long, at the rate of 3 and 5 miles respectively. When and where do they meet?

20. A train leaves A for B at 10·30 a.m. and travels at 24 miles an hour; 20 minutes after another leaves A for B at the rate of 30 miles an hour. Find when and where the latter will overtake the former.

CHAPTER XXVIII.

RATIO AND PROPORTION.

321. We have already seen in Arts. 238 and 239 that one quantity is compared with another of the same kind by considering how many times or what fraction the one is of the other; and that the relation which thus exists between the two quantities is called RATIO.

We have also seen that the ratio of A to B, two quantities of the same kind, may be expressed as A : B or as the abstract fraction $\frac{A}{B}$.

322. The quantities which form a ratio are called its TERMS; the first term in the ratio is known as the *antecedent*, and the second term the *consequent*.

The terms of a ratio must both be abstract or both concrete; and if concrete, they must not only be of the same kind but should be expressed in the same denomination. Art. 240.

323. Since a ratio may be expressed as a vulgar fraction, properties of vulgar fractions are true ratios.

For example, the terms of a ratio may be multiplied or divided by the same number without altering the value of the ratio.

Ex. (*a*) $2 : 3 = 6 : 9$, each term being multiplied by 3.
(*b*) $15 : 10 = 3 : 2$, each term being divided by 5.

Fractional ratios may, therefore, be simplified.

EXAMPLE. Simplify the ratio $\frac{25}{90} : \frac{35}{120}$.

$\frac{25}{90} : \frac{35}{120} = \frac{5}{18} : \frac{7}{24}$, reducing the fractions to lowest terms.

$= \frac{5}{3} : \frac{7}{4}$, multiplying the ratio by 6.

$= 20 : 21$, multiplying the ratio by 12.

Or $5 \times 4 : 7 \times 3 = 20 : 21$.

EXAMPLE. Which is the greater ratio 5 : 6 or 7 : 8 and by how much? Alter the first of the four numbers to make the ratios equal.

$$5 : 6 = \frac{5}{6} = \frac{20}{24}\text{; and } 7 : 8 = \frac{7}{8} = \frac{21}{24}.$$

∴ The second is the greater ratio by $\frac{1}{24}$.

To make the ratios equal, 5 should be altered to $5\frac{1}{4}$.

EXAMPLES.—ORAL.

1. By multiplying or dividing by the same number, show that the following ratios are equal:—

(1) 12 : 30 and 16 : 40. (2) 50 : 35 and 80 : 56.

(3) 3 : 7 and 2·4 : 5·6. (4) $\frac{6}{8} : \frac{5}{10}$ and 9 : 6.

(5) 6 : 5 and $\frac{3}{4} : \frac{5}{8}$. (6) $\frac{4}{6} : \frac{6}{8}$ and 32 : 42.

2. Find the value of x which makes the following ratios equal:—

(1) $\frac{15}{16} = \frac{x}{4}$. (2) $\frac{4}{5} = \frac{x}{10}$. (3) $\frac{4}{5} = \frac{28}{x}$. (4) $\frac{7}{8} = \frac{35}{x}$.

(5) $\frac{8}{9} = \frac{12}{x}$. (6) $\frac{5}{12} = \frac{4}{x}$. (7) $\frac{x}{24} = \frac{7}{2}$. (8) $\frac{x}{4} = \frac{20}{24}$.

(9) $\frac{x}{3} = \frac{a}{b}$. (10) $\frac{x}{5} = \frac{c}{d}$. (11) $\frac{x}{a} = \frac{b}{c}$. (12) $\frac{x}{m} = \frac{n}{p}$.

3. Which is the greater ratio and by how much?

(1) $\frac{1}{2}$ or $\frac{7}{12}$. (2) $\frac{2}{5}$ or $\frac{5}{12}$. (3) $\frac{9}{10}$ or $\frac{11}{12}$. (4) $\frac{7}{8}$ or $\frac{9}{10}$.

(5) 3*d*. : 1*s*. or 2*s*. 6*d*. : 10*s*. (6) 5 as. 4p. to R. 1 or 6*s*. 8*d*. to £1.

(7) $\frac{\text{1 ton 10 cwt.}}{\text{2 tons.}}$ or $\frac{\text{£1 5s.}}{\text{£1 10s.}}$. (8) $\frac{\text{125 m.}}{\text{1 Km.}}$ or $\frac{\text{35 g.}}{\text{2 Hg.}}$.

4. Find the ratio of—(1) the area of a square of side 5 in. to that of a square of side 10 in. (2) ,, ,, ,, 3 cm. ,, ,, ,, 9 cm. (3) the volume of a cube of side 2 in. to that of a cube of side 4 in. (4) ,, ,, ,, 3 cm. ,, ,, ,, 6 cm.

EXAMPLES. I.

1. Express in their simplest forms—

(1) 76 mds. : 95 mds. (2) 3·5 miles : 1,040 poles.

(3) 17 hrs. 45 min. 28 sec. : 5 days 4 hrs. 18 min. 16 sec.

(4) 0·75 of 2 tons 13 cwt. 3 qrs. 3 lbs. : 0·4 of 3 tons 5 cwts. 11 lbs.

2. Simplify the following ratios:—

(1) $1\frac{1}{2} : 0·03$. (2) $1·4 : 14·7$. (3) $\frac{36}{54} : \frac{21}{56}$. (4) $\frac{144}{168} : \frac{117}{72}$.

3. Which is the greater ratio, and by how much?

(1) $\frac{13}{25}$ or $0·53 : 1$. (2) $3·1417 : 1$ or $\frac{355}{113} : 1$.

(3) $\dfrac{\text{7 can. 12 mds. 4 v.}}{\text{8 can. 15 mds.}}$ or $\dfrac{\text{12 tons 9 cwt.}}{\text{15 tons 11 cwt. 1 qr.}}$

(4) $3\frac{4}{7} : 8\frac{1}{3}$ or $\frac{5}{6}$ of £5-8-6 : $\frac{3}{4}$ of £7-4-8.

4. The annual rainfall in Madura is 35 inches. If in the month of October 8·7 inches are recorded, what is the ratio of this to the annual rainfall?

5. One train travels at the rate of 25 miles in 1 hr. 30 min., and another goes at the rate of 35 miles in 2 hrs. Compare the speed of the trains.

6. Given that 915 metres are approximately equal to 1,000 yards, find the approximate ratio of 1 m. to 1 yd.

7. A map is drawn to a scale of 1″ to 20′. Find the ratio of any length on the map to the actual distance it represents.

8. Which is the more advantageous one to buy, a suit of clothes costing Rs. 15 and lasting for 6 months or one costing Rs. 12 and lasting for 5 months?

9. A bicyclist covers 20 miles in 1 hr. 40 min; another goes $22\frac{1}{2}$ miles in 1 hr. 45 min. Find the ratio of their speeds per hour.

10. A room measures 15′ by 10′, in the centre of which there is a carpet 5′ by 3′, and the remainder is covered with oil-cloth. Find the ratio of the carpet to the oil-cloth.

11. A and B buy an equal number of mangoes at Rs. 6-4 per 100; A sells them at 15 as. per dozen, and B at Rs. 1-10-8 per score. Find the ratio of their gains.

PROPORTION.

324. When the ratio of one quantity to the second is **equal** to the ratio of the third quantity to the fourth, **Proportion** is said to exist among the four quantities; so that proportion expresses the equality of two ratios.

EXAMPLE. 7 : 8 = 42 : 48.

This is also written 7 : 8 : : 42 : 48 ; and is read 7 is to 8 as 42 is to 48.

The order of terms in poroportion begins from the left. In the above proportion 7 is the first term, 8 is the second term, 42 is the third, and 48 is the fourth term.

325. Since ratios may be expressed as fractions, four quantities are in proportion when the first is of the same fraction of the second that the third is of the fourth.

The numbers 15, 20 : 33, 44 are in proportion, since 15 is $\frac{3}{4}$ of 20, and 33 is $\frac{3}{4}$ of 44 ; that is, since $\frac{15}{20}=\frac{33}{44}$.

The quantities that form a proportion are called proportionals.

326. In a proportion, if one ratio is formed by **two abstract numbers,** the other may be formed by **two concrete numbers,** for we know that $\frac{\text{concrete number}}{\text{concrete number}}=$abstract number.

EXAMPLE. 12 : 18 = Rs. 10 : Rs. 15 ; Rs. 18 : Rs. 21 = 12 : 14.

Similarly if one of the ratios in a proportion be formed from concrete numbers of one kind, the other ratio may be formed from concrete numbers of a different kind.

EXAMPLE. Rs. 15 : Rs. 25 = 6 mds. : 10 mds.

$$\frac{\text{Rs. }15}{\text{Rs. }25}=\frac{3}{5}\text{; and }\frac{6\text{ mds.}}{10\text{ mds.}}=\frac{3}{5}.$$

Also 6 yds. 2 ft., 16 yds. 2 ft. ; Rs. 2-8 as. Rs. 6-4 as., are proportionals since $\frac{6\text{ yds. }2\text{ ft.}}{16\text{ yds. }2\text{ ft.}}=\frac{20\text{ ft.}}{50\text{ ft.}}=\frac{2}{5}$, and $\frac{\text{Rs. 2-8 as.}}{\text{Rs. 6-4 as.}}=\frac{10\text{ qr. rs.}}{25\text{ qr. rs.}}=\frac{2}{5}$.

It will be noticed in these examples that the first term is of the same fraction of the second that the third is of the fourth.

Generally, we may state that when four quantities a, b, c, d, are proportionals, the proportion is expressed as follows :—

$$\frac{a}{b}=\frac{c}{d}\text{ or }a : b = c : d\text{ or }a : b : : c : d.$$

327. The first and fourth terms of a proportion are called the **extremes**; the second and third terms are called the **means.**

In a proportion formed by four quantities, the last term is known as the *fourth* proportional. When the second and third terms are the same, each is said to be a *mean* proportional to the other two.

Thus in the proportion, 8 : 12 : : 12 : 18, 18 is the fourth proportional; 12 is the mean proportional to 8 and 18; and 18 is said to be the third proportional to 8 and 12.

328. In a proportion, the product of the extremes is equal to the product of the means.

In Rs. 18 : Rs. 21 : : 12 : 14 Rs. 18 × 14 = Rs. 21 × 12.

In $a : b :: c : d$, $a \times d = b \times c$.

Proof.

The proportion $a : b :: c : d$ may be written $\frac{a}{b} = \frac{c}{d}$.

Multiplying each ratio by bd, $\frac{a}{b} \times bd = \frac{c}{d} \times bd$, i.e.

$\frac{a \times bd}{b} = \frac{c \times bd}{d}$ or $ad = bc$.

329. In a proportion any three of the four proportionals being given, the other may be found.

EXAMPLE 1. Find the fourth proportional to—

4*s*. 6*d*. : 17*s*. : : 6 lbs. :

Let x be the fourth proportional in lbs.

Then in the proportion 4*s*. 6*d*. : 17*s*. : : 6 lbs. : x, the product of the extremes being equal to the product of the means, we have $4\frac{1}{2} \times x = 17 \times 6$.

$\therefore \quad x = \frac{17 \times 6}{4\frac{1}{2}} = 22\frac{2}{3}$. The fourth proportional is $22\frac{2}{3}$ lbs.

EXAMPLE 2. What sum of money bears to Rs. 11-5-4, the ratio of 6 mds. 30 srs. to 8 mds. 20 srs.?

In other words, find the first term of the proportion x : Rs. 11-5-4 $= \frac{\text{6 mds. 30 srs.}}{\text{8 mds. 20 srs.}}$, where x denotes the required sum of money in rupees.

$$\frac{x}{11\frac{1}{3}} = \frac{270}{340}$$

$\therefore x = \frac{27}{34} \times 11\frac{1}{3}$, multiplying each ratio by $11\frac{1}{3}$.

$= \frac{27}{34} \times \frac{34}{3} = 9$ ∴ Sum reqd. = Rs. 9.

Example 3. Find the value of x in the proportion— 7·5 yds. : 12·75 yds. $= x : 8\cdot5s.$

$$\frac{7\cdot5}{12\cdot75}=\frac{x}{8\cdot5}.$$

$$\therefore \frac{7\cdot5}{12\cdot75}\times 8\cdot5 = x,\ \text{multiplying by } 8\cdot5.$$

$$\therefore x=\frac{63\cdot75}{12\cdot75}s. = 5s.$$

EXAMPLES. II.

1. Find the values of x in the following proportions :—

(1) 3 mds. 2 viss : 8 mds. 4 viss : : Rs. 26 : x.
(2) $7\frac{1}{2}$ tons : $12\frac{1}{2}$ tons = Rs. $22\frac{1}{2}$: x.
(3) 25 miles : $22\frac{1}{2}$ miles : : x : 1 hr. 21 min.
(4) 2·6 metres : 3·9 metres = x : 15 francs.
(5) $x : 7 = 540 : 420$. (6) x : Rs. 1·5 : : 9 yds. : 3 yds.
(7) £2-6-8 : x = 7 men : 11 men.
(8) 42 : x : : £7-15-2 : £9-19-6.

2. What weight has the same ratio to 2 tons 2 cwt. that 4 mi. 2 fur. bears to 10 mi. 4 fur. ?

3. The contents of two purses are in the ratio of 2 : 5. If the smaller contains Rs. 350, what does the larger contain ?

4. An estate is divided in the ratio of 5·25 to 0·025. If the smaller share is worth Rs. 3,471 8 as., find the value of the larger share at the same rate ?

5. The value of gold is fixed at £3-17-$10\frac{1}{2}$ per oz. What should be the price of silver per oz., if its value is fixed to that of gold in the ratio of 1 : $15\frac{1}{2}$?

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330. Tests of Proportionality.

i. If proportion exists among four quantities, the ratio of the first quantity to the second should, as has already been shown, be equal to the ratio of the third quantity to the fourth; or the product of the extremes should be equal to the product of the means.

EXAMPLES. III.

Find out which of the following form proportions :—

1. 20 : 21 : : Rs. 15 : Rs. $15\frac{3}{4}$.
2. 13 yds. : 25 yds. : : Rs. $12\frac{1}{2}$: Rs. $24\frac{1}{2}$.

3. $22\frac{1}{2}$ days : $17\frac{1}{4}$ days = 30 s. : 23 s.

4. $18\frac{3}{4}$ m. : $3\frac{1}{8}$ m. = 18 sec. : 3 sec.

---

ii. Proportionality may also be tested by drawing the graph.

The following table gives the wages paid to a carpenter and the number of days he worked :—

Wages in Rs.	$5\frac{1}{4}$	$9\frac{3}{4}$	$13\frac{1}{2}$	18.
No. of days.	7	13	18	24.

Plot the points $(7, 5\frac{1}{4})$, $(13, 9\frac{3}{4})$, $(18, 13\frac{1}{2})$, $(24, 18)$, taking 2 horizontal divisions to represent 1 day, and 2 vertical divisions to denote 1 rupee.

Draw the graph.

Observe that all the points lie in a straight line passing through the origin ; and the graph is a straight line.

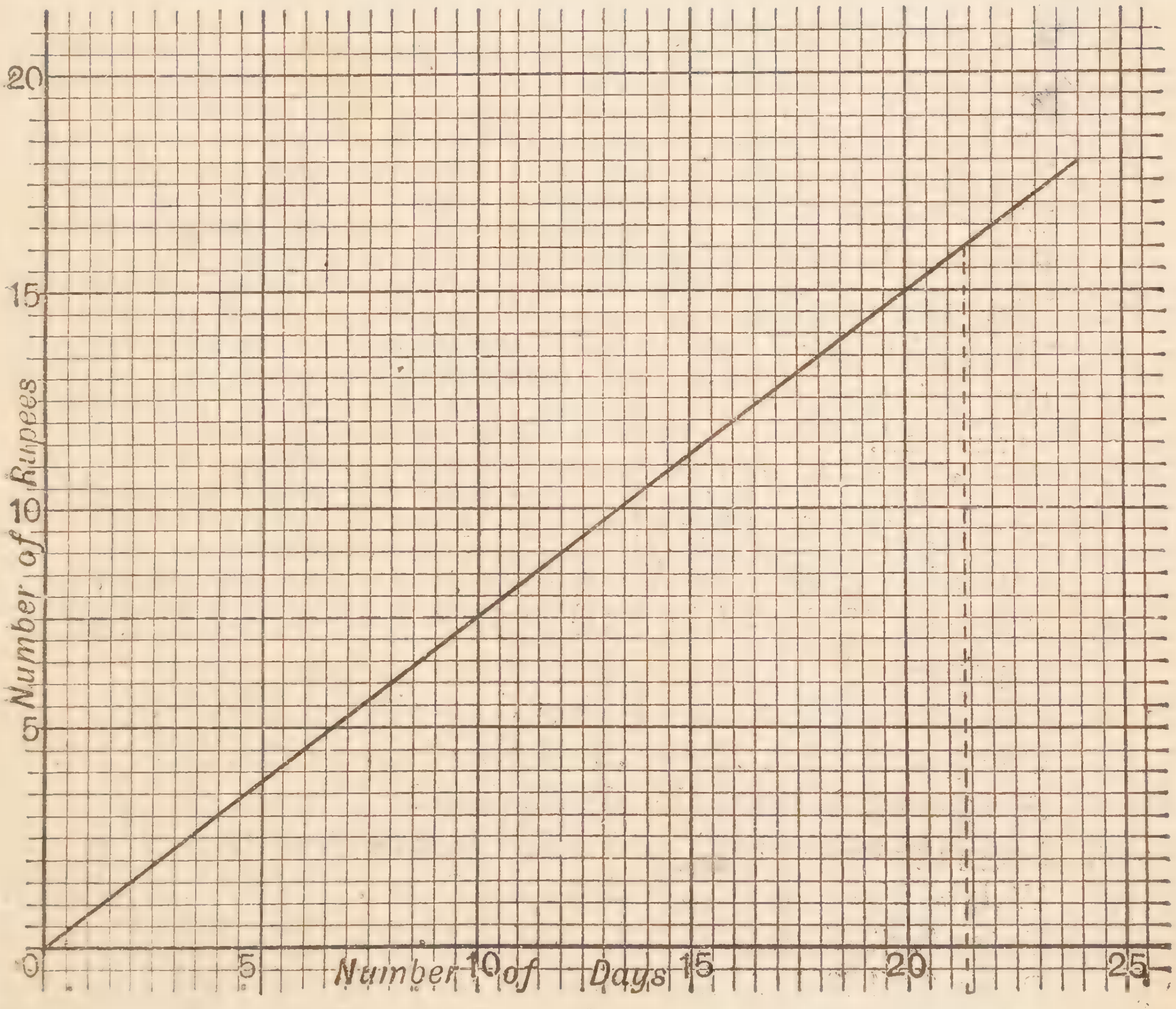

FIG 64

When two quantities are so related that one bears a constant ratio to the other, the graph showing the relation is a straight line.

The graph may be used to read off the wages for any number of days up to 24 and to find out the number of days he should work to secure a given sum as wages up to Rs. 18.

Thus for 10 days, he will get Rs. $7\frac{1}{2}$ and to get Rs. 16 he should work $21\frac{1}{3}$ days.

If the diagram be drawn on a large sheet of squared paper, the wages could be read off for any number of days.

331. We thus see that when a graph has been accurately drawn through carefully plotted points, we can use it to ascertain values of the variables at intermediate points. This process is called *Interpolation.*

Graphic Exercise.

Draw graphs—

1.

No. of yds.	4	6	8	12
Cost in Rs.	$2\frac{1}{2}$	$4\frac{1}{2}$	5	$7\frac{1}{2}$

Find if the cost is proportional to the number of yards.

2.

Number of working hours	6	7	8	9	10
Wages in annas ...	8	9	10	$10\frac{1}{2}$	$11\frac{1}{2}$

Examine if the increase is proportional to time.

3.

Distance travelled in miles	18	24	27	30
Time taken by S.I.R.	45 min.	52 min.	1 hr.	1 hr. 28 min.

Determine if the distance traversed is proportional to time occupied.

4.

2nd class fare S.I.R.	Rs. 1-9,	1-14,	2-3,	2-8,	2-13,	3-2
Distance travelled in miles	50	60	70	80	90	100

Is the fare proportional to distance?

5. Between Madras (Egmore) and Tiruvannamalai (143 mi.) parcel rates are—

Weight in seers not exceeding	5	10	20	30	40
Charges	4 as.	8 as.	12 as.	Re. 1	Re. 1-8.

Are the rates proportional to weight?

6. Luggage rates for 325 and 650 miles are given below—

Weight in seers ...	10 srs.	20 srs.	30 srs.	40 srs.
Rates for 325 mi. ...	12 as.	Rs. 1-4	Rs. 1-12	Rs. 2-4
Rates for 650 mi. ...	Rs. 1-4	Rs. 2-4	Rs. 3-8	Rs. 4-8

Draw the graphs on the same sheet and determine if the rates are proportional to weight.

332. We know that the sum of money a man gets as his wages is directly proportional to the number of days he works.

Now consider the following examples :—

A letter weighing 1 tola can be sent by post for half-anna stamp and a letter weighing 10 tolas can go for one anna. Here because when weight increases, the rate increases, it would be wrong to say the two quantities are proportional ; for we know a half-tola letter requires a half-anna stamp and for a letter of 2 tolas weight one anna stamp is to be affixed.

Again if a boy runs 100 yds. in 15 sec. and a mile race in 3 min. 30 sec., because time increases, as distance increases, it cannot be assumed that he will run 50 yds. in $7\frac{1}{2}$ sec. or 2 miles in 7 min. The rate of motion cannot be uniform and the time taken therefore cannot be proportional to distance.

Then again when a side of a square increases, we know that the area does not increase proportionately. The area of a square of side 6 in. is 36 sq. in. ; but the area of a square of 9 in. is 81 sq. in., 9 in. is $1\frac{1}{2}$ times 6 in. ; but 81 sq. in. is $\frac{81}{36}$ or $2\frac{1}{4}$ times 36 sq. in.

From these examples, it is clear that though two quantities may be so related that when one increases or decreases, the other also increases or decreases, they are not necessarily proportional.

EXAMPLES.—ORAL.

1. A letter weighing 1 tola can go for half-anna postage stamp. What is the postage to be paid for a letter weighing 5 tolas.

2. A boy weighed $2\frac{1}{2}$ stone when he was 5 ; and 5 stone when he was 10 years old. What will he weigh when he is 15 years old ?

3. A parcel weighing 40 tolas is charged 2 as. ; what is it for one weighing 100 tolas ?

4. A boy takes 3 min. 30 sec. to run a mile race ; will he have taken 1 min. 10 sec. to run the first one-third of the mile ?

5. A pound of sugar is sold for 2 as. ; is it correct to assume that cwt. will be sold for 112 × 2 as. ?

6. A field 100 yds. square has pasture sufficient for 12 sheep. Is it right to say that the pasture of a field 200 yds. square will be sufficient for the same 12 sheep for double the time ? What is the correct answer ?

SIMPLE PROPORTION.

EXAMPLE. If 27 yards of cloth be bought for Rs. 18, how many yards can be had for Rs. 20 at the same price?

333. Solution by the unitary method.

For Rs. 18, the number of yards bought = 27

∴ For Re. 1 ... ... $= 27 \times \frac{1}{18}$

∴ For Rs. 20 ... ... $= 27 \times \frac{20}{18}$

$= 30$

Multiplying ratio or fraction.

To get the required number of yds., 27 yds. has to be multiplied by the fraction $\frac{20}{18}$.

∴ $\frac{20}{18}$ is the **Multiplying fraction** or the **Multiplying ratio.**

Here the number of yds. is *greater than* 27, because Rs. 20 is *greater than* Rs. 18, and the multiplying ratio is 20 : 18.

If at the same price it is required to find how many yards can be purchased for Rs. 12, it is clear that the number of yards required $= 27 \times \frac{12}{18} = 18$.

Here Rs. 12 being less than Rs. 18, the number of yards is also less than 27 yds., and the multiplying ratio is $\frac{12}{18}$.

334. Solution by Proportion.

For Rs. 18 can be bought 27 yds.
For Rs. 20 ... ... ?

27 yds. should bear the same ratio to the required number of yds. that Rs. 18 has to Rs. 20.

Denoting the fourth term by x, the proportion is—

Rs. 18 : Rs. 20 : : 27 : x.

$\therefore \frac{18}{20} = \frac{27}{x}$

$\therefore 18 \times x = 20 \times 27$

$\therefore \quad x = 30$

Or, the product of the extremes being equal to the product of the means,

$18 \times x = 20 \times 27$; i.e. $x = 30$.

∴ The number of yards required = 30.

335. Graphic solution.

Let each horizontal and vertical division denote 1 yard and 1 rupee respectively.

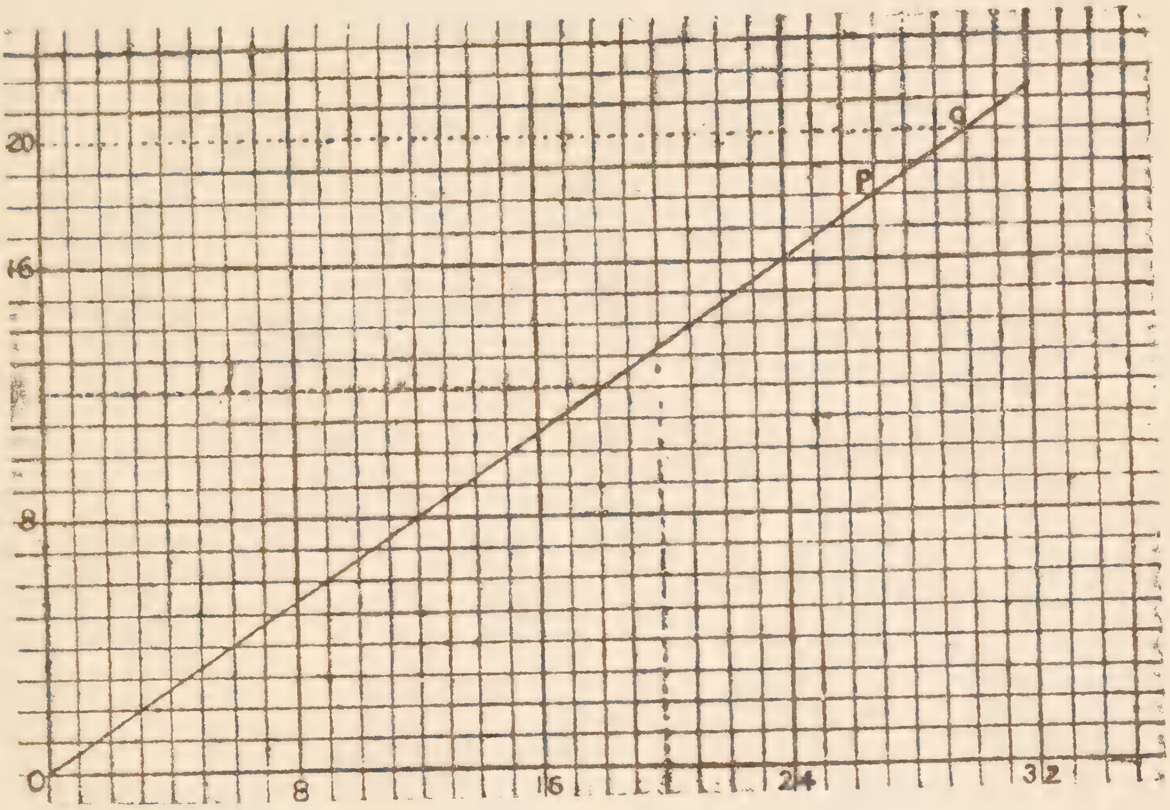

FIG. 65.

Since 27 yards can be bought for Rs. 18, plot the point P. (27, 18.) Join P with the origin by a straight line. Then OP is the graph.

To find how many yards Rs. 20 will buy, produce the graph OP to any length.

Then the point Q on the graph with ordinate 20 has for its abscissa 30.

∴ The number of yards required is 30.

Note.—This process is known as *exterpolation*.

Similarly for Rs. 12, the number of yards can be read off and is seen to be 18.

The cost of 20 yds. is seen to be Rs. $13\frac{1}{3}$.

336. We see that a larger sum of money buys a greater number of yards; and a smaller sum buys a smaller number of yards. The number of yards that can be bought is directly proportional to the sum of money laid out. This is therefore an example of **Direct Proportion**.

EXAMPLES.—ORAL.

Find by the method of multiplying ratios—

1. The length in number of cm. less than 15 in the ratio of 3 : 5
2. The No. of Rs. less than 24 in the ratio of 2 : 3.
3. The No. of mds. greater than 120 mds. in the ratio of 9 : 8
4. The area greater than 100 sq. ft. in the ratio of 7 : 5.
5. If 12 yds. of silk cost Rs. 15, find the cost of 16 yds.
6. If 5 inches represent 100 yds. in a certain plan, what does 3 in. represent ?
7. If 20 lbs. of tea cost £1 10*s.*, what is the cost of 17 lbs. ?
8. A bankrupt is able to pay only Rs. 9 for every Rs. 100 borrowed.

What will be received on a debt of Rs. 650 ?

EXAMPLES. IV.

Solve graphically—

1. If 25 yds. of cloth cost Rs. 4-11 as., what is the cost of 15 yds. ?

Use the graph to find the number of yards that can be bought for Rs. 10.

2. If 20 men of equal ability earn among them Rs. 15 15 as., how much do 12 of them earn ?

Hence find how many of them would earn Rs. 19 2 as.

3. If 5 kilograms are equal to 11 lb., find the number of kilograms in one cwt.

Use the graph to find the number of Kg. in a ton and the number of lbs. in 45·5 Kg.

4. If 18 gals. of beer cost £1 1*s.*, find the cost of 15 pints.

Solve the following by the method of multiplying ratio and verify the answers by any other method.

5. If the rent of 13 acres is Rs. 45 8 as., what is the rent of 85 acres ?

6. If 32 tons cost £400 4*s.*, find the cost of 18 tons.

7. If a man walks 68 miles in 4 days, in how many days will he walk 119 miles at the same rate ?

8. If 9 boys can do the work of 4 men, how many boys can do the work of 52 men ?

9. If 311 sheep sell for £585-1-4$\frac{1}{2}$, what is the value of 20 ?

10. If a man earns £191-12-6 a year, in what time will he earn £31 10*s.*

11. If the price of 2$\frac{2}{3}$ lbs. of tea be 15*s.* 4*d.*, what quantity can be bought for £4-0-6 ?

12. If 7 cwt. 26 lbs. cost £28 7*s.*, how much may be bought for £23-12-6 ?

Solve graphically—

13. If the yard is to the metre as 54 is to 59, find the number of metres in 100 yds.

14. Forty-five pages of a manuscript occupy 32 pages of print, how many pages of print would 100 pages of the same manuscript occupy ?

Hence find how many pages of the manuscript would occupy 60 pages of print.

15. In a graph 10 divisions represent 8 stone ; how many lbs. do 7 divisions represent ? How many divisions are represented by 10 stone ?

16. On a map, an inch represents 4 miles ; find the distance on the map which represents 15 miles. How many miles are represented by 6·75 inches ?

EXAMPLE. The weight of a copper wire is 3·2 lbs. to the yard. Find to the nearest tenth the number of kilograms in a metre, given 1 m. : 1 yd. = 35 : 32, and 1 Kg. = 2·2 lbs.

The metre being longer than the yard, the multiplying fraction is $\frac{35}{32}$.

∴ The weight of 1 m. = $3·2 \times \frac{35}{32}$ lbs. = 3·5 lbs.

Number of Kg. required $= 1 \times \dfrac{3·5}{2·2} = 1·6$ to the nearest tenth.

EXAMPLES. V.

To be worked in decimals—

1. The weight of a copper wire 400 ft. long is 13·08 lbs. If a piece 125 ft. long be cut out of it, what will be its weight to the nearest tenth of a pound?

2. A kilogram is 2·2 lbs.; find to the nearest tenth how many kilograms there are in 28 lbs.

3. A gallon of sea water weighs 10·27 lbs.; how many gallons are there in one cwt., to the nearest integer?

4. In a specimen of water, 1 gram contains 0·025 gram of dissolved matter, find the number of Kg. of the water that will give 1 Kg. of dissolved matter.

5. On a certain map 3·75 in. represents a distance of 80·5 miles. Find to the nearest tenth of an inch what length will represent 64·25 miles.

6. The height of the mercury barometer at the normal pressure of air is 760 mm. If 1 mm. = 0·03937 in., find to two decimal places the height in inches.

7. Find to the nearest gram the weight of 58·4 cubic cm. of silver. Given that equal volumes of silver and water are in the ratio of 10·5 : 1, and that 1 c. cm. of water weighs 1 gram.

8. The weight of a sheet of iron 4 ft. square is 4·8 lbs. Find the weight of a similar sheet 5·4 ft. square. (Answer correct to two decimal places.)

EXAMPLES. VI.

MISCELLANEOUS.

1. If 64 cm. of copper wire weighs 1·05 grain, what is the weight of 1 metre of the same wire?

2. If 53·625 metres of a wire of a certain diameter can be drawn out from 3·3 grams of gold, what length of similar wire can be drawn from 4·95 grams?

3. The maximum number of marks for a paper was fixed at 120. It was subsequently altered to 150. How many marks should a candidate now get, if formerly he had secured 94? Verify your answer graphically.

4. Six litres go to make 0·2 c. ft. How many litres are there in 1·75 c. ft.

5. 16 tons of coal cost £17 12*s*. and 11 tons cost £12 2*s*. Is the price proportional to the quantity?

6. If 1 cwt. of sugarcane produces 12 lbs. of sugar, what weight is required to produce 1 ton of sugar?

7. If the weight of a sheet of copper 4 ft. by 3 ft. be 120 lbs., what will be the area of a similar sheet of 1 ton weight?

8. If a cube of iron of 6 in. side weighs one ton and a half, find the weight of a block 3·5 c. ft in volume.

9. If one million cubic inches of air weigh 46·55 lbs., find to the nearest hundred the number of cubic inches weighing 21·6 lbs.

10. A person pays duty on 2 Kg. 620 g. of tobacco at 3*s*. 4*d*. per lb. If 1 Kg. = 2·2 lbs., find how much he pays.

11. A train goes up an incline of 1 in 120 at the rate of 30 miles an hour. How many feet has it risen at the end of 5 minutes?

12. A train takes 3 hrs. 48 min., including 12 min. stoppages, to travel a distance of 177 miles. What time will it take to go 118 miles without any stoppage, if the average rate of motion is the same?

INVERSE PROPORTION.

EXAMPLE. If 10 men can do a piece of work in 30 days, how many days will (1) 15 men, (2) 20 men, (3) 3 men, (4) x men of equal ability take to do it?

i. Solving by the Unitary Method.

10 men can do the work in 30 days.

$\therefore$ 1 man................................30×10 days.

$\therefore$ 15 men............................... $\frac{30 \times 10}{15}$ or 20 days.

$\therefore$ 20 men............. .... $\frac{30 \times 10}{20}$ or 15 days.

$\therefore$ 30 men............................... $\frac{30 \times 10}{30}$ or 10 days.

$\therefore$ x men............................... $\frac{30 \times 10}{x}$ days.

337. Since when the number of men *increases*, the corresponding number of days *decreases* and *vice versa*; this is not an example of direct proportion, but one of **inverse** proportion.

In a direct proportion, the *ratio* of each pair of values is equal; but in an inverse proportion, the *product* of each pair is equal.

In the above example, $10 \times 30 = 15 \times 20 = 20 \times 15 = 30 \times 10 = 300$

ii. By the method of multiplying ratio.

10 men can do the work in 30 days.

11....................................?

The number of men being *greater*, the work will be done in less time.

$\therefore$ The multiplying fraction (M.F.) is $\frac{10}{15}$,

$\therefore$ The number of days required is $30 \times \frac{10}{15}$ or 20 days.

iii. Solving as proportion.

Denoting the number of days required by x, we have $x : 30 = 10 : 15$.

$$\therefore \frac{x}{30} = \frac{10}{15} \qquad \therefore x = \frac{10}{15} \times 30 = 20.$$

338. Graph of Inverse Proportion.

Number of men at work	...	10	15	20	30
Number of days taken	...	30	20	15	10

Draw a graph to show how the two variables are related, and use it to find in how many days 25 men will do the work.

Take one horizontal division to denote 1 man, and one vertical division 1 day.

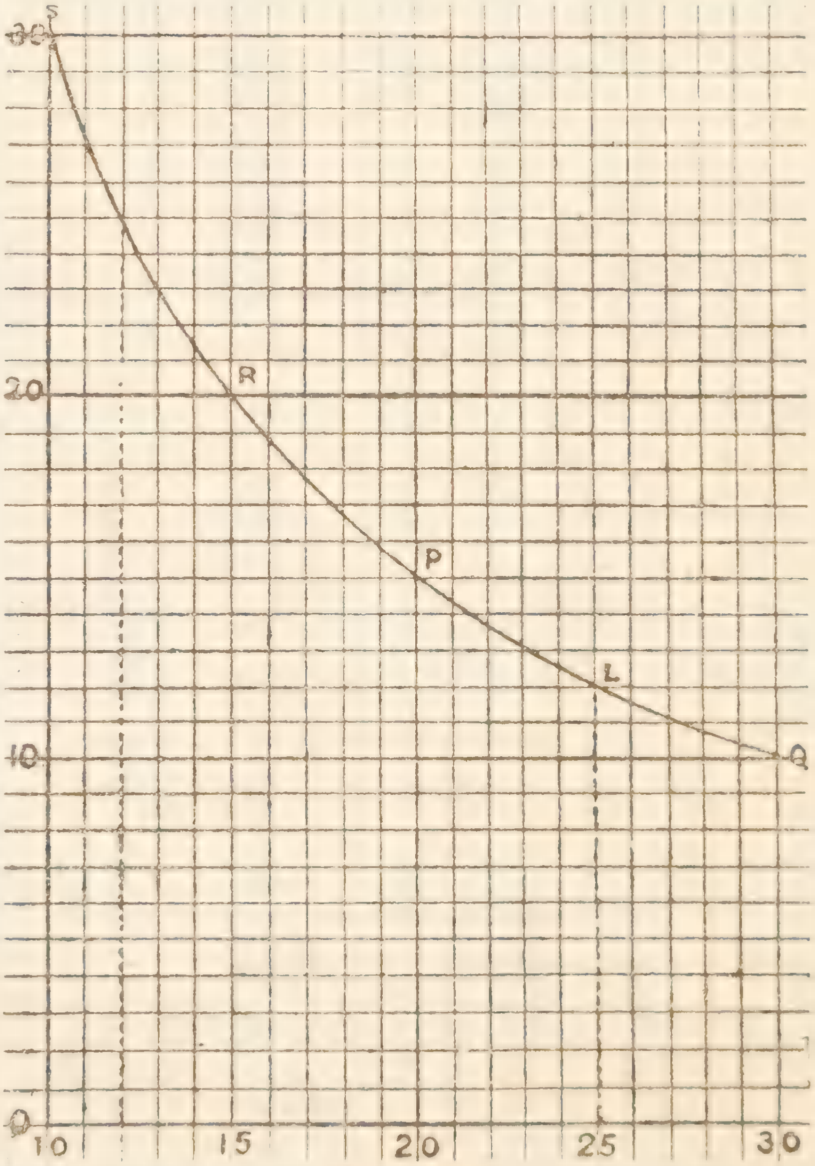

FIG. 66.

Plot the points S (10, 30), R (15, 20), P (20, 15) and Q (30, 10). Connect these points by a free hand curve and you get the graph required. Draw the vertical from abscissa 25 and it meets the graph at L, whose ordinate is 12.

∴ 25 men will do the work in 12 days. It is seen from the graph that 12 men are required to do the work in 25 days.

EXAMPLES.—ORAL.

Find in each case the M.F., and then give the answer.

1. If 15 men can reap a field in 5 hrs., in what time will double the number of men reap it ?

2. If 19 men can do a piece of work in 12 days, how long will three times the number of men take to do it ?

3. If 24 men can bale out the water in a well in 3 hrs., how long will 18 men require to bale it out ?

4. A man working 5 hrs. a day does a piece of work in 9 days ; how many hours a day must he work to finish it in $4\frac{1}{2}$ days.

5. If 8 men can dig a trench in 12 days, when they work 6 hrs. a day, in what time will they do it, when they work 8 hrs. a day ?

6. If when a soldier is allowed 15 oz. of bread a day, an army can hold out for 30 days, how long can it hold out when the daily allowance is 12 oz. ?

7. A person exchanged 20 cows for 14 oxen each worth Rs. 75. Find the price of a cow.

8. A room is 18 ft. long and 12 ft. wide ; how wide is another room of equal area, whose length is 16 ft. ?

9. If when wheat is 8*s*. a bushel, the penny-loaf weighs 6 oz., what ought it to weigh when wheat is 9*s*. a bushel ?

10. If a piece of cloth is 20 yds. long and 2 ft. 6 in. wide, how wide is another 15 yds. long and which contains as much cloth as the other ?

11. If 15 mds. be carried 40 miles for Rs. 6 4 as., how much ought to be carried for the same sum for 25 miles ?

Note.—Here Rs. 6 4 as. is of no consequence and should, therefore, be left out in the calculation.

12. If x men can do a piece of work in 10 days, how long will 5 x men take to do it ?

13. If 10 horses can plough a field in x hrs., in how many hours will 2 horses plough it ?

14. A train takes 12 hrs. to perform a journey at the rate of x miles an hour. What time will it take at $\frac{x}{2}$ miles an hour ?

EXAMPLES. VII.

1. If 15 cwt. 2 qrs. be carried 60 miles for Rs. 14 6 as., how far ought $3\frac{1}{4}$ cwt. to be carried for the same hire?

2. If an army can hold out for 15 days, if each soldier is allowed $1\frac{1}{4}$ lb. of bread a day, how long can it hold out if each is allowed $1\frac{1}{2}$ lb. a day?

3. A person exchanged 120 yds. of silk for 150 yds. of velvet at 1*s.* 6*d.* a yard. Find the price of silk per yard.

4. A man has a million dollars each worth 4*s.* 2*d.*, what is this in Indian money at 1*s.* 4*d.* per rupee?

5. If when rice sells at 4 as. a measure, the quarter-anna cake weighs one palam, what must it weigh when rice sells at 5 as. 4 p. a measure?

6. If 1 ton 12 cwt. be carried for 15*s.* for 36 miles, what weight must be carried for the same sum for 32 miles?

7. The front wheel of a carriage is 3 ft. high and the hind wheel 4 ft. high. If the latter revolves 480 times in going over a certain distance, how often will the former have revolved in the same distance?

8. How many sacks each containing 1·25 mds. of rice will be required for taking in 4,000 sacks of rice each containing $1\frac{1}{2}$ mds.?

9. A garrison of 1,200 men has provisions for 20 days; find by how many men it was reinforced, if the provisions were exhausted in 16 days.

10. A piece of work can be done by 15 men in 12 days; but some of the men being absent, the work was completed in 18 days. How many were absent?

11. A train travels a distance of 450 miles in 22 hrs. 30 min. How soon will a motor-car perform the same journey at the average rate of 25 miles an hour?

12. A cannon ball moving at the rate of 1,600 ft. per second hits the target in $5\frac{1}{2}$ sec. How long will it be before the sound is heard which travels at the rate of 1,100 ft. per second?

EXAMPLES. VIII.

Solve the following graphically :—

1. A wire 6 in. long can support without bending a weight of 3·6 Kg. When it is drawn out to 9 in., it supports 2·4 Kg., to 12 in. 1·8 Kg. and to 15 in. 1·44 Kg. What weight can it bear when it is 10 in. long ?

2. With 240 words in a page, a book comes up to 312·5 pages, with 300 words to 250 pages, and with 375 words to 200 pages. What will it come to with 350 words in a page ? (Answer to the nearest integer.)

3. When the pressure is 15 lbs. to the square inch the volume of a gas is 100 c. cm. ; when it is 12 lbs., 125 c. cm., 20 lbs., 75 c. cm., and 10 lbs., 150 c. cm. What is the volume when the pressure is 18 lbs. to the sq. in. ?

4. A mass of metal can be beaten out into a sheet 80 sq. ft. in area having a thickness of 1·5 in. ; into 120 sq. ft. with 1 in. thickness ; into 240 sq. ft., 0·5 in. thick, and 480 sq. ft., 0·25 in. thick. Find its area when the thickness is 0·375 in.

EXAMPLE. If 2,000 men have provisions for 95 days, and if after 15 days, 400 men go away, find how long the remaining provisions will serve the number left.

There are now only (2,000—400) men or 1,600 men, and the provisions should last for (95—15) days or 80 days. For 2,000 men, the provisions left would be sufficient for 80 days ; but since there are only 1,600 men, the provisions will last for a longer time.

∴ The M. F. is $\frac{2000}{1600}$.

$$\therefore \text{Required time} = \frac{2000 \times 80}{1600} \text{ days} = 100 \text{ days.}$$

EXAMPLES. IX.

1. If 1,000 men have provisions for 85 days, and if after 17 days 150 go away, find how long the remaining provisions will serve the number left.

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2. 20 men have to do a piece of work in 9 days; but at the end of 4 days, 5 men leave. How long will the work be in hand?

3. A ship having a crew of 26 persons, carries provisions for 21 days; after having been at sea for 11 days, they pick up a party from a wreck, and it is found that the provisions will be exhausted in 5 days; how many were taken from the wreck?

4. If 480 men have provisions for 16 days; and if after 6 days, some men leave, and if the remaining provisions now last for 16 days, find how many left.

EXAMPLES. X.

MISCELLANEOUS.

1. How many men can be kept in a famine camp at the rate of 1 an. 8 p. a head for the same sum as will support 120 men in jail at 2 as. 11 p. a head?

2. A train goes 20·2 miles in 48 min. How many kilometres is this per hour? 1 Km.=$\frac{5}{8}$ mile.

3. A pipe delivers 198·8 litres of water in 21 min. Find in kilograms the weight of water obtained in $1\frac{1}{2}$ hrs.

4. A pound of powder costs 4*s.* and the charge for a gun is $2\frac{1}{4}$ drams; how many shots will 6*s.* 9*d.* worth of powder furnish?

5. If 7·5 in. represent on a map 118·5 miles, what is the R.F. (in miles to the inch). What distance is represented by 3·5 in.?

6. If the weights of equal volumes of mercury and glycerine be as 11 to 1, what will be the height of a glycerine barometer, when the mercurial barometer stands at 29·45 in. (Answer to the nearest inch.)

7. The weight of a piece of wood 8·4 metres long is 100 Kg. What should a part 6·25 metres long weigh?

8. A steamer going at the rate of 18 knots per hour finishes her passage in 192 hours. How long would it take at the rate of 12 knots per hour?

9. How many oranges at £·084375 a dozen ought to be given for 378 eggs at ·0625*s.* each?

10. A tank is filled in 36 min. by a tap admitting 25 gals. of water a minute. How long would another tap admitting 40 gals. a minute take to fill it?

11. A borrowed of B, Rs. 400 for $6\frac{1}{2}$ months; afterwards A would requite B's kindness by lending him Rs. 910; how long should he lend it for?

12. 30 men can do a piece of work in 40 days; how many men assisted by 12 boys can do the same in 25 days, supposing that in a given time 2 men can do as much work as 3 boys?

13. If 3 cows or 7 horses can eat the produce of a field in 29 days, in how many days will 7 cows and 3 horses eat it up?

14. If 10·8 litres of wine is sold for £2-0-6, what is the value of one kilolitre?

15. 35 women can do as much work as 20 boys, and 16 boys as much as 7 men; how many women can do the work of 13 men?

16. If one cubic inch of lead weighs $6\frac{1}{2}$ oz., find the weight of one c. dm. of lead, it being given 1 c. cm. : 1 c. in. = 1 : 16·4.

17. If 24 men can finish a piece of work in 12 days; and if after 6 days, 16 men leave the work, in what time will the remaining men finish the work?

18. If in a town of 2,275 people, the average daily consumption of water is 23,318·75 gals., what would be the average daily consumption in a town of 3,120 people at the same rate?

19. If 6 horses and 5 cows consume 3 tons of hay in 15 days, how many tons of hay will be consumed by 10 horses and 15 cows in the same time, if 2 horses eat as much as 5 cows?

20. 18 men and 10 boys can do a certain work in 39 days; how long will 9 men and 15 boys take to do it, if the work of 3 men be equal to that of 5 boys?

CHAPTER XXIX.

COMPOUND PROPORTION.

339. Examples in Compound Proportion involve two or more applications of simple proportion.

In simple proportion, the answer is obtained, as we know, by multiplying the given term which is of the same kind as the answer by a single ratio (M.F.); but in compound proportion, the multiplying ratios (M.F.'s) are more than one, each of which affects the value of the required term.

Example 1. If 16 horses can plough 128 acres in 8 days, how many acres will 12 horses plough in 5 days?

By Unitary Method.

16 horses can in 8 days plough 128 acres.

1 horse ,, 8 days ,, $\frac{128}{16}$ acres.

12 horses ,, 8 days ,, $\frac{128 \times 12}{16}$ acres.

12 horses ,, 1 day ,, $\frac{128 \times 12}{16 \times 8}$ acres.

12 horses ,, 5 days ,, $\frac{128 \times 12 \times 5}{16 \times 8}$ acres or 60 acres.

By Multiplying Fraction.

16 horses in 8 days can plough 128 acres.

12 horses in 5 days can plough ?

12 horses can plough less number of acres than 16 horses.

∴ M.F. is $\frac{12}{16}$.

They can plough in 5 days, less number of acres than in 8 days.

∴ M.F. is $\frac{5}{8}$.

∴ The required number of acres is the product of 128 by $\frac{12}{16}$ and by $\frac{5}{8}$.

∴ Number of acres required $= 128 \times \frac{12}{16} \times \frac{5}{8} = 60$.

Observe that the answer is got by multiplying the given term, which is of the same kind as the answer, successively by the M.F. obtained from each pair of corresponding quantities.

340. Graphic Illustration.

Example. The length of a rectangle is 10 in. and the breadth 8 in. Find the area formed by increasing its length by 2 in. and decreasing the breadth by 2 in.

Let each division represent 1 in. Fig. 67.

In the figure, A represents the rectangle 10 in. by 8 in.; and its area is 80 sq. in.

Increase the length of A by 2 in., the breadth remaining unaltered and you get the rectangle B.

Increased length is to original length $= 12 : 10$ or $\frac{6}{5}$.

∴ Area of B = area of A $\times \frac{6}{5} = 80$ sq. in. $\times \frac{6}{5}$.

Decrease the breadth by 2 in. and you get the rectangle C. Diminished breadth is to original breadth $= 6 : 8$ or $\frac{3}{4}$.

$$\therefore \text{Area of C} = \text{Area of B} \times \tfrac{3}{4} = \text{Area of A} \times \tfrac{6}{5} \times \tfrac{3}{4}.$$
$$= 80 \text{ sq. in.} \times \tfrac{6}{5} \times \tfrac{3}{4} = 72 \text{ sq. in.}$$

The required area is obtained by multiplying the area of A successively by the fractions $\frac{6}{5}$ and $\frac{3}{4}$.

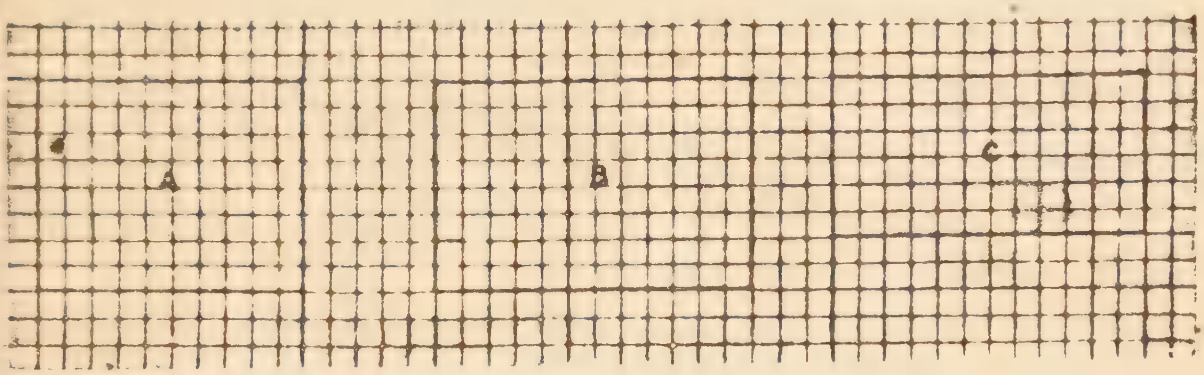

FIG. 67.

Practical Exercise.

Find Graphically the areas of the following rectangles, and shew how each illustrates the method of working an example in compound proportion.

1. 12 in. by 8 in. Length and breadth both increased by 4 in.; scale 0·1″ to 1″.

2. 30 ft. by 20 ft. Length increased by 6 ft. and breadth by 4 ft.; scale 1″ to 10′.

3. 8 yds. by 8 yds. Length increased by 2 yds. and breadth decreased by 2 yds.; scale 0·5″ to 1 yd.

4. 54 cm. by 36 cm. Length decreased by 6 cm. and breadth increased by 4 cm.; scale $\frac{1}{10}$ in. to 1 cm.

EXAMPLES. I.

1. If the railway fare for 10 persons for 444 miles is Rs. 46-4 as., what must be paid for 120 miles for 37 persons?

2. The cost of keeping 25 horses being at the rate of Rs. 113-0-4 per week, what will be the cost of keeping 13 horses for 61 days?

3. If the carriage of 1 cwt. 3 qrs. 21 lbs. for $52\frac{1}{4}$ miles costs Rs. 8-11-4, what will be charged for $2\frac{1}{2}$ tons for $46\frac{1}{2}$ miles?

4. If the carriage of 60 cwt. for 20 miles cost Rs. 14 8 as., what weight can be carried 30 miles for Rs. 5-7 as.?

5. If 24 lbs. of cotton make 115 yds. of cloth 1 yd. wide, how much cloth $1\frac{1}{4}$ yds. wide ought 12 oz. to make?

6. If 15 men can dig a trench in 100 days working 6 hrs. a day, how many hours a day must 8 men work to dig it in 120 days?

7. It costs Rs. 7 to support a family of 10 persons for 28 days, when rice sells at 7 measures for the rupee; how much will it cost, when rice is sold at 9 measures for the rupee, to keep a family of 15 persons for 48 days?

8. If 270 measures of rice are required to support 24 men for 30 days, allowing each man 3 ollocks a day, find how long 36 men can be supported on 540 measures, each man being allowed $2\frac{1}{2}$ ollocks.

EXAMPLE 2. If 2,000 men besieged in a town with provisions for 9 weeks, allowing each man 1 lb., be reinforced by 400 men more and have their daily allowance reduced to 12 oz., how long will the provisions last?

State thus:

There are now 2,000 men + 400 men or 2,400 men

For 2,200 men at 1 lb. each, the provisions will last for 9 wks.

For 2,400 men at 12 oz. each, the provisions will last for ?

(1) The third term is 9 weeks.

(2) More men, less time. M.F. $= \frac{2000}{2400}$.

(3) Less quantity being given, daily provisions last longer. M.F. $= \frac{16}{12}$.

$$\therefore \text{ Required time} = 9 \times \frac{2000}{2400} \times \frac{16}{12} \text{ wks.} = 10 \text{ wks.}$$

EXAMPLES. II.

1. If 1,000 men besieged in a town with provisions for 5 weeks, allowing each man 16 oz. a day, be reinforced by 500 men more, and have their daily allowance reduced to $6\frac{2}{3}$ oz., how long will the provisions last?

2. The inhabitants of a village numbering 1,200 have provisions which will last for 200 days, when each person is allowed 4½ ollocks a day. If 400 of them are sent to another village and the supply be made to last for 240 days, what should each man's daily allowance be?

3. A garrison 1,000 men had water-supply for 30 days allowing each man 2½ gals. a day. Shew that the water will be sufficient for the garrison for 20 days, if each man is allowed 3 gals., it being reinforced by 250 men.

4. If the six-penny loaf weighs 4·35 lbs. when wheat is at 5·75*s.* a bushel, what weight of loaf, when wheat is at 18·4*s.* a bushel, ought to be had for 18*s.* 1·6*d.* ?

EXAMPLE 3. If 24 coolies receive Rs. 58-8 as. for 6 days' work, what should 36 coolies and 8 boys receive for 8 days' work, supposing a boy to do ⅓ the work of a man?

Since 1 man's work = that of 3 boys;
24 men's work = that of 3 × 24 boys or 72 boys;
36 men's work = that of 3 × 36 boys or 108 boys;
36 men's + 8 boys' work = (108 + 8) boys' or 116 boys' work.

State thus:
72 boys get for 6 days' work Rs. 58½.
116 boys ,, 8 days' ,, ,, ?

(1) The 3rd term is Rs. 58½.
(2) More boys, more wages. M.F. $=\frac{116}{72}$.
(3) More days, more wages. M.F. $=\frac{8}{6}$.

$$\therefore \text{ Required sum} = \text{Rs. } 58\tfrac{1}{2} \times \frac{116}{72} \times \frac{8}{6} = \text{Rs. } \frac{377}{3} = \text{Rs. } 125\text{-}10\text{-}8.$$

EXAMPLES. III.

1. If 36 coolies receive Rs. 60-8 as. for 18 days' work, what should 48 coolies and 18 boys receive for 24 days' work supposing a boy to do ⅓ the work of a man?

2. The wages of 18 workmen amount to £79 4*s.* for 22 days; what will be the wages of 25 apprentices for 45 days if for every 2*s.* 6*d.* that a workman gets, an apprentice receives 2*s.*?

3. If 4 men earn as much in a day as 7 women, and 1 woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earning of 8 men and 6 women for 10 days ?

4. If 6 horses eat 3 tons of hay in 15 days, how long will it take 18 sheep to eat 5 acres of grass ; 2 horses consuming as much as 7 sheep, and one acre producing 3 tons of grass, each ton being as satisfying as $1\frac{1}{2}$ tons of hay ?

EXAMPLE 4. A wall 720 yds. long has to be built in 36 days ; and it is found that 24 men have completed only 180 yds. in 12 days. How many additional men must be employed that the wall may be finished in the time required ?

In 12 days, 24 men have built 180 yds.

Find how many men must be employed to complete the remaining 540 yds. in the remaining 24 days.

State thus :

In 12 days 180 yds. are built by 24 men.
In 24 days 540 yds. are built by ?
(1) 3rd term 24 men.
(2) More days, less men. M.F. $= \frac{12}{24}$.
(3) More yds., more men. M.F. $= \frac{540}{180}$.
∴ Number of men required $= 24 \times \frac{12}{24} \times \frac{540}{180} = 36$.
∴ Number of additional men reqd. $= (36 - 24)$ men $= 12$ men.

EXAMPLES. IV.

1. A wall 700 yds. long is to be built in 29 days. At the end of 11 days, 18 men had built only 220 yds. ; how many additional men should be employed to finish the work in time ?

2. 250 men are employed on a railway embankment, $1\frac{1}{2}$ mi. long, which they are expected to finish in 4 wks. But at the end of 1 week, it is found that they have only finished 520 yds. How many more men must be engaged to finish it in time ?

3. 21 men are employed to reap 72 acres in 12 days ; and it is found that in 5 days they have reaped 40 acres. How many men must be withdrawn to finish reaping just in time ?

4. 51 men are employed to do a piece of work in 15 days; in 10 days' time they finish $\frac{3}{4}$ of the work. How many should be withdrawn so that the work might be finished just in time?

EXAMPLES. V.

MISCELLANEOUS.

1. If 7 fires burning 10 hrs. a day consume 4 tons 10 cwt. of coal in 30 days, how much coal is consumed by 20 fires in 12 days burning for 14 hrs. a day?

2. How much paper is required for 2,000 copies of a book of 25 sheets, if 99 reams are required for 5,000 copies of a book of 9 sheets?

3. If with a pace of 2 ft. 9 in. I walk a mile in 20 min., how long will it take me to walk the same distance, if I quicken my steps in the proportion of 5 to 4, but decrease the length of my pace to 2 ft. 8 in.

4. A boat is propelled by 8 oars which take 10 strokes per minute; and it goes at the rate of 9 miles an hour. Find the rate of a boat propelled by 6 oars which take 8 strokes per min. when 5 strokes of the latter are equivalent to 6 of the former.

5. Some correspondence occupies $12\frac{1}{2}$ pages of a book. The page contains 2 columns of 63 lines, with an average of 9 words to the line. How many pages will be required, if the matter is set up in single columns, with 55 lines to the page and an average of 12 words to the line?

6. The wages paid in a weaving concern of 480 looms is £167 4*s*. for $5\frac{1}{2}$ days. What will be the wage-bill at the same rate for a concern of 330 looms for $8\frac{1}{2}$ days?

7. If 5 men can perform a piece of work in 12 days of 10 hrs. each, how many men will perform a piece of work four times as large in a fifth part of the time, if they work the same number of hours a day, supposing that 3 of the second set can do as much work in an hour as two of the first set?

8. A railway company, having reduced the second class fare from $1\frac{1}{2}$*d*. to $1\frac{1}{4}$*d*. per mile, found that the number of persons who travelled second class had increased in the proportion of 4 to 3, and that the average distance they travelled remained the same; if the total receipts from second class passengers amounted to £15,570 before the reduction, what did they amount to afterwards?

9. If 100 men can make an embankment 2 miles long in 20 days, how much overtime must 120 men work in order to finish an embankment 3 miles long in 24 days? Twelve hours is supposed to be a regular day's work.

10. If 12·25 sq. yds. of a fine sand filter can filter 1,200 gals. of water in 15 hrs., find the area of a similar filter that can filter 6,750 gals. of water in 20 hrs.

11. An iron plate 10·2 cm. long, 1075 cm. broad and 0·75 cm. thick weighs 2,040 grams; find the thickness of another iron plate 12·5 cm. long and 8·6 cm. broad, which weighs 2,500 grams.

12. An iron bar 5 ft. long, $2\frac{1}{2}$ in. wide and $1\frac{3}{4}$ in. thick weighs 48·5 lbs.; what will be the thickness of a bar of the same metal 7 ft. long, 2 in. wide, which weighs 77·6 lbs.?

13. Each page of a book of 560 pages contains 40 lines, and each line on an average 41 letters. How many pages will there be in another edition of the book consisting of pages of 60 lines, each line one quarter longer, and printed in type occupying more space on the line in the proportion of 6 to 5 than that used in the other edition?

14. A person can read a book containing 110 pages, each of which contains 30 lines and each line on the average 12 words, in $2\frac{3}{4}$ hrs.; how long will it take him to read a book having 200 pages, each of which contains 36 lines and each line on the average 15 words?

15. A book consists of $21\frac{3}{4}$ sheets of 16 pages, each page containing 38 lines; how many sheets of 24 pages will it come to, if each page contained 32 lines, the lengths of the lines being as 28 : 29?

16. If the carriage of 150 feet of wood, that weighs 3 stone per foot, cost Rs. 30 for 40 miles, how much will the carriage of 54 feet of wood, that weighs 8 stone per foot, cost for 25 miles?

17. A farmer engaged a number of men and boys to reap corn. The men were to receive each 4*s*. a day, and the boys 2*s*. 6*d*. If the work of 3 boys was equal to that of 2 men, and 4 men and 5 boys could reap 22 acres in 4 days, what sum should 6 men and 7 boys receive for reaping 48 acres?

18. A boat with 2,720 sq. ft. of sail spread travels 51 knots in $8\frac{1}{2}$ hrs. when the wind gives a pressure of 0·125 lb. per square foot. How far will it go in $2\frac{2}{3}$ hrs. with a sail area of 7,650 sq. ft. and a wind pressure of 0·5 lb. per sq. foot, assuming the speed to be proportional to the area of the sail and the wind pressure?

19. If it takes 3 min. to read over 2 pages of a book containing 30 lines in a page with an average of 10 words in a line, how many pages of another book can be read in 20 min., when there are 50 lines in a page and 12 words in a line?

20. An engine while driving machinery burns coal at the rate of 1 ton 10 cwt. in 6 hrs. When the machinery is not in motion, the con-

sumption of coal is only $\frac{2}{5}$ of this rate ; how much coal will the engine burn in 150 hrs. during $\frac{1}{12}$ of which the machinery is at rest ?

CHAPTER XXX.

PROPORTIONAL DIVISION.

341. When a given quantity is divided into several parts, if the ratios between the parts are known, the parts themselves may easily be determined.

If, for example, Rs. 10 is divided into two parts in the ratio of 2 : 3, it is divided into 5 parts in all ; the first part contains $\frac{2}{5}$ of Rs. 10 or Rs. 4, and the second part contains $\frac{3}{5}$ of Rs. 10 or Rs. 6.

Or, if Rs. 69-5-9 has to be divided among 3 persons A, B and C into 3 parts proportional to $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$, the ratios may be made integral by multiplying the fractions by 12, the L.C.M. of the denominators.

Thus $\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = 6 : 8 : 9$. Total No. of parts $= 23$.

$\therefore$ A gets $\frac{6}{23} \times$ Rs. 69-5-9 or $6 \times$ Rs. 3-0-3 ; i.e. Rs. 18-1-6.

B gets $\frac{8}{23} \times$ Rs. 69-5-9 or $8 \times$ Rs. 3-0-3 ; i.e. Rs. 24-2-0.

C gets $\frac{9}{23} \times$ Rs. 69-5-9 or $9 \times$ Rs. 3-0-3 ; i.e. Rs. 27-2-3.

EXAMPLE 1. An alloy is made of copper and zinc in the ratio of 90 : 10. Find the weight of each in 1 cwt. 8 lbs. of the alloy.

$90 : 10 = 9 : 1$.

In $9 + 1$ or 10 parts of the alloy, 9 parts or $\frac{9}{10}$ is copper and $\frac{1}{10}$ is zinc.

$\therefore$ In 1 cwt. 3 lbs. or 120 lbs., $\frac{9}{10} \times$ 120 lbs. or 108 lbs. is copper and $\frac{1}{10}$ of 120 lbs. or 12 lbs. is zinc.

EXAMPLES.—ORAL.

1. Divide Rs. 15 between A and B in the ratio of 2 : 3.

2. Divide £63 into two parts in the ratio of 3 : 4.

3. Water consists of hydrogen and oxygen in the ratio of 1: 8 by weight. Find the weight of each in 18 grams.

4. Air is a mixture of oxygen and nitrogen in the ratio of 21 : 79 by volume. Find the quantity of each in one litre of air.

5. The two adjacent sides of a rectangle are in the ratio of 5 : 4 ; and the sum of the two sides is 27 inches. Find the length and breadth.

6. In a mixed school the number of boys and girls are in the ratio of 17 : 3. Find the number of girls, if the strength of the school is 120.

7. Divide 24 plantains among three boys in parts proportional to 1, 2 and 3.

8. A profit of Rs. 1,000 is divided among three partners in the ratio of 2 : 3 : 5. What is each man's share.

9. The perimeter of a triangle is 30 cm. If the sides are proportional to 4, 5 and 6, find the length of each side.

EXAMPLES. I.

1. Luggage weighing 20 mds. 25 srs. is put into two vans, A and B, in the ratio of 12 : 13. Find the weight in each.

2. The amounts contributed to a certain business by A and B are in the ratio of $1\frac{1}{2}$ to 0·03. The year's profits amount to £1,308 3*s*. What should each get?

3. Divide 1,000 plantains among A, B and C, so that for every 10 that A gets B may get 9 and C, 6.

4. Divide Rs. 650 between A, B and C in the proportion of $2\frac{1}{2}$, $1\frac{2}{3}$ and $1\frac{1}{4}$.

5. In a company of 266 persons there are 4 times as many men as children, and twice as many women as children. How many of each are there?

6. An annual tax of Rs. 2,255 is levied on 4 villages A, B, C and D, and the rate to be paid by each of the villages A, B and C is to the rate to be paid by D as 3 : 2. What are the annual payments due from the villages?

7. Rupees 580 is to be raised from 4 villages containing 250, 300, 400 and 500 inhabitants, in proportion to the population. What has each to pay?

8. A punitive police force of 750 has to be distributed among 4 villages in proportion to their populations, viz. 1,215, 1,755, 2,835 and 4,320 respectively. Find the number in each.

9. A sovereign consisting of gold and copper in the ratio of 22 : 2 weighs 123·274 grains. Find to the nearest grain the amount of pure gold in 1,000 sovereigns.

10. English gunpowder is a mixture of nitre, charcoal and sulphur in the ratio of 75 : 15 : 10. How much of each is required to manufacture 1 Kg. ?

11. A can do a piece of work in 6 days, B in 9 days and C in 12 days. They work together and are paid Rs. 29 4 as. What does each get ?

12. For a work that A does in 7 hrs., B requires 9 and C 8 hrs. If together they finish a job which fetches Rs. 11 15 as., how much do B and C together get more than A ?

13. Divide Rs. 480 between A and B, giving A Rs. 20 more than twice as much as B.

14. Divide Rs. 600 between A, B and C, so that A may have Rs. 40 more than B, and B twice as much as C.

15. Nitre and sulphur are mixed together in a mass of 120 lbs. in the proportion of 7 : 3. How much nitre must be added to the mass so that the proportion may be 11 : 4 ?

16. One cubic yard of concrete consists of three 2 cwt. bags of cement, $\frac{1}{2}$ c. yd. of sand and 1 c. yd. of stone. Find the amount of materials required for a trench 40 ft. long, 4 ft. 9 in. wide filled with concrete to a depth of 18 inches.

EXAMPLE 2. If for the month of March a certain number of coolies receive Rs. 467-14-6 as wages, the men receiving 5 as. 6 p. and the women 3 as. a day each, and there are twice as many women as men, find the number of women.

March has 31 days.

∴ The daily wages of the men and women = Rs. 467-14-6 ÷ 31.
= Rs 15-1-6 = 483 half-annas.

By the question, for every man there are 2 women.

The daily wages of 1 man is 5 as. 6 p. ; and of that 2 women is 6 as.

The total daily wages = 23 half-annas.

23 half-annas = sum received by 1 man and 2 women.

∴ 483 half-annas = sum received by 21 men and 42 women.

∴ No. of women required = 42.

EXAMPLES. II.

1. In a week of 6 working days, a certain number of coolies receive Rs. 252. If a man earns Re. 1 4 as. and a boy 12 as. a day, and there are thrice as many boys as men, find the number of boys.

2. In the month of April a certain number of men and twice as many women together earn Rs. 300; each man earns 12 as. and each woman 4 as. a day. Find the number of women.

3. A certain number of sheep and four times as many hens together cost Rs. 89-6 as. If a sheep cost Rs. 4-4 as. and a hen 10 as. 6 p., how many of each were bought?

4. A certain number of pagodas and 5 times as many half-rupees amount to Rs. 102. Find the number of each coin.

EXAMPLE 3. Divide Rs. 480-6-8 among 7 men, 9 women and 3 boys, so that each woman may have $\frac{4}{5}$ of a man's share and each boy $\frac{3}{4}$ of a woman's share.

$\frac{4}{5}$ of a man's share = 1 woman's share.
∴ 1 man's share = $\frac{5}{4}$ of a woman's share.
∴ 7 men's shares = $\frac{5}{4} \times 7$ or $\frac{35}{4}$ of a woman's share.
1 boy's share = $\frac{3}{4}$ of a woman's share.
∴ 3 boys' shares = $\frac{3}{4} \times 3$ or $\frac{9}{4}$ of a woman's share.
∴ The shares of 7 men, 9 women and 3 boys
$= (\frac{35}{4} + 9 + \frac{9}{4})$ women's = 20 women's shares.

20 women's shares = Rs. 480-6-8.

$$\therefore 1 \text{ woman's share} = \text{Rs. } \frac{480\text{-}6\text{-}8}{20} = \text{Rs. } 24\text{-}0\text{-}4.$$

∴ $\frac{5}{4}$ of a woman's or 1 man's = Rs. 24-0-4 × $\frac{5}{4}$ = Rs. 30-0-5.
∴ $\frac{3}{4}$ of a woman's or 1 boy's = Rs. 24-0-4 × $\frac{3}{4}$ = Rs. 18-0-3.

EXAMPLES. III.

(1 *to* 4 *to be taken orally.*)

1. Divide Rs. 80 among 3 men and 5 women, giving each man five times as much as a woman.

2. Divide Rs. 1,000 among 30 men and 30 women, giving each man three times as much as each woman.

3. Divide £10 5*s*. among 7 boys and 9 girls, giving each girl one-third as much as a boy.

4. 12 sheep and 15 goats together cost Rs. 138. If a goat costs half as much again as a sheep, find the value of each.

5. Divide Rs. 500 among 20 men, 30 women and 40 children, so that a man may have twice as much as a woman, and that a woman may have thrice as much as a child.

6. A gentleman distributed £41-5-0 among 12 men, 16 women and 30 children; to every man he gave twice as much as to a woman, and to every woman three times as much as to a child. What did each receive?

7. Divide £1,400 among A, B and C, so that A may have thrice as much as B, and that B may have five times as much as C.

8. Divide Rs. 568 among A, B and C, so that $\frac{2}{5}$ of A's share may be equal to $\frac{1}{6}$ of C's, and $\frac{1}{2}$ of B's may be equal to $\frac{2}{3}$ of A's.

9. Divide Rs. 2,882-8-0 among 42 men, 54 women and 18 boys, so that each woman may have $\frac{4}{5}$ of a man's share, and each boy $\frac{3}{4}$ of a woman's share.

10. Divide £2-13-8 among 7 men, 5 women and 17 boys, giving each woman $\frac{2}{3}$ of each man's share, and each boy $\frac{2}{3}$ of each woman's share.

11. Divide Rs. 180 among 15 men, 20 women and 30 children, so that a man and a child may receive as much as 2 women, and all the women may get Rs. 60.

12. Divide £5-16-10$\frac{1}{2}$ among A, B and C, so that for every 5*d*. which A receives, B may receive 6*d*., and C half as much more as A and B together.

EXAMPLE 4. A bag contains Rs. 57-13-9. There are twice as many rupees as there are pies, and 1$\frac{1}{2}$ times as many 4-anna pieces as there are rupees. How many coins of each sort are there?

For every 1 pie the bag contains, there are 2 rupees and $1\frac{1}{2}$ times 2 or three 4-anna pieces.

Ratio of the number of coins = 1 : 2 : 3.
Ratio of their values in pies = 1 : 384 : 144.
Total parts = 529.
The bag contains Rs. 57-13-9 or 11,109 pies.

∴ Number of pies = $\frac{1}{529}$ × 11,109 or 21 ; number of Rs. = 42 and number of four-anna pieces = 63.

EXAMPLES. IV.

1. A bag contains Rs. 93 12 as. There are thrice as many half-rupees and five times as many quarter-rupees as there are rupees. How many coins of each sort are there ?

2. A purse contains Rs. 37 8 as. There are twice as many 4-anna pieces as there are rupees, and $1\frac{1}{2}$ times as many 2-anna pieces as there are 4-anna pieces. Find the number of each coin.

3. I have in hand £40 10*s*. There are half as many half-guineas as there are half-sovereigns ; and the number of half-crowns is 4 times the number of half-guineas. Find the number of each coin.

4. A certain number of carpenters, half that number of masons and $\frac{3}{4}$ that number of smiths are paid Rs. 1,425 for a month of 30 days. The wages of a carpenter is 10 as., of a mason 6 as., and of a smith 8 as. a day. How many are there of each ?

EXAMPLE 5. A horse-keeper was engaged for a month of 30 days for Rs. 5 and 21 meas. 3 oll. of rice ; he left at the end of 10 days and received 2 as. 8 p. and the 21 meas. and 3 oll. of rice. How many measures of rice did the master consider as equal in value to a rupee ?

The horse-keeper was engaged for 30 days , but he worked only for 10 days or $\frac{1}{3}$ of the time.

∴ He was entitled to $\frac{1}{3}$ of his pay and $\frac{1}{3}$ of the rice.

i.e.......................... $\frac{1}{3}$ of Rs. 5 or Rs. 1-10-8 and $\frac{1}{3}$ of 21 meas. 3 oll. or 7 meas. and 1 ollock of rice.

But he got 21 meas. 3 oll. and 2 as. 8 p.

∴ (Rs. 1-10-8 — 2 as. 8 p.) or Re. 1 8 as. was considered equal in value to (21 meas. 3 oll. — 7 meas. 1 ollock) or 14 meas. 2 oll.

Since Rs. 1-8 or Rs. $\frac{3}{2}$ was considered = 14 meas. 2 oll.

Re. 1...................... = $\frac{2}{3}$ × 14 meas. 2 oll.

= 9 meas. 4 oll. or $9\frac{1}{2}$ meas.

EXAMPLES. V.

1. A horse-keeper was engaged for 8 months for Rs. 38 and some measures of rice; but he left at the end of 5 months and received Rs. 19 and all the rice. Find the value of the rice.

2. A servant was engaged for 12 months for 6 guineas and some clothes: he worked for 4 months only and got £1-5-6 and the clothes. What were the clothes worth?

3. A servant was engaged for 10 months for Rs. 37 8 as., and 250 measures of rice. He left at the end of 7 months and received Rs. 13 12 as. and all the rice. How many measures were considered equivalent to one rupee?

4. A cook was hired for Rs. 30-8-0 and 360 measures of rice for the year 1884. He worked from the first of January to the end of the last day of August, and received Rs. 9-5-4 and 350 measures of rice. How many measures of rice were considered equivalent to one rupee?

CHAPTER XXXI.

MISCELLANEOUS PROBLEMS.

EXAMPLE 1. A grocer exchanges 320 bags of coffee each containing 1 cwt. 2 qrs. 14 lbs. at 2*s*. a lb. for 256 bags of sugar each containing 1 cwt. 7 lbs. together with £4,554-13-4. Find the price of a lb. of sugar.

The value of the coffee at 2*s*. a lb. = 320 × 182 × 2*s*. = £5,824.

Also the value of 256 × 119 lbs. of sugar + £4,554-13-4 = £5,824.

= The value of 256 × 119 lbs. or 30464 lbs. of sugar alone.

= £5,824 — £4,554-13-4

= £1,269-6-8 = 304640*d*.

∴ The value of 1 lb. of sugar = 10*d*.

EXAMPLES. I.

1. A bookseller exchanged 250 copies of Euclid worth Re. 1 12 as. each for 175 copies of Algebra and Rs. 43 12 as. What is the price of a copy of Algebra?

2. A trader exchanged 120 mds. of jaggery at $6\frac{1}{4}$ as. a viss for 1 ton 2 cwt. of sugar together with Rs. 82. Find the price of a lb. of sugar.

3. A dealer exchanged 120 chairs, each worth Rs. 6 4 as., for 80 tables each worth Rs. 8 2 as., and 16 boxes. Find the price of a box.

4. A person exchanged 1 cwt. of coffee at 2*s.* 3*d.* a lb. for 100 lbs. of tea at 2*s.* 6*d.* a lb. and some sugar worth 6*d.* a lb. How many lbs. of sugar did he get?

EXAMPLE 2. Four lbs. of tea are worth 9 lbs. of coffee and 3 lbs. of coffee are worth 5 lbs. of sugar; how many pounds of tea can be bought for Rs. 5 10 as., if a lb. of sugar be worth 4 as.?

3 lbs. of coffee = 5 lbs. of sugar.
∴ 1 lb. ... ... = $\frac{5}{3}$ lb. of sugar.
∴ 9 lbs.... ... = $\frac{5}{3} \times 9$ lbs. or 15 lbs. of sugar;
and 4 lbs. of tea = 9 lbs. of coffee.
∴ 4 lbs. ... ... = 15 lbs. of sugar.
∴ 4 lbs. of tea = 15 × 4 as.
∴ 1 lb. ... ... = 15 as.
Since 15 as. is the cost of 1 lb. of tea.
1 an. is the cost of $\frac{1}{15}$ lb. of tea.
∴ Rs. 5-10 as. or 90 as. is the cost of $\frac{90}{15}$ lbs. of tea or 6 lbs. of tea.

EXAMPLES. II.

1. If ·75 of the value of an ox = ·875 of the value of a cow, how many oxen can be bought for Rs. 1,260, if a cow is worth Rs. 54?

2. Four lbs. of tea are worth 9 lbs. of coffee, and 6 lbs. of coffee are worth 15 lbs. of sugar; how many lbs. of sugar are equal to 56 lbs. of tea?

3. If 2 horses cost the same as 7 bulls, and 3 bulls the same as 5 sheep, and if the average price of a sheep is Rs. 2-7-2$\frac{2}{5}$, what is the value of 120 horses?

4. The price of an orange is $\frac{2}{3}$ that of a mango, and the price of 11 mangoes is the same as that of 18 apples. If 4 apples cost 3 as. 1 p., find how many oranges can be bought for Rs. 6-5-9.

EXAMPLE 3. I distribute a sum of money among a certain number of boys, and find that if I give Rs. 5 each, I have not money enough by Rs. 8; but if I give Rs. 4 each, Rs. 6 remains. Find the number of boys, and the sum of money I have.

If I give Rs. 5 each, I have to pay all that I have and I want Rs. 8 more. But if I give Rs. 4 each, there is a remainder of Rs. 6, and I save in addition the Rs. 8, which I wanted: that is, in all I save Rs. (8 + 6) or Rs. 14.

Since Rs. (5 − 4) or Re. 1 is saved from what is given to 1 boy,
Rs. 14..............................14 boys.

∴ There are 14 boys; and I have Rs. 4 × 14 + Rs. 6; that is, Rs. 62.

EXAMPLES. III.

1. A person meeting a certain number of beggars gave them 2 as. each and had 8 as. left. If he gave them 2 as. 6 p. each, he would require 12 as. more. How many were there?

2. A person gave 2 as. 8 p. each to a certain number of beggars and had 10 as. 8 p. left. He found that he would require 8 as. more to enable him to give them 4 as. each. How many were there, and how much had he with him?

3. I have a certain sum of money to be distributed among a number of people. If I give them 5 as. each, I shall spend Rs. 2 too little; but if I give them 6 as. each, I shall spend Re. 1 too much. What sum have I?

4. A dealer buys 80 tons of coal, and after selling them at 1*s*. 6*d*. per sack, finds that he has gained £4. Had he sold them at 1*s*. 4*d*. per sack, he would have lost £6. Find the number of sacks, and the weight of each.

Example 4. I engage a carpenter for 60 days on condition that he should get 10 as. 8 p. for every day he works and forfeit 7 as. 4 p. for every day he is absent. In the end he was paid Rs. 23 2 as. Find the number of days he was absent.

If he worked 60 days he should get 60 × 10 as. 8 p. or 640 as. But he gets only Rs. 23 2 as. or 370 as.

∴ He loses (640 — 370) as. or 270 as.

If he is absent, he forfeits 7 as. 4 p., in addition to his daily pay, i.e. he loses in all 10 as. 8 p. + 7 as. 4 p. or 18 as. a day.

Since he loses 18 as. if he is absent 1 day.

he loses 1 anna if he is absent $\frac{1}{18}$ day.

∴ He loses 270 as. if he is absent $\frac{1}{18} \times 270$ days or 15 days.

EXAMPLES. IV.

1. I engage a man for 25 days on condition that he should receive 8 as. for every day he works and forfeit 4 as. for every day he is absent. He received in all Rs. 8 12 as. How many days was he absent?

2. A man was hired for 40 days, agreeing to get Re. 1 10 as. for each day he worked and pay a fine of 10 as. for every day he did not work; at the end of 40 days he got Rs. 29. How many days was he idle?

3. A man was engaged for a month of 30 days on the understanding that he is to get 12 as. for each day he worked and to lose 8 as. for every day he was absent. At the end of the month he received Rs. 15. How many days did he work?

4. A carpenter was engaged for 50 days on condition that he should receive 14 as. for every day he worked and lose a certain sum of money every day he was idle. He received Rs. 19 8 as., having been idle 24 days. What did he lose on each day he was idle?

Example 5. A gentleman distributed in charity 100 shillings among 100 women and children, giving to each woman 14*d*. and each child 9*d*. How many women and how many children were there?

If all were women, he would have to give 100 × 14*d*. or 1,400*d*.
But he had only 100*s*. or 1200*d*.
∴ He would require 1400*d*.—1200*d*. or 200*d*. more.
Since 14*d*.—9*d*. or 5*d*. is the saving on 1 child.
1*d*.......................$\frac{1}{5}$ child.
200*d*.......................$\frac{200}{5}$ or 40 children.
∴ There were 40 children and (100—40) or 60 women.

EXAMPLES. V.

1. A gentleman, on the occasion of the marriage of his daughter, distributed Rs. 650 among 1,000 persons consisting of men and women Each man received 12 as. and each woman 8 as. Find the number of men and women.

2. 100 boys and girls subscribe Rs. 16 14 as. Each boy subscribes 3 as., and each girl 2 as. 3 p. How many boys were there and how many girls ?

3. A person bought 120 copies of Euclid and Algebra for Rs. 250. The price of a copy of Euclid is Rs. 2 12 as. and that of a copy of Algebra Rs. 1 8 as. How many of each did he buy ?

4. On a certain railway the second and third class fares are 7 pies and 3 pies respectively. A man who travels 100 miles and possesses Rs. 2-6-6 spends it in going part of the distance second class and the remainder third class. How far did he travel in second class ?

ONE-HOUR PAPERS—THIRD SERIES.

REVISION EXERCISES.

I.

1. A mass of pure silver weighs as much as Rs. 25. The mass together with Rs. 15 weighs $1\frac{1}{2}$ seers. What is the weight of the mass of silver ?

2. For Rs. 3-8-8 you can buy 30 mangoes and 50 guavas. The price of a mango and a guava is 1 a. 9 p. Find the price of each.

3. How many teak plants 14 ft. 6 in. long and $9\frac{1}{2}$ in. broad will be required for the floor of a hall 76 ft. long and 29 ft. broad? Draw a diagram to shew how the planks must be placed to prevent wastage

4. The cost of feeding 8 men, 6 women and 15 children for 30 days comes to Rs. 105. For how many days can 27 men, 21 women and 45 children be fed for Rs. 626-1-0, it being assumed that 2 men eat as much as 3 women, and 2 women as much as 5 children ?

5. A closed packing case measuring externally 32, $25\frac{1}{2}$ and $15\frac{1}{2}$ inches, the wood being $\frac{3}{4}$ in. thick, weighs 75 lbs. Find the weight of a cubic foot of the same wood.

II.

1. One hundred coolies, men and women, earn Rs. 95-14-2 in a week (7 days). The daily wages of a man and a woman are 2 as. 6 p. and 1 a. 8 p. respectively. How many women are there ?

2. A room whose height is 11 ft. and length twice its breadth takes 143 yds. of paper 2 ft. wide for its four walls. How much carpet will it require ?

3. If 1869 sovereigns are coined at the mint for 40 lbs. troy of standard gold, and 1,939 oz. at the mine are worth £7,000, find the ratio of the value of gold at the mint to that at the mine. (12 oz. = 1 lb. Troy.)

4. If two carts with two bullocks each can carry as much as five carts with one bullock each, and if a cart with one bullock can carry as much as 7 men, how many carts of two bullocks will be required to carry $59\frac{1}{2}$ candies, assuming that a man can carry 4 mds. ?

5. Space passed by a falling body is given in the following table :—

No. of seconds	1	2	3	4	5
Space in feet	16	64	144	256	400

Draw a graph and from it find the number of feet passed through in 3·5 sec.

III.

1. If a gallon contains 277·29 c. in., how many gallons are there in a rectangular cistern, 13′ by 7′ 6″ by 6′ 7″ ?

2. In a certain village containing 1,500 inhabitants, the supply of rice is just sufficient to allow a measure a day to each inhabitant for three weeks. A number of people are, however, sent to the village for relief from another district ; when it is found that by giving each person only $\frac{3}{4}$ of a measure a day, the supply will now last for exactly 20 days. How many additional people were sent to the village ?

3. A man engaged a cooly for a month of 30 days for Rs. 6 4 as. and 20 measures of rice ; he left at the end of 6 days and received 12 measures of rice and 4 as. How many measures of rice were considered equivalent to one rupee ?

4. If 40 lbs. troy of standard gold is coined into 1,869 sovereigns, and the proportion of pure gold to alloy in standard gold is 22 : 2, find to the nearest grain the weight of pure gold in a sovereign. (1 lb. troy contains 5,760 grs.)

5. If 24 oxen plough 6 acres in 10 days working 8 hrs. a day, how many acres will 20 horses plough in 15 days working 6 hrs. a day, supposing 3 horses can do as much as 10 oxen?

IV.

1. If 8 lbs. of coffee cost as much as 5 lbs. of tea, and 7 lbs. of coffee as much as 40 lbs. of sugar, what is the price of each per lb., when 1 lb. of tea, of coffee, and of sugar together cost 4*s*. 7½*d*.?

2. A and B can together do a piece of work in 30 days; at the end of 18 days, B goes away and A finishes it in 20 days. If Rs. 20 is paid for the work, what does each get?

3. If a certain number of workmen can do a piece of work in 25 days, in what time will $1\frac{2}{3}$ of that number do a piece of work twice as great; supposing that 2 of the first set can do as much in an hour as 3 of the second set in $1\frac{1}{2}$ hrs., and that the second set work half as long a time as the first set?

4. The water in a tank 16 ft. long and 7 ft. wide is 4 ft. deep. If a cubic foot of water weighs 1,000 oz., find in tons the weight of water in the tank.

5. Find the cost of matting the floor of a room 8 yds. by $7\frac{1}{2}$ yds. with mats 9 ft. by 2 ft. at the rate of 9 as. 6 p. per mat. How should the mats be spread so that the wastage may be the least?

V.

1. A bill for 37 lbs. of beef and 24 lbs. of mutton amounted to Rs. 21-7-2; the mutton cost 1 an. 2 p. more per lb. than the beef. Find the price of the beef per lb.

2. Fifteen horses having four feeds a day can be kept for two months for 16 guineas; what will be the cost of keeping 20 cows for five months giving them 3 feeds a day (a horse's feed being $\frac{5}{9}$ of a cow's)?

3. The price of 1 lb. of tea and 9 lbs. of sugar is Rs. 1-9-6; and for Rs. 2-5-6 may be bought 1 lb. of tea and 15 lbs. of sugar. If the cost of 3 lbs. of coffee is the same as that of 20 lbs. for sugar, for how much is coffee sold per lb.?

4. If A's money is to B's as 3 : 7, and if B pays A Rs. 345, the ratio becomes 3 : 5, find how much money each had.

5. A gallon of water weighs 10 lbs., and a cubic foot of water weighs 1,000 oz. If a rectangular cistern 8 ft. by 6 ft. contains 240 gals., find the depth of the water in the cistern.

VI.

1. Find by Practice the value of 37 bags of coffee each containing 6 mds. 5 viss 35 pals. at Rs. 9-6-6 a maund.

2. Two ducks are worth 3 chickens, and 2 geese cost the same as 8 ducks; what is the price of 5 geese, 4 ducks and 3 chickens, if 2 chickens are bought for 11 annas?

3. If 24 men in $2\frac{1}{2}$ days of $12\frac{1}{2}$ hrs. each can dig a trench 139·75 yards long, $4\frac{1}{2}$ yds. wide, and $2\frac{1}{2}$ yds. deep, what length of trench will 90 men dig in $4\frac{1}{5}$ days of $9\frac{2}{3}$ hrs. long, the trench being $4\frac{7}{8}$ yds. wide and $3\frac{1}{5}$ yds. deep?

4. The cost of feeding a horse for a week, when gram is 10 p. a measure, is Re. 1 4 as. more than when gram is 7 p. a measure. What will it cost when gram is 9 p. a measure?

5. A room is 19 ft. 5 in. long and 16 ft. 7 in. broad, and the cost of painting the walls at 7 as. 6 p. per sq. yd. is Rs. 43 3 as. Required the height of the room.

VII.

1. For a certain sum of money you can buy 40 mangoes or 90 oranges. If the price of a mango and an orange is 3 as. 3 p., find the price of each.

2. A workman was hired for 40 days at 3 as. 4 p. a day for every day he worked; but with this condition that for every day he did not work he was to forfeit 1 anna 4 pies; and on the whole he received Rs. 2-12-8. How many days did he work?

3. If a certain quantity of provisions serve 1,850 men for 36 days at the rate of $2\frac{1}{4}$ lbs. a day for each man, how long ought the same provisions to last if the number is increased by 370, and the rate decreased by $\frac{3}{4}$ lb. a day?

4. Divide Rs. 925 among A, B, C and D, giving B $\frac{11}{12}$ of A's portion, C $\frac{3}{8}$ of B's, and D half as much as B and C together.

5. A litre of water weighs one kilogram; what is the weight of five cubic centimetres of mercury, it being known that mercury is 13·6 times as heavy as water?

VIII.

1. Find the number of cubic inches in 10 c. cm. if 1 c. in. is equal to 0·0164 litre.

2. If 20 men, 40 women and 50 children receive Rs. 350 for 7 weeks' work, all working 8 hrs. a day; and if 2 men receive as much as 3 women or 5 children, what sum will 30 women earn in 3 weeks working 6 hrs. a day?

3. The monthly supply of paper for a printing press costs Rs. 100 more when the 50 lbs. paper is Rs. 8 4 as. a ream than when it is Rs. 8 2 as. a ream. What will it cost when the paper is selling at Rs. 7 14 as. a ream?

4. A person meeting a certain number of beggars gave them 4 as. each and had 10 as. left; he found that he would require 8 as. more to enable him to give them 6 as. each. How many were there?

5. A district map is 3 ft. by 2 ft. and is on a scale of 6 in. to the mile. How many square miles does it represent?

IX.

1. A coffee dealer bought 2 kinds of coffee at 13 as. 4 p. a viss and 12 as. 7¼ p. a viss; and mixed with every 3 lbs. of the former kind 4 lbs. of the latter, and sold the mixture at 13 as. 3 p. a viss, and gained thereby Rs. 10-3-4. How many lbs. of each did he buy?

2. Find by practice the value of 17 cwt. 3 qrs. 22 lbs. at £ 4-6-7½ per cwt.

3. If 40 tons of flour are sufficient to feed 800 men for 30 days, how long will 85 tons last 1,000 men at the same rate?

4. A person after paying out of his income a tax of 9 pies in the rupee and 0·75 of the remainder for expenses has Rs. 167-12-0 left. Find his gross income.

5. If 8,250 c. ft. of road material be spread over a quarter of a mile of road 40 ft. wide, what is the thickness of the material?

X.

1. Find the cost of 4 candies 2 mds. 3 viss 16 pals. of sugar at Rs. 3-7-5 a maund.

2. I bought an equal number of apples and oranges for Rs. 24, the price of an apple being equal to that of 2 oranges. I sold the apples alone, at a gain of an anna each, for Rs. 20. Find the number of oranges bought.

3. If 6 men can copy 125 pages in 3 days working 10 hours a day, how many women must be employed to copy 1,500 pages in 20 days of 12 hours each, supposing 3 men to do in 5 hrs. as much as 2 women can do in 10 hours?

4. A cistern is 2 m. 40 cm. long, 2 m. wide and 75 cm. deep. How many litres of water can it hold?

5. Examine the correctness or otherwise of the following rule: 'To convert price in farthings per lb. to price in £ per ton, multiply by 2⅓.' Express 1½*d*. per lb. in £ per ton by applying the rule.

XI.

1. If 1 lb. of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons, what is the value of 1 lb. of tea, when a lemon is worth a penny?

2. A trader buys 1,680 lbs. of sugar at Rs. 32 a cwt. and sells it at Rs. $7\frac{1}{2}$ a md. What does he gain or lose? 1 viss = 3 lb. 2 oz.

3. If 10 compositors, who can set 3 letters in 5 seconds, finish 27 pages in an hour and a half, how many compositors, who can set 5 letters in 6 seconds, will complete 50 pages of the same book in an hour?

4. In a school dormitory, a room for 4 boys is 20 ft. by 16 ft. by 12 ft. How much (1) floor space, (2) air space is there for each boy?

5. A man bought 20 ponies at Rs. 120 each. Two died. At what price must he sell each of the remainder to gain Rs. 300 on the whole?

XII.

1. $\frac{1}{5} + \frac{1}{12} = \frac{17}{60}$; use this to find the value of $2\frac{17}{60}$ times 7 ft. 6 in.

2. If $\frac{3}{5}$ of a bushel of wheat be worth $\frac{3}{4}$ of a bushel of barley, and 1 bushel of barley be worth $\frac{2}{3}$ of a crown, how many quarters of wheat can be bought for £15?

3. A man, his wife and 3 children earn Rs. 20 10 as. a week. The wife earns twice as much as each child, and the man three times as much as his wife. Find the man's daily earning. (A week = 6 working days.)

4. If 5 men can do a piece of work in 12 days of 10 hrs. each, how many men will perform a piece of work four times as great in a fifth part of the time, if they work the same number of hours a day, supposing 3 of the second set can do as much work in an hour as 5 of the first set?

5. Between what limit must the area of a rectangular room lie, whose length is 28 ft. and breadth 20 ft., correct within ± 1 foot.

XIII.

1. Find to the nearest pie the cost of a draft in London for £ 1,050-12-6 at 1*s*. $3\frac{15}{16}$*d*. per rupee.

2. A motor-car travels one kilometre in 35 seconds. Find to two places of decimals the speed of the car in miles per hour; given that 1,000 yds. = 915 metres.

3. On a map drawn to a scale of 1 : 3,600, the sides of a rectangular field are 2·2 in. and 1·21 in. Find the area of the field in acres.

4. Work out in the most intelligent way you can—

   (1) Find the average of the following barometric readings in inches:—29·66, 29·59, 29·60, 29·67, 29·68, 29·72, 29·70.

   (2) If an article costs Rs. 2-15-9, find the cost of 99.

   (3) If 15 tons can be bought for £ 27-10-5, how much would $16\frac{1}{2}$ tons cost.

5. The railway from A to B is 3 mi. 6 fur., and the average gradient is 1 in 160. Find in feet the difference in height of A and B.

XIV.

1. Find the cost of 10 can. 8 mds. 5 viss 16 pals. of sugar at Rs. 3-8-9 a maund, by the shortest method.

2. If 12 men and 9 boys do a piece of work in 25 days, how many men assisted by an equal number of boys can do twice the work in 9 days working only half as long a day; 2 men doing as much work as 3 boys?

3. A person rents 25 acres for Rs. 76; the land consists of two sorts; the better sort he rents at Rs. 4 per acre, and the worse at Rs. 2-8 as. an acre. How many acres did he rent of each sort?

4. A contractor undertakes to do a piece of work in 60 days, and immediately employs upon it 30 men; at the end of 48 days, the work is only half done; required the number of additional men necessary to fulfil the contract.

5. A railway station covers 216 acres; if the length is $2\frac{1}{4}$ miles, find the average breadth.

XV.

1. If 50 gals. of water weighs 525 lbs., and a cubic inch of water weighs 7 oz., find the number of gallons in a tank 23 ft. by 20 ft. by 4 ft.

2. Use the formula $(a+b)^2 = a^2 + 2ab + b^2$ to find to the nearest unit the value of $(635{\cdot}07)^2$, given that $(635)^2 = 403225$.

3. Prices of terrestrial globes of different sizes are given below:

Diameter	2″	3″	5″	6″	9″	12″
Price in *s. d.*	6/	8/6	15/	18/6	30/6	45/

Find graphically the price of globes of diameter 8 in. and 10 in.

4. If the mass of the third satellite of Jupiter be 0·000085 of the mass of the planet and 0·02947 of the mass of the Earth, find the mass of the Earth in terms of the mass of the planet to three places of decimals.

5. A garrison of 1,140 men has provisions for 40 days. After 16 days on full rations, it is reduced by 152 men; but the existing provisions are increased by $\frac{1}{12}$ by capture. For how many days can the reduced garrison now hold out?

CHAPTER XXXII.

PERCENTAGES.

342. The fraction $\frac{3}{100}$ denotes, as we know, that the unit is divided into 100 equal parts and 3 of such parts are taken, the unit being any quantity, simple or compound.

If the unit is Rs. 1,000 $\frac{3}{100}$ of the unit means that Rs. 1,000 is divided into 100 equal parts and 3 of these parts are taken.

If the unit taken is 5 can. 10 mds., $\frac{3}{100}$ of the unit indicates 3 parts out of every 100 parts contained in 5 can. 10 mds.

The term **per cent.** is used to denote such parts ; per cent. from Latin *per centum* meaning ‘for a hundred’.

The vulgar fraction equivalent to 3 per cent. is $\frac{3}{100}$ and the equivalent decimal is ·03.

$$\text{per cent.} = \frac{6\frac{1}{4}}{100} = \frac{6{\cdot}25}{100} = {\cdot}0625.$$

The symbol % is used to denote per cent. ; sometimes the letters p.c. stand for per cent.

EXAMPLE 1. Find the value of $3\frac{1}{2}\%$ of Rs. 2,500.

$$3\tfrac{1}{2}\% \text{ of Rs. } 2{,}500 = \frac{3\frac{1}{2}}{100} \times \text{Rs. } 2{,}500 = 3\tfrac{1}{2} \times \text{Rs. } 25 = \text{Rs. } 87\ 8 \text{ as.}$$

$$5\tfrac{1}{4}\% \text{ of } £12\text{-}5\text{-}7 = \frac{5\frac{1}{4}}{100} \times \text{Rs. } 12{\cdot}279 = 5\tfrac{1}{4} \times £0{\cdot}12279.$$

$$£0{\cdot}1228 = 5\tfrac{1}{4} \times £0{\cdot}1228 = £0{\cdot}6447 = 12s.\ 10\tfrac{3}{4}d.$$

5 to the nearest penny.

$\frac{1}{4}$ | 0·6140
| 0·0307
£0·6447

EXAMPLES.—ORAL.

1. Express as vulgar fractions in their lowest terms—
   50% ; 25% ; 10% ; $12\frac{1}{2}\%$: $8\frac{1}{3}\%$; $6\frac{1}{4}\%$; $16\frac{2}{3}\%$; $11\frac{1}{9}\%$; $33\frac{1}{3}\%$.
2. Express as decimals—
   5% ; 9% ; 10% ; 25% ; 35% ; 40% ; 65%.
3. Find the value of—
   - (*a*) 5% of £300, Rs. 500, £800, Rs. 40, £50, Rs. 70, £250.
   - (*b*) 4% of 300 Km., 250 tons, 150 yds., £50, Rs. 75.
   - (*c*) 3% of 50 ac., Rs. 150, £200, Rs. 750.
   - (*d*) 6% of Rs. 150, Rs. 250, Rs. 1,500, Rs. 2,200.
   - (*e*) 9% of 200 guin., Rs. 50, £25, 75 yds., 350 metres, 675 links.
   - (*f*) 12% of Rs. 200, Rs. 150, £75, £425, Rs. 650.

EXAMPLES. I.

(*The first four questions are to be done orally.*)

1. Five per cent. of the boys in a school of 420 are Mahomedans. How many Mahomedans are there ?

2. Gunpowder contains 75% of nitre; how many lbs. of nitre are there in 120 lbs. of gunpowder?

3. In a school of 240, 15% are girls; how many are boys?

4. $12\frac{1}{2}\%$ of the boys in a class of 40 read Sanskrit; how many read other languages?

5. Find the fractions equivalent to—
(1) $17\frac{1}{2}\%$. (2) $13\frac{1}{3}\%$. (3) $66\frac{2}{3}\%$. (4) $54\frac{2}{7}\%$.

6. Express as decimals—
(1) $6\frac{1}{2}\%$. (2) $12\frac{1}{4}\%$. (3) $13\frac{3}{4}\%$. (4) $17\frac{1}{8}\%$.
(5) $16\frac{2}{3}\%$. (6) $\frac{5}{7}\%$. (7) $23\frac{2}{11}\%$. (8) $6\frac{2}{13}\%$.
the last four to 3 decimal places.

7. Find the value of—
(1) 39% of Rs. 2-1-4. (2) $4\frac{1}{2}\%$ of £2,356-13-4.
(3) $13\frac{3}{4}\%$ of 1 mi. 2 fur. (4) $15\frac{7}{8}\%$ of 4 Kg.

8. Find to the nearest pie the value of—
$\frac{3}{8}$, 5·8, 14, 120·5 per cent. of Rs. 31,560.

9. Find the value to the nearest pie or penny—
(1) 4% of £528 4*s*. (2) $3\frac{1}{2}\%$ of Rs. 1,234 2 as.
(3) 9% of Rs. 726 12 as. (4) $4\frac{7}{8}\%$ of £86-5-4.

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343. A **percentage** of a quantity is that part of the quantity expressed in hundredths.

EXAMPLE 1. What percentage is denoted by $\frac{1}{6}$?

Expressing $\frac{1}{6}$ as hundredths, the percentage required
$= \frac{1}{6} \times 100 = \frac{50}{3} = 16\frac{2}{3}$.

EXAMPLES.—ORAL.

What percentage is denoted by the following fractions?

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}, \frac{3}{5}, \frac{1}{12}, \frac{1}{20}, \frac{9}{25}, \frac{1}{9}, \frac{1}{3}, \frac{3}{40}, \frac{3}{4}, \frac{2}{3}$; 0·25, 0·05; 20, $\frac{x}{25}, \frac{x}{50}$.

344. Since fractions represent ratios, a ratio may be expressed as a percentage.

EXAMPLE 2. What percentage is 4 as. 4 p. of 6 as. 8 p. ?

4 as. 4 p. = 52 p. ; 6 as. 8 p. = 80 p.

Expressing 4 as. 4 p. as a fraction of 6 as. 8 p., we get $\frac{52}{80}$ or $\frac{13}{20}$.

$\therefore$ Percentage required $= \frac{13}{20} \times 100 = 65$.

In other words, find the number that bears the same ratio to 100 that 4 as. 4 p. has to 6 as. 8 p. ?

Denoting the number required by x, we have $\frac{x}{100} = \frac{52}{80}$.

$\therefore x = \frac{52}{80} \times 100 = 65$. The percentage required is 65.

345. Graphic Solution.

Let each horizontal and each vertical division denote 4 pies.

Plot the point P (80, 52) and join OP. The graph is a straight line as this is an example of direct proportion.

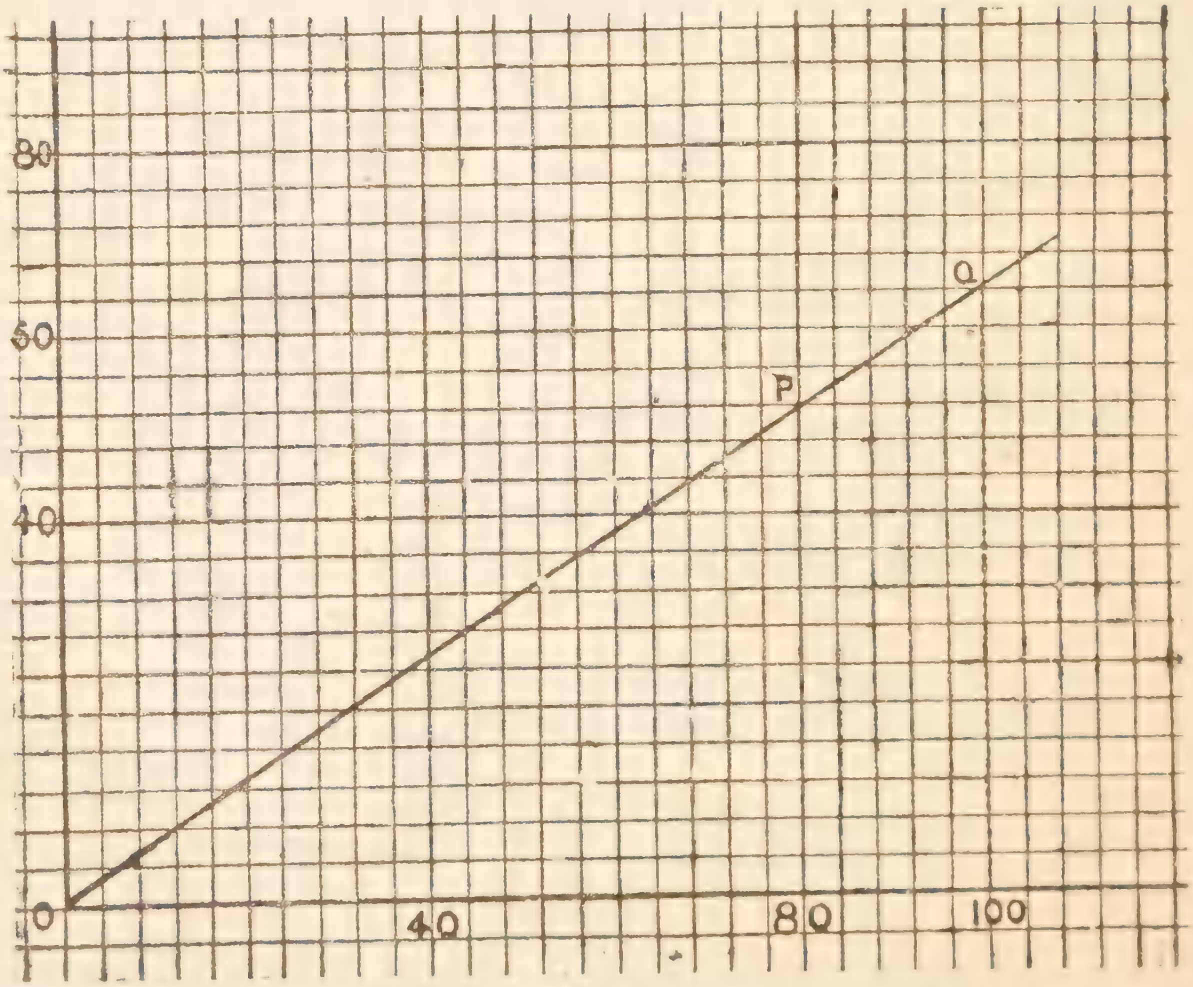


FIG. 68.

The vertical drawn from the point 100 meets OP, produced at Q. The ordinate of Q is seen to be 65.

∴ The percentage required is 65.

EXAMPLES.—ORAL.

What percentage is—

1. Rs. 17 of Rs. 34. 2. £8 of £32. 3. Rs. 9 of Rs. 45.
4. 17*s*. 6*d*. of £3 10*s*. 5. 12 as. of Rs. 2-4 as. 6. 5*s*. of 7*s*. 6*d*.
7. Rs. 1½ of Rs. 30. 8. 2 viss of 1 md. 9. 3 dm. of 1 metre.
10. 1½ hrs. of 1 day. 11. 4 fur. of 1 mi. 12. 5 as. 4 p. of Rs. 2.
13. x of 100. 14. x of 150. 15. x of y.

EXAMPLES. II.

(1 *to* 4 *to be done orally, and* 13 *to* 16 *graphically.*)

1. In a town of 50,000 people, 2,000 died of cholera in a certain year. What is the percentage of mortality?

2. Out of 25 boys that went up for Matriculation from a certain school, 12 were successful. What is the percentage of success?

3. From a salary of Rs. 120 a month, a person spends Rs. 100. What percentage of his salary does he save?

4. In a school of 250 boys, 180 know to read English. What is the percentage?

What percentage is—

5. £15-7-6 of £250. 6. 11 chains of 4 miles.
7. 5 mds. 4 viss of 2½ can. 8. 324 metres of 3·6 Km.

How much per cent. is—

9. 15 of 180. 10. 27½ of 110.
11. 2·125 of 15. 12. 18·875 of 60.

13. A bicycle is bought for £15; and it costs 9*s*. to maintain it in order every month. How much is this per cent.?

14. For selling books worth Rs. 131 4 as., a bookseller gets Rs. 19 11 as. commission. What is that per cent.?

15. Out of a maximum of 510 marks, a candidate obtains 306 marks. What percentage of the maximum does he get?

16. In a class of 32 boys, 6 are Brahmins and the rest are Christians. Find the percentage of each.

Example 3. Of what sum of money is Rs. 113 12 as. $22\frac{3}{4}\%$.

$\frac{22\frac{3}{4}}{100}$ of the required sum = Rs. 113 12 as.

$\frac{22{\cdot}75}{100}$ .... = Rs. 113·75

0·2275 .... = Rs. 113·75

$\therefore$ 1 or .... = Rs. $\frac{113{\cdot}75}{{\cdot}2275}$ = Rs. 500.

EXAMPLES. III.

(1 *to* 8 *are to be done orally.*)

1. Of what sum of money is Rs. 5 ten per cent. ?
2. Of what amount is Rs. 15 five per cent. ?
3. Of what length is 10 yds. four per cent. ?
4. Of what distance is 15 miles $12\frac{1}{2}$ per cent. ?
5. 15*s*. is 75% of what sum ?
6. 28 marks are 35 per cent. of what maximum ?
7. 24 tons is 8 per cent. of what weight ?
8. Of how many days is 20 hrs., $16\frac{2}{3}\%$?

9. There are 25 boys in a certain class ; and these form 5 per cent. of the total number in the school. Find the strength of the school.

10. If 81 marks are 60% of the maximum, what is it ?

11. If £720 is 0·4% of the value of an estate, find its value.

12. A bankrupt is worth £875 which is 17·5 per cent. of his debts. Find the amount of his debts.

Example 4. In a town containing 15,000 inhabitants, 800 were attacked with cholera ; and of these 125 died. Find what percentage of the population was attacked, and the percentage of mortality (1) to the population, (2) to those attacked.

If out of 15,000, 800 were attacked, what is the rate for 100 ?

Reqd. percentage $= \frac{100 \times 800}{15{,}000} = \frac{16}{3} = 5\frac{1}{3}$.

(1) If out of 15,000, 125 died, what is the rate for 100 ?

Reqd. percentage $= \frac{125 \times 100}{15{,}000} = \frac{5}{6}$.

(2) If out of 800 attacked, 125 died, what is the rate for 100 ?

Reqd. percentage $= \frac{100 \times 125}{800} = \frac{125}{8} = 15\cdot625.$

EXAMPLES. IV.

1. In a certain election out of 800 voters, 450 promised to vote for A ; but only 375 actually voted. What percentage
   (1) of the voters promised to A ?
   (2) of those who promised actually voted ?
   (3) of the total votes did A get ?

2. From a class of 32 boys, 21 were sent up for Matriculation ; and of these 15 passed the examination. Find the percentage of (1) the number sent up to the total number ; (2) of the successful candidates to the number (*a*) sent up, (*b*) in the class.

3. Out of 2,350 cases of small-pox, 340 persons died ; and out of 3,250 cases of cholera, 460 persons died. Find the rate per cent. of mortality in (1) each case, (2) in the whole number of sick people.

4. The strength of 5 schools in a town in 1887 was 80, 100, 120, 150 and 180 respectively. In 1888, the three former had increased by 40, 30 and 20 per cent. respectively ; while the two latter had decreased by $16\frac{1}{2}$ and 25 per cent. respectively. Find the increase or decrease in the strength of the 5 schools together.

EXAMPLE 5. Between the years 1871 and 1881 the population of the Madras Presidency increased by $29\frac{1}{6}$ per cent. and in the latter year it was 31 millions. What was it in 1871 ?

For every $129\frac{1}{6}$ persons in 1881 there were 100 persons in 1871.

Number reqd. $= \frac{100}{129\frac{1}{6}} \times 31$ millions.

Number reqd. $= \frac{21 \times 100 \times 6}{775}$ millions $= 24$ millions.

EXAMPLES. V.

(1 *to* 4 *to be taken orally.*)

1. The population of a village is 1,100, and it is 10 per cent. more than it was 5 years ago. What was it then?

2. In a school there were 240 boys; and during the year the numbers increased by 5 per cent. Find the numbers at the end of the year.

3. If after a deduction of 25 per cent. in a bill, I pay Rs. 120, what was the amount of the original bill?

4. If a person spends Rs. 60 and saves 20 per cent. of his income, what is it?

5. The population of a town increases 10 per cent. every ten years. What was the population 10 years ago, if it now contains 22,000 inhabitants?

6. The capital of a trading company was increased 35 per cent. between 1871 and 1881; in 1881 it was Rs. 2,70,135; what was it in 1871?

7. The profits of a company decreased by 33·11 pen cent. in 1901. If in this year the profits were Rs. 1,07,024; what were they in 1900?

8. The population of a town in 1891 was 16,808; and it had increased by 12½ per cent. in 1901. What was it in 1901? At the same rate of increase, what was the population in 1911?

EXAMPLES. VI.

MISCELLANEOUS.

✓1. Find the percentage of error in the following cases correct to one decimal place:—

(*a*) If a metre 39·3701 in. long is taken as 40 in.

(*b*) One Kg., which is equal to 2·2046 lbs. is taken as 2⅕ lb.

(*c*) When a 4 lb. weight is too light by 11½ drams.

(*d*) 'To convert miles per hour to feet per second, multiply by 1½.'

✓2. Of a silver coin $\frac{37}{40}$ is pure silver, the rest is alloy. What percentage of the coin is (1) pure silver, (2) alloy?

3✓ A gold mine yields 18 dwt. of gold for every ton of quartz crushed. Express this as a percentage. 1 dwt = 24 grs. and 7000 grs. = 1 lb. av.

4. If 500 c. cm. of air, when taken into the lungs, contain 0·03 er cent. of carbonic acid, but when breathed out contain 4 per ent. ; how many c. cm. of air have been replaced by carbonic acid ?

5. The birth-rate of a town is 40 per 1000, and the death-rate 22·5 per 1000 in a certain year. Find the increase per cent. in the opulation.

6. The capital of a company is 6½ lakhs of rupees; and its rofits are Rs. 25,000. If a dividend of 3½ per cent. on the capital is aid, what sum is carried to the reserve fund ?

7. Two casks contain respectively 50 gals. and 30 gals. of pirits. The former contains 60 per cent. of alcohol and the latter 50 per cent. of alcohol. When the two are mixed, how much per ent. of the mixture is alcohol ?

8. Standard gold is 24 carat. 22 carat gold contains 22 parts gold and 2 parts alloy. What is the percentage composition ?

9. If a town of 4,000 people increases in 5 years by 5 per cent., and in the next 5 years by 5 per cent., what is the percentage increase for the ten years ?

10. Of 25,000 persons, 30·5 per cent. can read and write, 50·4 per cent. can do neither, and the rest can only read. Find the number in each class.

11. The percentage of boys in the first three forms of a certain school of 60 boys is 65, and 75 in a school of 68. What is the percentage in the two schools together ?

12. Water is composed of 88$\frac{8}{9}$ per cent. by weight of oxygen and rest of hydrogen. How many lbs. of each are there in 9 c. ft. of water, if 1 c. ft. of water weighs 1000 oz. ?

Use contracted methods in working the following questions :—

13. In a certain year the total revenue was 144 million pounds sterling, and the tax on tea produced £5,790,000. What per cent. of the whole revenue to the nearest whole number, was produced by the tea tax ?

14. Imports into Great Britain were for the year 1900 as shown below. Find, to the nearest integer, what percentage of the total imports came from each of the sources named :

British Possessions	£109,530,635	France ...	£ 53,618,656
United States ...	£138 789,261	Other countries.	£221,136,611

15. Population of—

England and Wales, 1891...	29,002,525	1901 ...	32,577,843
Scotland ,, ...	4,025,647	,, ...	4 472,103

Find which population was increasing at the greater proportional rate during 1891—1901; and the increase per cent. to 2 decimal places.

	Eng. & Wales.	Scotland.	Ireland.
16. Estimated population in 1903...	33,378,000	4,579,000	4,414,000
No. of children in primary schools.	3,037,000	669,000	482,000.

Find in each case the number of children as a percentage of the estimated population to the nearest whole number.

CHAPTER XXXIII.

SIMPLE INTEREST.

346. **Interest** is the sum of money paid for the use of money. The sum lent is called the **principal.** Interest is calculated at the rate of so much **per cent.** for a year; that is, it is reckoned by the sum paid for the use of £100 or Rs. 100 for one year.

Per cent. = *per centum* = for a hundred. The symbol % is used to denote per cent.

The words **per annum,** meaning **for a year,** are either expressed or understood.

347. Interest is of two kinds:—**Simple** and **Compound.**

Simple Interest is that which is reckoned only on the original principal; but **Compound Interest** is that which is paid not only for the use of the original sum, but also for the use of the interest as it becomes due.

EXAMPLE. Find the interest on Rs. 450 for one year at $3\frac{1}{2}$ per cent.

The question, in other words is—If the interest on Rs. 100 is Rs. $3\frac{1}{2}$, what is the interest on Rs. 450? This is a case of simple proportion.

$$\text{Reqd. interest} = \frac{3\frac{1}{2}}{100} \times \text{Rs. } 450.$$

$$\therefore \text{ Interest reqd.} = \text{Rs. } \frac{7}{200} \times \frac{450}{1} = \text{Rs. } \frac{63}{4} = \text{Rs. } 15 \;\; 12 \text{ as.}$$

Hence, to find the interest on a given sum for one year, multiply the principal or sum given by the rate per cent. and divide the product by 100.

In other words find how many hundreds there are in the given sum; and multiply the number of hundreds by the rate per cent.

In 450 there are $4\frac{1}{2}$ hundreds; and the interest is therefore Rs. $4\frac{1}{2} \times 3\frac{1}{2}$ or Rs. 15 12 as.

EXAMPLES.—ORAL.

Find the interest for 1 year on—

1. £800 at 8%. 2. Rs. 1,000 at 12%. 3. Rs. 900 at 10%.
4. £1,500 at 7%. 5. Rs. 2,500 at 6%. 6. Rs. 250 at 12%.
7. £450 at 10%. 8. Rs. 775 at 8%. 9. £575 at 10%.
10. Rs. 525 at 6%. 11. £675 at 4%. 12. Rs. 750 at 12%.
13. Rs. 900 at $3\frac{1}{4}\%$. 14. £1,200 at $2\frac{1}{2}\%$. 15. Rs. 1,800 at $3\frac{1}{4}\%$.
16. £ x at 4%, 9%, 12%, $6\frac{1}{4}\%$.

348. To find the interest on any sum for one year at 5 per cent.

As 5% is $\frac{1}{20}$, take one-twentieth of the given sum.

For English money, take a shilling for every 20*s*. (£1); and a penny for every 20 pence.

For Indian money, take a rupee for every 20 rupees, an anna for every 20 annas, and a pie for every 20 pies.

EXAMPLE 1. Find the interest for one year at 5 per cent.
(*a*) on £5,764-13-4.
(*b*) on Rs. 1,381 4 as.

(*a*) £5,764-13-4 = £5,764 + 160*d*.
Interest required = 5,764*s*. + 8*d*. = £288-4-8.
(*b*) Rs. 1,381 4 as. = Rs. 1,380 + Rs. 1-4 = Rs. 1,380 + 20 as.
Interest required = Rs. 69 + 1 anna = Rs. 69-1-0.

EXAMPLES.—ORAL.

Find the interest for 1 year at 5 per cent. on—

1. Rs. 650. 2. £750. 3. Rs. 950. 4. £527.
5. £633. 6. £451. 7. £326 10*s*. 8. Rs. 400 12 as.
9. Rs. 100-1-8. 10. Rs. 300-3-4. 11. Rs. 500 5 as. 12. Rs. 660-6-8.
13. Rs. 780-8-4. 14. £540-11-8. 15. £620-15-0. 16. £880-6-8.

EXAMPLES I.

Find the interest for one year on—

1. Rs. 1,875 at $3\frac{3}{4}\%$. 2. £2,650 at $3\frac{1}{2}\%$.
3. Rs. 15,675 at $4\frac{1}{2}\%$. 4. £2,925 at $6\frac{1}{4}\%$.

Find the interest for one year at 5% on—

5. Rs. 871-5-8. 6. £653-6-8. 7. Rs. 551-7-4. 8. £234-5-5.

349. The simple interest for any number of years is obtained by multiplying the interest for one year by the number of years given.

EXAMPLE 2. Find the simple interest on Rs. 8,675 for 4 years at 12 per cent. per annum.

The interest for 1 yr. $= \text{Rs. } \dfrac{8675 \times 12}{100} = \text{Rs. } 86{\cdot}75 \times 12 = \text{Rs. } 1{,}041.$

$\therefore$ The interest for 4 yrs. $= \text{Rs. } 1{,}041 \times 4 = \text{Rs. } 4{,}164.$

Or, taking this as an example in compound proportion, we may proceed thus :

Int. reqd. $= \text{Rs. } 12 \times \dfrac{8675}{100} \times \dfrac{4}{1} = \text{Rs. } 12 \times 86{\cdot}75 \times 4 = \text{Rs. } 4{,}164.$

350. The Interest I, for any number of years N, for a given sum or principal P, at a given rate per cent. R, is therefore $= \dfrac{P \times R \times N}{100}$.

EXAMPLES.—ORAL.

Find the interest on—

1. Rs. 200 at 4% for 5 years.
2. £500 for 3 yrs. at 6%.
3. £750 at 3% for 4 yrs.
4. Rs. 1,050 at 9% for 2 yrs.
5. Rs. 875 at 12% for 3 yrs.
6. £625 at 8% for 10 yrs.
7. £x at 5% for 12 yrs.
8. Rs. y at 10% for 5 yrs.
9. £100 for 2 yrs. at 3½%.
10. Rs. 500 for 4 yrs. at 3%.
11. £800 for 3½ yrs. at 4%.
12. Rs. 200 for 4½ yrs. at 3%.
13. £50 for 4 yrs. at 7½%.
14. £1,000 for ½ yr. at 4%.
15. Rs. 500 at x% for 4 yrs.
16. Rs. 450 at 5% for y years.

EXAMPLES. II.

Find the interest on—

1. Rs. 575 at 9% for 4 yrs.
2. £625 at 2½% for 6 yrs.
3. Rs. 850 at 12% for 3¾ yrs.
4. £1,225 at 6¼% for 3½ yrs.

Find the interest at 5%, with the least amount of work, on—

5. £1,234 for 5 yrs.
6. £2,345-7-6 for 2 yrs.
7. Rs. 2,356-0-8 for 3 yrs.
8. Rs. 1,978-3-4 for 3 yrs.

Example 3. Find to the nearest penny the interest on £3,040-13-4 for 10½ yrs. at 2¾ per cent.

$$\pounds 3{,}040\text{-}13\text{-}4 = \pounds 3{,}040{\cdot}666 \ldots\ldots$$

$$\text{Interest reqd.} = \frac{\pounds 3040{\cdot}666}{100} \times 10\tfrac{1}{2} \times 2\tfrac{3}{4}$$

$$= \pounds 30{\cdot}4067 \times 10\tfrac{1}{2} \times 2\tfrac{3}{4}$$

$$2\tfrac{3}{4} = 3 - \tfrac{1}{4}.$$

$$\begin{array}{r|r}
 & \pounds 30{\cdot}4067 \\
 & 10 \\ \hline
\tfrac{1}{2} & 304{\cdot}067 \\
 & 15{\cdot}203 \\ \hline
 & 319{\cdot}270 \\
 & 3 \\ \hline
\tfrac{1}{4} & 957{\cdot}810 \\
\text{Subtracting} & 79{\cdot}817 \\ \hline
 & \pounds 877{\cdot}993 = \pounds 877\text{-}19\text{-}10\tfrac{1}{4}.
\end{array}$$

Note.—The interest for any number of months is obtained by expressing it as a fraction of a year.

EXAMPLES. III.

Find to the nearest penny or pie, the interest on—

1. £2,367 8*s*. for 4 years at 3 per cent.
2. Rs. 2,952 10 as. for 2 years at 3¾ per cent.
3. Rs. 38,756-7-9 for 2 years 6 mo. at 12 per cent.
4. Rs. 5,089-10-8 for 2 years at 8 per cent.
5. Rs. 816-0-4 for 6 years at 5½ per cent.
6. Rs. 20,367-14-8 for 4 years 6 mo. at 6½ per cent.
7. Rs. 1,000-8-4 for 3 years 6 mo. at 4½ per cent.
8. £567,890-16-8 for 4 years 3 mo. at 3 per cent.

---

351. The principal with the interest added to it is called the **Amount.**

If the amount be required, it may be obtained either by adding the interest to the principal or by multiplying the amount of £1 or Re. 1 for the given time at the given rate per cent. by the principal.

Caution.—Never multiply the amount of 100 for one year by the number of years given. For if the amount of 100 at 5 per cent. in 1 year is 105, the amount of 100 for 3 years is not 315.

Example 4. Find the amount on £5,666-13-4 for 3½ years at 4¼ per cent. per annum.

1st method. The interest $= \frac{£5,666·666}{100} \times 3\frac{1}{2} \times 4\frac{1}{4} = £56·6666 \times 3\frac{1}{2} \times 4\frac{1}{4}$

£56·666
3
170·000
$\frac{1}{2}$ 28·333
198·333 = Product
4 by $3\frac{1}{2}$.
793·332
$\frac{1}{4}$ 49·583
5666·667 = Principal
£6509·582 = Amount
= £6,509-11-8.

2nd method.

£5666·667
1·14875
5666·667
566·667
226·666
45·333
3·966
0·283
£6,509·582 = £6,509-11-8.

Int. on £1 for $3\frac{1}{2}$ yrs. at $4\frac{1}{4}$%

$= \frac{3\frac{1}{2}}{100} \times 4\frac{1}{4} = \frac{3·5}{100} \times 4·25$

$= ·035 \times 4·25 = ·14875.$

$\therefore$ Amount of £1 = 1·14875.

352. We have therefore A (amount) $= P + \frac{PRN}{100}$ ... (1).

$$P \times = \left(1 + \frac{RN}{100}\right) \text{ ... (2).}$$

EXAMPLES. IV.

Required the interest and amount of—

1. Rs. 625 at 6 per cent. for $2\frac{1}{2}$ years.
2. Rs. 850 at 9 per cent. for 3 yrs. 9 mo.
3. Rs. 2,362 8 as. for $3\frac{3}{4}$ years at 4 per cent.
4. £1,345 10*s*. for $4\frac{1}{2}$ years at 5 per cent.

Find the amount of—

5. £2,708-2-6 for 6 years at $2\frac{1}{2}$ per cent.
6. £2,345-12-6 for 5 years at $3\frac{3}{4}$ per cent.
7. £2,666-13-4 at $3\frac{1}{2}$ per cent. for $4\frac{1}{2}$ years.
8. Rs. 1,200-10-8 at $6\frac{1}{4}$ per cent. for $3\frac{3}{4}$ years.

---

353. Since interest is directly proportional to the principal or sum and to time or number of years, the graph showing the variation of interest with the principal and of interest with the number of years is a straight line.

The interest on Re. 1 at 5 per cent. for different periods is given below—

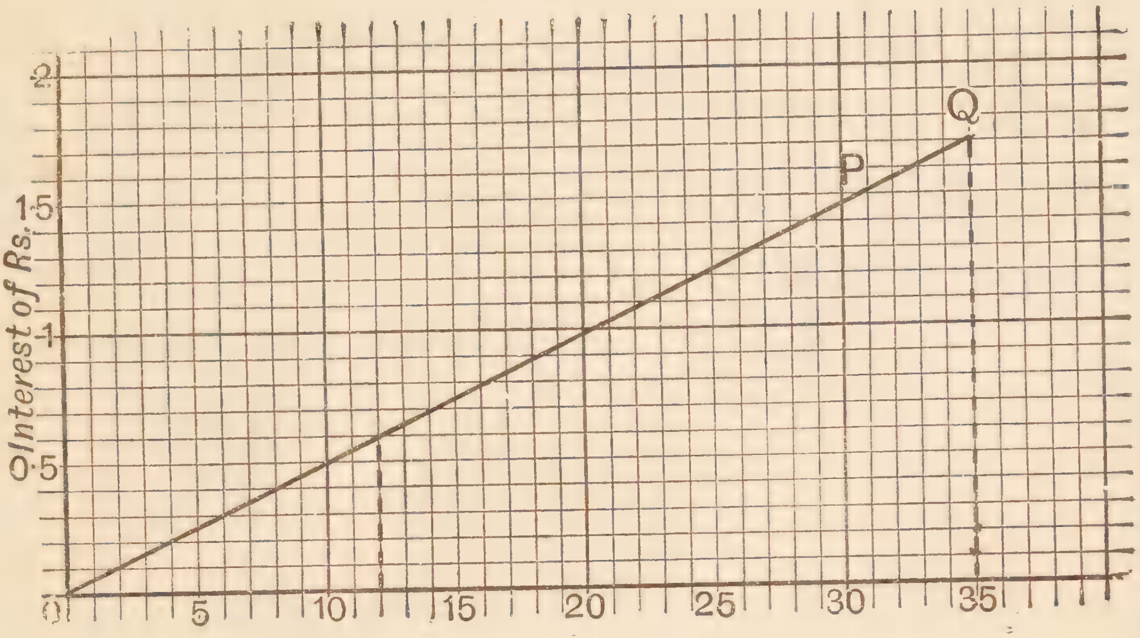
Number of years.	0	5	10	15	20	25	30
Interest ...	0	·25	·5	·75	1·0	1·25	1·5

Draw the graph and use it to find the interest of (1) Rs. 625 for 12 years and (2) Rs. 240 for 35 years.

Let each horizontal division denote 1 yr. and each vertical division represent one-tenth of a rupee.

Plot the points and draw the graph OP.

OP gives the interest of Re. 1 at 5 per cent. for any number of years up to 30.



NUMBER OF YEARS.

FIG. 69.

(1) When the number of years is 12, the interest on Re. 1 is seen from the graph to be 0·6.

∴ The interest on Rs. 625 for 12 years is Rs. 625×·6 or Rs. 375.

(2) Since the graph is a straight line, it may be produced to any length and used to find the interest for any number of years.

Produce O P to Q to meet the vertical from 35 years.

The interest on Re. 1 for 35 yrs. is seen to be 1·75.

∴ The interest on Rs. 240 for 35 years is Rs. 240 × 1·75 or Rs. 4 20

354. Since amount = Principal + Interest, where principal is constant and interest varies with the number of years, the graph showing the variations of amount and time is a straight line.

The above graph may be used to read off the amounts of Re. 1 in any given years, and therefore to find out the amount of any sum.

When the number of years is 0, the amount is Re. 1; therefore the co-ordinates of the origin are (0, 1).

To find the amount of Re. 1 in any given year, we have only to add Re. 1 to the interest.

Thus the amount of Re. 1 in 5 years is Re. 1·25, in 15 years Re. 1·75, in 25 years Rs. 2·25.

Use the graph to find the amount of (1) Rs. 120 in 10 years and (2) Rs. 1,875 in 35 years.

The amount of Re. 1 in 10 years can be seen from the graph to be 1·5.

∴ The amount of Rs. 120 in 10 years = Rs. 120 × 1·5 = Rs. 180.

Similarly the amount of Re. 1 in 35 years is 2·75.

∴ The amount of Rs. 1,875 in 35 yrs. = Rs. 1,875 × 2·75 = Rs. 5,156·25 = Rs. 5,156 4 as.

Note.—The same graph may be used to read off the interest of £1 or the amount of £1 at 5 per cent.

Practical Exercise.

1. Use the graph on page 359 to find (1) the interest, (2) the amount of (*a*) £80 for 3 years. (*b*) £250 for 8 years. (*c*) £725 for 12 years. (*d*) £140 for 21 years.

2. If the interest on Rs. 250 at 7½ per cent. for 4 years is Rs. 75, find graphically the interest on (1) Rs. 120, (2) Rs. 380 at the same rate per cent. for the same time.

3. If the interest on a given sum is £144 for 10 years, find graphically the interest on the same sum for (1) 8 years, (2) 25 years at the same rate per cent.

4. Construct a graph to show the amounts of £1 or Re. 1 at 10 per cent. for any number of years, and use it to find the amounts of any sums of money mentioned by the teacher.

5. Construct a graph to show the interest of £1 or Re. 1 at 4 per cent. for any number of years, and use it to find the interest of any sum of money.

---

355. Methods of shortening labour by means of Aliquot parts. We have already seen how interest at 5 per cent. may easily be calculated (Art. 348).

Ex. 5. 1. Interest at $2\frac{1}{2}$ per cent. = $\frac{1}{2}$ the interest at 5 per cent.

2. Interest at $7\frac{1}{2}$ per cent. = Interest at 5 per cent. + $\frac{1}{2}$ the interest at 5 per cent.

3. Interest at 6 per cent. = Interest at 5 per cent. + $\frac{1}{5}$ of the interest at 5 per cent.

4. Interest at $3\frac{3}{4}$ per cent. = Interest at $2\frac{1}{2}$ per cent. + $\frac{1}{2}$ the interest at $2\frac{1}{2}$ per cent.

5. Interest at 3 per cent. = Interest at $2\frac{1}{2}$ per cent. + $\frac{1}{5}$ of the interest at $2\frac{1}{2}$ per cent.

6. Interest at 4 per cent. = Interest at 5 per cent. — $\frac{1}{5}$ of the interest at 5 per cent., and so on for other rates per cent.

EXAMPLES. V.

(*Solve with as little work as possible.*)

1. Find the interest on—
   - (1) £1,875 at $12\frac{1}{2}$ per cent. for 3 years.
   - (2) Rs. 1,021-4-0 at $7\frac{1}{2}$ per cent. for 2 years.
   - (3) £402 10*s*. at $2\frac{1}{2}$ per cent. for 3 years.
   - (4) £230-6-8 at 3 per cent. for 5 years.
2. Find the amount of—
   - (1) £1,875 at 9 per cent. for 3 years.
   - (2) £870-12-6 at 6 per cent. for $2\frac{1}{2}$ years.
   - (3) Rs. 551-10-8 at $3\frac{3}{4}$ per cent. for 4 years.
   - (4) Rs. 2,025-8-4 at 4 per cent. for $3\frac{1}{2}$ years.

356. In calculating interest for a certain number of days, express the number of days as the fraction of 365 days (a year).

If weeks are given, calculate 52 weeks for a year.

When months and days are given, if the months are not named, 30 days are reckoned to the month.

Note.—In reckoning the number of days from one given day to another, the rule is to include one of them only in the calculation.

Thus from March 15th to April 10th will be 26 days.

EXAMPLE 6. Find the interest to the nearest pie, on Rs. 600 at 4 per cent. per annum, from the 4th of January to the 11th of August, 1889.

Jany. (31 – 4) =	27
February	28
March	31
April	30
May	31
June	30
July	31
August	11
	219

Number of days = 219, i.e. $\frac{219}{365}$ year or $\frac{3}{5}$ year.

Interest required = Rs. 600 × $\frac{4}{100}$ × $\frac{3}{5}$.

= Rs. 6 × 0·6.

= Rs. 14·4.

= Rs. 14-6-5 to the nearest pie.

EXAMPLES. VI.

Calculate the interest on—

1. £700 for 2 years 146 days at 4½ per cent.
2. £250 for 3 years 219 days at 6 per cent.
3. Rs. 1,022 from Jan. 26, 1859 to Sep. 13, at 4½ per cent.
4. 219 guineas from July 15 to Dec. 25, at 2¼ per cent.
5. Rs. 7,137 from Feb. 10, 1856 to Aug. 24, at 3⅛ per cent.
6. £610 from Jan. 7, 1888 to Oct. 27, at 5 per cent.
7. Find the interest at 4 per cent. per annum on Rs. 595-9-0 for 4 years 17 weeks reckoning 52 weeks equal to a year.
8. A person bought a house on the 3rd of March for £1,496 10*s.*; and agreed to pay for it with interest on the purchase money at 4⅜ per cent. on the 1st of October, what sum will he have to pay?

---

357. If the number of days, when expressed as a fraction of a year is not capable of being reduced to an easy fraction, multiply the principal by the number of days, and the product by twice the rate per cent., then divide the result by 73,000. The quotient is the interest required.

Instead of dividing by 365 × 100, we divide by 2 × 36,500 or 73,000 because we multiply the rate per cent. by 2.

358. The shortest method to divide by 73,000. The **Third, Tenth** and **Tenth** Rule.

Suppose we divide 73,000 by itself. Move the decimal point of the quantity to be divided 5 places to the left thus:

the quantity to be divided 5 places to the left thus:	·73000
Take $\frac{1}{3}$ of this,	·24333
Take $\frac{1}{10}$ of this $\frac{1}{3}$,	·02433
Take again $\frac{1}{10}$ of this $\frac{1}{10}$.	·00243
Adding the four items, we get	1·00009

When we proceed thus the quotient is approximately 1·0001, which is too large by $\frac{1}{10000}$

This error is about one farthing in excess for every £10 or one pie in excess for every Rs. 50.

Example. Find the interest (1) on Rs. 635 for 237 days at 3 per cent. (2) on £1,028-17-6 for 148 days at $4\frac{1}{2}$ per cent.

$$\text{Interest reqd. (1)} = \text{Rs. } \frac{635 \times 6 \times 237}{73{,}000} = \text{Rs. } \frac{635 \times 1{,}422}{73{,}000}$$

$$(2) = £\frac{1028{\cdot}875 \times 9 \times 148}{73{,}000} = £\frac{1028{\cdot}875 \times 1{,}332}{73{,}000}.$$

$$\begin{array}{llrr}
(1) & \text{Rs. } 0{\cdot}00635 & & \\
\hline
 & 6{\cdot}35 & = \times & 1000 \\
 & 2{\cdot}54 & = \times & 400 \\
 & {\cdot}127 & = \times & 20 \\
 & {\cdot}0127 & = \times & 2 \\
\hline
 & 9{\cdot}0297 & = \times & 1{,}422 \\
\frac{1}{3} & 3{\cdot}0099 & & \\
\frac{1}{10} & 0{\cdot}3009 & & \\
\frac{1}{10} & 0{\cdot}0300 & & \\
\hline
 & \text{Rs. } 12{\cdot}3705 & &
\end{array}$$

= Rs. 12-5-11 Ans.

There is no deduction for error.

$$\begin{array}{lr}
(2) & £0{\cdot}0102|8{,}875 \\
\hline
 & 10{\cdot}2887 \\
 & 3{\cdot}0866 \\
 & {\cdot}3086 \\
 & {\cdot}0206 \\
\hline
 & £13{\cdot}7045 = \times\ 1332 \\
 & 4{\cdot}5681 \\
 & {\cdot}4568 \\
 & {\cdot}0456 \\
\hline
 & £18{\cdot}7750 = £18\text{-}15\text{-}6
\end{array}$$

Since £18 is nearly £20 deducting half a penny for error, £18-15-$5\frac{1}{2}$ Ans.

EXAMPLES. VII.

1. Find the interest on Rs. 725 for 157 days at $4\frac{1}{2}$ per cent.

2. What is the interest on £2,875 for 182 days at $2\frac{1}{2}$% ?

3. Find the amount of Rs. 1,500 from 1st January to 28th October, 1909, at 9 per cent.

4. Calculate the amount of Rs. 4,536 8 as. from 30th June to 31st December, at $3\frac{1}{4}$ per cent.

359. We have seen how Interest (or amount) may be found when principal, Time (number of years) and Rate per cent. are given.

In all questions of Interest any three of these four being given, the fourth may be found.

360. To find P when I, N and R are given.

EXAMPLE. What sum will produce as interest Rs. 45 in $3\frac{1}{3}$ yrs. at 4 per cent. per annum?

This is simply an example in compound proportion and may be worked thus:

$$\text{Reqd. sum} = \text{Rs. } 100 \times \frac{45}{4} \times \frac{1}{3\frac{1}{3}} = \text{Rs. } 100 \times \frac{45}{4} \times \frac{3}{10}$$

$$= \text{Rs. } \tfrac{775}{2} = \text{Rs. } 387 \text{ } 8 \text{ as.}$$

When I, R and N are given, $P = \dfrac{100 \times I}{N \times R}$.

EXAMPLES. VIII.

What sum will produce as interest—

1. Rs. 141 12 as. in $2\frac{1}{4}$ years at 9 per cent. per annum?
2. Rs. 133 in 19 years at $3\frac{1}{2}$ per cent. per annum?
3. What sum will produce 15 as. interest at the rate of 4 pies per rupee per mensem?
4. What sum will produce Rs. 1-6-6 interest at the rate of 3 pies per rupee for a week?

---

361. To find P when A, R and N are given.

EXAMPLE. What sum will amount to Rs. 754 in 9 years at 5 per cent.?

Interest on Rs. 100 for 9 yrs. at 5 per cent. per annum is Rs. 45

Therefore the amount of Rs. 100 in 9 yrs. is Rs. 145.

∴ The amount of Re. 1 is Rs. 1·45.

If when Rs. 1·45 is the amount the principal is Re. 1, what is the principal, when Rs. 754 is the amount?

$$\text{Sum reqd.} = \text{Rs. } \frac{754}{1{\cdot}45} = \text{Rs. } 520.$$

$$P = \frac{A}{\text{Amount of R. 1}}.$$

Or find P from the formula, $A = P \times \left(1 + \dfrac{NR}{100}\right)$. Art. 352.

EXAMPLES. IX.

What sum will amount to—

1. £1,075 10*s*. in $2\frac{1}{2}$ years at 5 per cent. ?

2. Rs. 5,635 8 as. in 8 years at $4\frac{1}{2}$ per cent. ?

3. What sum lent at 5 per cent. will produce in 2 years the same amount of interest as Rs. 1,550 will produce in 3 years at 4 per cent. ?

4. What sum lent at 4 per cent. will produce in 3 years the same amount of interest as Rs. 540 will produce in 2 yrs. at 5 per cent. ?

---

362. To find N when P, R and I are given.

Example. In what time will Rs. 525 gain Rs. 73 8 as. interest at 4 per cent. ?

In other words : If Rs. 100 gains an interest of Rs. 4 in 1 year, in what time will Rs. 525 gain an interest of Rs. 73 8 as. ?

This is an example in compound proportion and may be worked thus :

$$\text{No. of yrs. reqd.} = 1 \times \frac{73\frac{1}{2}}{4} \times \frac{100}{525} = \frac{147}{2 \times 4} \times \frac{4}{21} = \frac{7}{2} = 3\tfrac{1}{2}.$$

$$\therefore N = \frac{100 \times I}{P \times R}.$$

363. To Find N when A, P and R are given.

Example. In what time will Rs. 850 amount to Rs. 1,020 at 9 per cent. ?

The interest on Rs. 850 = Rs. 1,020 − Rs. 850 = Rs. 170.

Proceeding as in the previous example, the No. of yrs. = $2\frac{2}{9}$.

EXAMPLES. X.

1. How long must Rs. 450 be placed at interest at $6\frac{1}{4}$ per cent. to gain Rs. 112 8 as. ?

2. In what time will the interest on Rs. 1,500 amount to Rs. 219 6 as. at $4\frac{1}{2}$ per cent. ?

3. How long will Rs. 1,250 take to gain Rs. 75 interest at 1 anna per mensem per Rs. 10.

4. In what time will Rs. 50 double itself at 12 per cent. ?

Find in what time—

5. Rs. 1,125 amounts to Rs. 1,282 8 as. at 7 per cent.
6. £250 tribles itself at 10 per cent. per annum.
7. £356-6-8 amounts to £409-15-8 at 4½ per cent.
8. Rs. 2,275 amounts to Rs. 2,673 2 as. at 5 per cent.

364. To find R when P, I and N are given.

Example. If Rs. 150 produce Rs. 108 interest in 6 years, what is the rate per cent. per annum ?

In other words : If Rs. 108 is the interest on Rs. 150 for 6 yrs., what is the interest on Rs. 100 for 1 yr. ? This may be worked as an example in compound proportion thus :

Required rate per cent. $= 108 \times \frac{100}{150} \times \frac{1}{6} = 12.$

$$\text{Hence } R = \frac{100 \times I}{P \times N}.$$

365. To find R when P, A and N are given.

Example. At what rate per cent. will £750 amount to £870 in 5 years ?

Interest on £750 for 5 yrs. = £870 — £750 = 120.

Proceeding as in the previous example, the rate per cent. is 3⅕.

EXAMPLES. XI.

1. A person borrows Rs. 15, and pays 15 as. a month for the loan of it ; what is the rate per cent. per annum ?

2. If Rs. 5,718 produce Rs. 800-8-3·84 interest in 3½ years, what is the rate per cent. per annum ?

3. The interest on £250 for 73 days amounts to £11 15*s*. Find the rate per cent. per annum.

4. The interest on Rs. 1,525 from May 15 to October 8, is Rs. 30 8 as. Find the rate per cent.

At what rate per cent. will—

5. Rs. 400 amount to Rs. 480 in 5 years ?
6. Rs. 1,160 amount to Rs. 1,334 in 6 years ?
7. Rs. 788-13-4 amount to Rs. 1,479 1 an. in 5 years ?
8. Rs. 2,630 amount to Rs. 3,090 4 as. in 3½ years ?

EXAMPLES. XII.

MISCELLANEOUS.

1. Find the interest on a lakh of Rs. for $4\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.

2. Find the amount of Rs. 2,125 for 2 yrs. 147 days at 3 pies per rupee per week. (1 year = 52 weeks).

3. What sum of money will amount to Rs. 3,761 14 as. in $3\frac{1}{2}$ years at $4\frac{1}{2}$ per cent.?

4. A person borrows Rs. 355-11-8; and at the end of 18 months has to pay Rs. 419-12-2. What is the rate of interest per annum?

5. In what time will the interest on any sum of money amount to the principal at $3\frac{1}{2}$ per cent. per annum?

6. What will be the interest on Rs. 150 at 1 anna a rupee per month amount to in 5 years, and how much is that per cent.?

7. If the interest on Rs. 1,000 for 5 years be Rs. 250, what will be the interest on Rs. 3,500 for one year and six months at the same rate per cent.

8. If an investment of £75 becomes £78 15*s*. in 8 months, what sum invested at the same rate of interest will become £201-17-6 in 10 months?

9. What will Rs. 3,650 amount to in 4 years 2 months at Rs. 3-6-8 per cent. per annum? In what time would a sum of money double itself at the above rate?

10. If £450 amount to £523 10*s*. in 1 year and 8 months, calculate the rate per cent.

11. What sum of money at 4 per cent. will secure the same income as Rs. 25,475 at $4\frac{1}{2}$ per cent.?

12. If Rs. 450 amount to Rs. 540 in 4 years, what sum will amount to Rs. 637 8 as. in 5 years at the same rate?

13. A certain sum of money in 10 years at $3\frac{1}{2}$ per cent. amounts to Rs. 727-0-6. In how many years will it amount to Rs. 840 2 as. at 4 per cent.?

14. A certain sum of money at simple interest amounts to Rs. 505·2 in 2 years, and to Rs. 589·4 in 4 years. Find the sum and the rate per cent.

15. A certain sum of money placed at simple interest amounted in 7 months to Rs. 8,080-3-10·08, and in 10 months to Rs. 8,159-3-2·4. Required the sum and the rate per cent.

16. A person has a certain sum of money put out at $4\frac{1}{2}$ per cent. simple interest. After paying an income-tax of 4 pies in the rupee, he finds his income to be Rs. 1,410 a year. Find the sum.

CHAPTER XXXIV.

PROBLEMS ON PROFIT AND LOSS.

366. Profit or Loss is usually calculated on the cost price.

To find the gain or loss per cent. when the cost price and the selling price are given.

EXAMPLE 1. Goods bought for £8-6-8 were sold for £9-13-4; what was the gain per cent.?

The gain on £8-6-8 or £$8\frac{1}{3}$, = £9-13-4 — £8-6-8 = £1-6-8 = £$1\frac{1}{3}$.

If the gain on £$8\frac{1}{3}$ is £$1\frac{1}{3}$, what is the gain on £100?

Reqd. percentage = £ $\frac{100 \times 1\frac{1}{3}}{8\frac{1}{3}}$ = £16.

That is, the gain per cent. is 16.

EXAMPLES. I.

(1 to 6 are to be done orally.)

1. An article bought for Rs.5 is sold for Rs.6. What is the gain %?

2. Bought oranges at 6 as. per dozen and sold them at 8 as. per dozen. Find the gain per cent.

3. Out of a cask containing 50 gals., 12 gals. leaked out. What was the loss per cent.?

4. Twenty cows were bought at Rs. 50 each; but two of them died. What was the loss per cent.?

5. A book bought for Rs. 12 8 as. had to be sold for Rs. 12. Find the loss per cent.

6. A pair of oxen has bought for Rs. 150 and sold for Rs. 156 4 as. Find the gain per cent.

7. Goods bought for £126-13-4 were sold for £150. Find the gain per cent.

8. One maund of sugar bought for Rs. 16-10-8 had to be sold at Rs. 1-14-8 per viss. What was the loss per cent.?

Find the gain or loss per cent. in the following cases:—

9. A gross of buttons costing 10 as. is sold at a pie each.

10. Sugar bought at Rs. 266 per ton is sold at 2 as. per lb.

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367. To find the **Cost Price** when the selling price and the gain or loss per cent. is given.

Example 2. A person sold his land for Rs. 373 12 as., and thereby made a profit of 15 per cent. What did he pay for it?

What is sold for Rs. 115 was bought for Rs. 100.

∴ What is sold for Re. 1 was bought for Rs. $\frac{100}{115}$.

∴ What is sold for Rs $373\frac{3}{4}$ was bought for Rs. $\frac{100}{115} \times \frac{1495}{4}$ or Rs. 325.

∴ The cost price is Rs. 325.

EXAMPLES.—ORAL.

Find the cost price in each case—

Selling price.	Gain or loss %.	Selling price.	Gain or loss %.
1. Rs 110	10	5. Rs. 69	15
2. £76	– 5	6. £700	$-12\frac{1}{2}$
3. Rs. 500	25	7. Rs. x	5
4. £96	– 4	8. £ y	– 10

EXAMPLES. II.

(*1 to 4 to be done orally.*)

1. A man sold a book for Rs. 15 gaining thereby 25 per cent. What did he pay for it?

2. By selling goods for £12-13-4, a person gains $9\frac{1}{11}$ per cent.; find the cost price.

3. By selling an article for 11*s*. 11*d*. a trader gains 10 % of the cost price. What is the prime cost of the article?

4. By selling an ox for Rs. 150, a person loses 20 per cent. on its cost price. What was the original cost?

✓ 5. A person sold a house for Rs. 3,000 losing thereby $16\frac{2}{3}$% on what he paid for it. What price did he pay for it?

✓ 6. Find the cost per cwt., if the sale of 4 tons for £5-7-$2\frac{1}{4}$ gives 5 per cent. profit.

✓ 7. By selling eggs at 8 as. per dozen, 20 per cent. is lost. What did they each cost?

8. One kilometre of silk sold for £180 yields a profit of $6\frac{1}{4}$ per cent. Find the cost per metre.

Example 3. A person buys an article for Rs. 28; at what price must he sell it so as to gain $33\frac{1}{3}$ per cent.?

To gain $33\frac{1}{3}\%$ what is bought for Rs. 100 must be sold for Rs. $133\frac{1}{3}$.

$\therefore$ What is bought for Re. 1 must be sold for Rs. $\frac{133\frac{1}{3}}{100}$ or Rs. $\frac{4}{3}$.

$\therefore$ What is bought for Rs. 28 must be sold for Rs. $28 \times \frac{4}{3}$ or Rs. $\frac{112}{3}$.

$\therefore$ The selling price is Rs. 37-5-4.

EXAMPLES.—ORAL.

Find the selling price in each case—

Cost price.	Profit or loss %.	Cost price.	Profit or loss %.
1. Rs. 100	10	5. Rs. 300	$6\frac{1}{4}$
2. £150	— 5	6. £160	— $37\frac{1}{2}$
3. Rs. 90	$33\frac{1}{3}$	7. x	25
4. £120	— $16\frac{2}{3}$	8. y	— 10

EXAMPLES. III.

1. Tea bought at £5 12*s.* 6*d.* a cwt. is sold so as to gain $22\frac{1}{2}$ per cent. How much is it sold for?

2. A carriage costs Rs. 1,020; at what price must it be sold to gain $16\frac{2}{3}$ per cent.?

3. A table bought for Rs. 7 8 as. had to be sold at a loss of 25 per cent.; find the selling price.

4. A person buys a house for Rs. 1,350; and he sells it at a loss of $6\frac{2}{3}$ per cent. What was it sold for?

EXAMPLE 4. If by selling a cow for Rs. 20 4 as., I lose 20 per cent., what must I have sold her for to gain 25%?

When the selling price is Rs. 80, the cost price is Rs. 100.

$\therefore$ When the selling price is Rs. $20\frac{1}{4}$, the cost price is Rs. $\frac{100}{80} \times \frac{81}{4}$ or Rs. $\frac{405}{16}$.

Again, when the cost price is Rs. 100, the selling price must be Rs. 125.

$\therefore$ When the cost price is Rs. $\frac{405}{16}$, the selling price must be Rs. $\frac{125}{100} \times \frac{405}{16}$.

or Rs. $\frac{2025}{64}$, i.e. Rs. 31-10-3.

EXAMPLES. IV.

1. If 15 per cent. be lost by selling a horse for £340, what must I have sold him for to gain $12\frac{1}{2}$ per cent.?

2. By selling an article for £21, I lose 30 per cent. For what price must I have sold it to gain 30 per cent.?

3. A person by selling an article for Rs. 50, gains 25 per cent. What price must he put upon it so as to lose 10 % ?

4. If by selling a house for Rs. 2,050, I gain $2\frac{1}{2}$ per cent., for what must I sell it so as to lose 8 per cent. ?

EXAMPLE 5. If 35 per cent. be gained by selling sugar-candy at Rs. 270 a candy, how much per cent. will be gained or lost by selling it at Rs. 2 a viss ?

When the selling price is Rs. 135, the cost price is Rs. 100.
∴ When the selling price is Rs. 270, the cost price is Rs. 200.
The selling price of 1 candy at Rs. 2 a viss is Rs. 320.
∴ The gain on Rs. 200 is Rs. 120.
∴ The gain on Rs. 100 is Rs. 60.
∴ The gain per cent. = 60.

EXAMPLES. V.

1. If 4 per cent. be gained by selling sugar at Rs. 26 per cwt., how much per cent. will be gained or lost by selling it at 4 as. a lb. ?

2. By selling goods at £540 a ton, $12\frac{1}{2}$% is gained ; what will be gained or lost per cent. by selling them at 5*s*. a lb. ?

3. If 5 per cent. is gained by selling goods at Rs. 42 a maund, what per cent. will be gained by selling them at 2 as. 3 p. per palam ?

4. If 20 per cent. is lost by selling a number of sheep at Rs. 50 a dozen, how much per cent. will be gained or lost by selling them at Rs. 105 per score ?

EXAMPLE 6. A farmer bought 24 sheep at Rs. 6 each and sold 16 of them at Rs. 6 8 as. each ; at what price must he sell each of the rest to gain $16\frac{2}{3}$ per cent. on the whole ?

The cost price of 24 sheep at Rs. 6 each = Rs. 144.

Since he wants to gain $16\frac{2}{3}$ per cent., what cost Rs. 100 must be sold for Rs. $116\frac{2}{3}$.

∴ What cost Rs. 144 must be sold for Rs. $\frac{144 \times 116\frac{2}{3}}{100}$ or Rs. 168.

∴ The selling price of 24 sheep is Rs. 168.

But 16 sheep have been sold for Rs. $6\frac{1}{2} \times 16$ or Rs. 104.

∴ The remaining 8 sheep must be sold for Rs. 64.

∴ 1 sheep must be sold for Rs. 8.

EXAMPLES. VI.

1. A person bought 20 sheep at Rs. 8 each, and sold 12 of them at Rs. 9 each; at what price must he sell each of the rest to gain 20 per cent. on the whole?

2. A merchant bought 12 cwt. of tea at Rs. 42 per cwt., and sold 6 cwt. for Rs. 336; at what price per lb. must he sell the remainder, if he wants to gain $11\frac{1}{9}$ per cent. on the whole?

3. A person bought 120 sheep for Rs. 540. The cost of feeding them for 3 months came to Rs. 60. He sold 80 of them for Rs. 400; what should he sell the remaining sheep at, so that he may gain 5 per cent. on the whole?

4. A tradesman having goods worth £60 sold $\frac{1}{3}$ of them at a loss of 5 per cent. What must he sell the remainder for, so as to gain 25 per cent. on the whole?

EXAMPLE 7. An article on being sold at a profit of $11\frac{1}{4}$ per cent. yields 3 as. 9 p. more than if it were sold at a loss of $7\frac{1}{2}$ per cent. Find its cost.

When sold at $11\frac{1}{4}$ % profit, it is sold for $\frac{111\frac{1}{4}}{100}$ or $\frac{89}{80}$ of its cost price.

When sold at $7\frac{1}{2}$ % loss, it is sold for $\frac{92\frac{1}{2}}{100}$ or $\frac{37}{40}$ of its cost price.

The difference between the two selling prices is $\frac{89}{80} - \frac{37}{40}$ or $\frac{3}{16}$ of its cost price.

$\frac{3}{16}$ of the cost price = 3 as. 9 p.

The cost price = 3 as. 9 p. × $\frac{16}{3}$ = Re. 1 4 as.

EXAMPLES. VII.

1. An article when sold at a profit of 25 per cent. brings Rs. 9 more than when it is sold at a loss of 20 per cent. Find the cost price.

2. A table when sold at a profit of $12\frac{1}{2}$ per cent. fetches Rs. $12\frac{1}{2}$ more than when it is sold at a loss of $12\frac{1}{2}$ per cent. Find its cost price.

3. Find the cost price of a sheep which when sold at a loss of $6\frac{3}{4}$ per cent. gives 2*s*. 6*d*. less than when it is sold at a gain of $13\frac{1}{4}$ per cent.

4. A house when sold at a loss of $11\frac{1}{9}$ per cent. fetches £300-6-9 less than when sold at a gain of $18\frac{8}{9}$ per cent. Find the cost price.

EXAMPLES. VIII.

MISCELLANEOUS.

1. A contractor bought 250 sheep, and sold them for £532-5-10 at a gain of $16\frac{2}{3}$ per cent. What was the cost price of each sheep?

2. Sold some goods for £52 5*s*. losing thereby 5 per cent. How much did I buy them for?

3. The sales of a bookseller amount to Rs. 25,000; $\frac{1}{4}$ of the sales are made at a profit of 25 per cent., $\frac{7}{10}$ at a profit of $16\frac{2}{3}$ per cent., and the remainder at a loss of 25 per cent. Find the cost of stock sold.

4. A person got $12\frac{1}{2}$ per cent. profit by selling an article for Rs. $12\frac{1}{2}$. What did it cost him?

5. A merchant buys 180 kalams of paddy at Rs. 3 2 as. a kalam; 45 per cent. of it is damaged, and he sells it at a loss of 6 per cent., the rest he sells at a profit of 25 per cent. What does he gain?

6. A rice merchant buys 1,600 kalams of paddy at Rs. 3 a kalam; 3·125 per cent. is wasted. He sells 48 per cent. of the remainder at Rs. 4 8 as. a kalam, 30 per cent. at Rs. 2 8 as. a kalam and the rest at cost price. Does he gain or lose by the transaction?

7. A man buying goods by means of false scales, defrauds to the extent of 15 per cent. in buying and 15 per cent. in selling. Find his whole gain per cent.

8. A merchant buys two kinds of tea at 9*s*. 6*d*. and 6*s*. 4*d*. per lb. respectively, and mixes them equally. What must be the selling price per lb. of the mixture that he may gain 15 per cent. of the outlay ?

9. By selling a horse for Rs. 570, a person loses 5 per cent. ; what would have been his loss or gain per cent. if he had sold the horse for Rs. 712 8 as. ?

10. What is the cost price of a book which when sold at a profit of $24\frac{3}{8}$ per cent. fetches Re. 1 4 as. more than when it is sold at a loss of $15\frac{5}{8}$ per cent. ?

11. A merchant bought 500 Kg. of sugar at $2\frac{1}{2}d.$ per Kg. and sold $\frac{2}{5}$ of it at 3*d*., $\frac{1}{4}$ of it at $2\frac{3}{4}d.$, and the remainder at $2\frac{1}{4}d.$ Find his gain per cent.

12. A merchant sells goods to a tradesman at a profit of 60 per cent. ; but the man becoming an insolvent pays only 2 as. in the rupee. How much per cent. does the merchant gain or lose ?

ONE-HOUR PAPERS.—FOURTH SERIES.

REVISION EXERCISES.

I.

1. A bicyclist rode from A to B, a distance of $25\frac{1}{2}$ miles in 2 hrs. 15 min. What was his average speed in feet per second to the nearest whole number ?

2. Compare the size of a cawni and an acre. A field is 209 yards long by 352 yards wide ; find its area in acres.

✓3. A gold mohur and a rupee each weighs 180 grains. Find the number of rupees that a gold mohur is worth, the price of silver being 32*d*. per oz. troy, and the price of gold being worth £16-14-6 per lb. troy.

✓4. A country consists of two parts, one of which contains 26,865 square miles, and has a population of 160 to the square mile ; the other contains 83,580 square miles and has a population of 123 to the square mile. Find the average population per square mile for the whole country.

✓5. In a certain school, containing 453 boys, there are 71 Brahmans, 210 other Hindus, and 48 Muhammadans, and the rest are Christians. What percentage of the whole attendance do the Christians form ?

II.

1. Multiply Rs. 265-12-3 by 242, and divide the result by 73, using the most intelligent method.

2. The number of acres in a district is 931,000. What is the area which represents this district on a map of India which is drawn to a scale of 1 inch to 10 miles. Answer to the nearest square inch.

3. If 24 coolies raise an embankment $\frac{1}{2}$ mile long, 4 yards broad and $5\frac{1}{2}$ feet high in 35 days of 12 hours each, in how many days of 9 hours each will 150 coolies raise an embankment $\frac{3}{4}$ mile long, $5\frac{1}{2}$ yards broad and 4 ft. high?

4. A man travelled 72 miles in one day, half as many more on the second day, and as many miles on the third day as he had done on the two previous days taken together. What percentage of the whole journey did he travel on the second day?

5. A journey of 132 miles can be performed either by river steamer or by train. The charge by steamer is at the rate of 1*s.* for 20 miles, and by train at the rate of one penny per mile. I cannot afford to pay more than 8*s.* for the journey. What is the least distance which I must travel by steamer?

III.

1. The cost of bamboo tatty for enclosing a rectangular shed 22 yards long and 17 yards wide is Rs. 40 at $6\frac{1}{2}$ sq. yards a rupee. If the sides are of uniform height, find the height.

2. A certain number of men, women and boys can dig a tank 40 feet long, 20 feet broad, and 8 feet deep in 6 days. How long will it take the same number of men, women and boys of twice the ability to dig a tank of the same length and breadth, but 12 feet deep?

3. A man owning in England a piece of land 550 yds. long and 440 yds. wide, sells it at £25 an acre; and exchanging the sum for Indian money at Rs. 15 per pound buys land in India at Rs. 300 a cawnie. How many cawnies can he buy?

4. A man borrows £1,358 13*s.* 4*d.* at $12\frac{1}{2}$ per cent. per annum. What sum will he have to pay back at the end of $3\frac{1}{2}$ years.

5. Thirty per cent. of the inhabitants of a village are Brahmins; half of the remainder are Sudras; one-fifth of the then remainder are Mahomedans; and the rest are Pariahs. If the number of Pariahs exceeds the number of Mahomedans by 420, what is the number of the Sudras?

IV.

1. Find what length of paper $\frac{7}{8}$ of a yard wide will be required to cover the walls of a room, whose length is 27 ft. 5 in., breadth 14 ft. 7 in. and height 12 ft. 10 in.?

2. If 125 men can make an embankment 100 yards long, 20 ft. wide and 4 ft. high in 4 days working 12 hours a day, how many men must be employed to make an embankment 1,000 yards long, and 16 ft. wide and 6 ft. high in 3 days, working 10 hours a day?

3 Divide £6,842-14-5 among A, B and C, so that A may have £568-14-4 more than B, and B may have £721-18-2 more than C.

4. A money-lender charges 4 pies interest on a rupee for a month. Find the rate per cent. he charges, and the interest he will receive on Rs. 250-5-4 for 2 years 3 months.

5. The area of a river basin is 4,082 sq. miles and the annual rainfall 31 inches. The river discharges 220,000 c. ft. per minute. What percentage (to the nearest integer) of the rain that falls is discharged by the river?

V.

1. One thousand men besieged in a town have provisions for 5 weeks at the rate of 16 oz. a day for each man. How long will the provisions last, if they are reinforced by 500 additional men and the rate is reduced to 6$\frac{2}{3}$ oz. a day for each man?

2. The length and the breadth of a room are measured by means of a walking stick and are estimated at 15 ft. and 12$\frac{1}{2}$ ft. In making the calculation it is assumed that the stick is 3 ft. long. The actual length is 2 ft. 9 in. Find the errors in the measurements of the room and in the area to the nearest sq. ft.

3. A railway return ticket costs Re. 1 10 as. a day; and the annual ticket Rs. 183 12 as. How many days in the year must a man travel to save money by taking an annual ticket instead of paying for each day?

4. In a certain year the national expenditure was £183,592,264. Of this the army cost £91,710,000, the navy £29,520,000, and the civil service £28,500,000. Express each of these as a percentage of the whole to the nearest integer.

5. What sum will amount to £546 5*s*. in 3 years at 5 per cent. per annum simple interest.

VI.

1. The population of the United Kingdom was 41,609,000 in 1891, and the national income derived from taxes on the sale of intoxicants was £31,428,000. Calculate to the nearest penny the average amount contributed per head in this way.

2. Which goes the faster, a train travelling at 40 miles an hour or a steamer going at 33 knots? A knot is a speed of 6,080 ft. per hour.

3. A block of glass measures in inches $5\frac{3}{32}$ by $2\frac{7}{8}$ by $1\frac{5}{16}$. The material weighs 1·6 oz. per cubic inch. Find the weight of the block to the nearest ounce.

4. The members of a musical society have the option of paying either 5*s*. each time they attend a concert; or an annual subscription of one guinea and 2*s*. 6*d*. each time they attend a concert. How many concerts must the latter class of members attend during a year, before they can gain any advantage over the former class?

5. Divide Rs. 3,580 among A, B, C and D, so that B may get Rs. 20 more than A, C Rs. 40 more than A and B together, and D Rs. 80 less than B and C together.

VII.

1. The number of spectators at a football match was 15,568; the sum of 6*d*. was charged for admission, and 1,245 of the persons admitted paid in addition 1*s*. each for grand stand tickets. The total receipts were £423-7-6. How many were admitted without tickets?

2. Assuming that the length of an iron rod in summer expands by $\frac{1}{4000}$ of its length in winter, find the expansion from winter to summer of a bar equal in length to $\frac{2}{3}$ of a mile.

3. A man sells a horse for Rs. 414 and loses 8 per cent. on what the horse cost him. What was the original cost?

4. 40 horses can plough a field in a certain number of days; and 36 horses require 1 day more. How long would one horse take to do the same?

5. A piece of cloth is measured with a yard measure which is 1 in. too short and appears to be 36 yards long. What is its true length?

If the same piece of cloth covers the floor of a room 8 yds. 1 foot by 5 yds. 9 in., find its breadth.

VIII.

1. A sum of money is divided among A, B and C; A gets one-seventh of the money, and B 8 as. 9 p. If B's share is to C's share as 7 is to 5, find the amount received by each.

2. A square courtyard 35 ft. in the side has to be paved with square tiles 8 in. in the side. How many tiles will be used and what space will be left uncovered at the edges, if the tiles cannot be divided.

3. What is the internal area in square inches of the cross section of a pipe which holds 3 c. ft. in 8 yds. of its length?

4. If 28 men working 9 hrs. a day can do a piece of work in 17 days, find in what time 51 men working 7 hrs. a day can do a piece

of work twice as great, supposing 4 men of the second set to do as much work as 3 men of the first set.

5. The manufacturer of an article makes a profit of 25 per cent., the wholesale dealer makes a profit of 20 per cent., and the retail dealer makes a profit of 12 per cent. What is the cost, to the manufacturer, of an article which is retailed for Rs. 56.

IX.

1. Find the price of a steel casting weighing 103 tons 5 cwt. 3 qrs. at £5-8-6 per ton. If the steel weighs 488 lbs. per c. foot, what is the volume of the casting ?

2. It was calculated that in boring a tunnel the rock yielded 1 cm. of its substance to the drill per minute. The whole length of the tunnel is 19·1 Km. How long would it have taken to bore the tunnel, if the work had been entirely uninterrupted? Answer to the nearest tenth of a year.

3. Show that, in finding by Practice the price of a number of articles at £11-18-4 each or at £2-12-6 each, one line of multiplication and one line of division is sufficient.

4. If 1 c. ft. of water is equal to 6·25 gals. or 62·321 lbs., find the number of gals. and the weight of water in a cistern $4' \times 2\frac{1}{2}' \times 3'$.

5. A dishonest tradesman uses two different stones for a maund weight, one for buying and the other for selling articles. He buys a quantity of ghee as 6 maunds and sells the same as 7 maunds. The stone with which he buys being 5 per cent. heavier than a true maund, find the true weight of the stone used by him in selling.

X

1. How many bricks $9'' \times 4\frac{1}{2}'' \times 3''$ can be carried in a truck whose load is 5 tons ? The bricks weigh 145 lbs. per c. foot. Answer to 2 significant figures.

2. A certain sum of money put out at simple interest amounts in two years to £3,932-5-0 and to £4,119-10-0 in 4 years. Find the sum and the rate per cent.

3. By selling a horse for Rs. 888-4-0 a person lost $6\frac{1}{2}$ per cent.; how much per cent. would he have lost or gained, if he had sold the horse for Rs. 1,000-8-0 ?

4. An army of 2,000 men had provisions for 36 days, but after 16 days it was reinforced, and the provisions were then exhausted in 8 days. Required the number of men in the reinforcement.

5. Two trains start from A and travel towards B which is 200 miles from A, the one travels at 20 miles an hour and leaves A at 6.30 a.m.,

and the other at 8 a.m., and overtakes the former at C, 50 miles from B. Find the rate of the second train.

XI.

1. Find by Practice the weight of 272 rails each weighing 12 maunds 6 viss $3\frac{3}{8}$ seers.

2. Eighty men working 7 hours a day can do a piece of work in 48 days; 200 boys working 4 hours a day can do the same work in 56 days. In how many hours will 6 men do the work done by 24 boys in 15 hours?

3. If at simple interest Rs. 240 amount to Rs. 300 in 3 years; in how many years will Rs. 400 amount to Rs. 600 at the same rate of interest?

4. In a mixture of oil and ghee, the ghee is 40 per cent. To 30 maunds of this mixture a certain quantity of oil is added, and then the ghee is $33\frac{1}{3}$ per cent. How many maunds of oil were added?

5. There are two cogged wheels with 5 and 24 teeth respectively working into each other. If the first makes 16 complete revolutions in 10 sec., how many revolutions will the second make in one minute?

XII.

1. Find the weight to the nearest Kg. of an iron rod of square section 10 metres long and 2·3 cm. broad. One cubic centimetre of iron weighs 7·207 grams.

2. A man bought a horse for £37 10*s*. and sold him so as to gain $33\frac{1}{3}$ per cent. after allowing a discount of 10 per cent. on the selling price. At what price was the horse sold?

3. The interest for 1 year on £3,060 after an income-tax at 6*d*. per £ is deducted from it amounts to £99 9*s*. Find the rate per cent.

4. A certain field could be reaped by 7 men in a certain time, and 5 boys could do as much as two men; find how many boys would be required in addition to 30 men for the reaping of a field of twice the size in a third part of the time.

5. A sets out from a certain place and walks at the rate of $3\frac{3}{4}$ miles an hour. B follows on horseback 20 minutes later at the rate of $8\frac{1}{2}$ miles an hour; how soon will he overtake A?

XIII.

1. Find decimally the value of ·275 of £13-2-6; and reduce 4*s*. 9*d*. to the decimal of £1 by the shortest method possible.

2. Find the cost of 239 cwt. 3 qrs. 8 lbs. at £7-2-4 per cwt. Also express the result in rupees, annas and pies considering £1 as equivalent to Rs. 15.

3. A half-sovereign is made up of $\frac{11}{12}$ gold and $\frac{1}{12}$ alloy. If the weight of gold in it is 56·5 grs., find the weight of the coin to 3 decimal places, if the alloy is $\frac{7}{13}$ as heavy as gold.

4. If 80 Kg. of tea at 2·75 fr. the kilogram are mixed with 20 Kg. at 9·5 fr., at what price per kilogram should the mixture be sold to gain 10 per cent.?

5. A cistern is constructed to hold 100 gals., and the base of the cistern is a square yard. What is the depth of the cistern to the nearest inch? 1 c. ft. = 6·23 gals.

XIV.

1. Find the rent, to the nearest penny, of a field containing 71 ac. 1 ro. 15 po. at £2-12-6 an acre.

2. In a certain year, the total revenue was £144,000,000, and the tax on tea produced £5,790,000. What percentage of the whole revenue to the nearest whole number was produced by the tea tax?

3. A tradesman's takings for a week are £25. What is his net profit, if he marks his goods 20 per cent. above cost price and pays for rent labour, etc., 10 per cent. of his takings?

4. A steel plate 2 m. long, 1 m. 8 cm. broad and 2. cm. thick weighs 339·3 Kg. How much more would it weigh, if it had been made 3 cm. broader and 2 mm. thicker. Answer to the nearest gram.

5. In how many years will Rs. 503 double itself at 5 per cent.? Verify your answer graphically.

XV.

1. A motor-car was timed over a measured quarter mile, and the time given as 34 sec. The error made in taking the time may have been 1 sec. Within what limits does the speed, in miles per hour, lie?

2. The following statements contain obvious errors :—

(1) $(0{\cdot}339697)^2 = 1{\cdot}153937.$

(2) $\dfrac{27^2 \times 7 \times 23}{8 \times 22 \times 227} = 24{\cdot}07.$

Give reasons to show that they are wrong.

3. A beam having a square section is 9 ft. long and weighs $3\frac{1}{2}$ cwt. A cubic foot of the substance of the beam weighs 32 lbs. What is the thickness of the beam?

4. Milk and water are mixed so that a gallon of the mixture contains a quart of water. What percentage of the mixture is water?

5. The weight of 1 c. in. of water is 253·17 grs. and that of 1 c. in. of air 0·31 gr. Find to 3 places of decimals, how many cubic inches of water are equal in weight to 1 c. ft. of air.

XVI.

1. A solid expands so that every linear measurement is increased in the ratio of 1·0054 : 1. In what ratio is the volume increased?

2. A mile = 1·609 Km. Find (1) the relative error to one significant figure; (2) the percentage error to the nearest tenth, if we take a kilometre as $\frac{5}{8}$ of a mile.

3. 'To calculate the circumference of a circle, from $3\frac{1}{2}$ of the diameter subtract one eight-hundredth part.' Show that this rule is approximately correct; and apply it to find the circumference of a circle whose diameter is 0·05847″.

4. On a certain map an area of 1,694 sq. mi. is represented by a rectangle 2·5 in. by 1·4 in. On what scale is the map drawn?

5. A sum of money amounts in 3 yrs. to Rs. 690 at 5 per cent. What will be its amount in 5 yrs.?

XVII.

1. 'To convert tons to kilograms, multiply the weight in tons by 1,000 and add $\frac{1}{50}$ of the product to the result.' Find to the nearest kilogram the error in thus converting a weight of $53\frac{1}{2}$ tons. A ton is 1016·05 Kg.

2. On a certain map a road 1,320 yds. long is represented by a length of $18\frac{3}{4}$ in. Determine the scale of the map. What area on the map would represent an area of $\frac{1}{10}$ sq. mile?

3. In a piece of fine linen there are 180 parallel threads to the inch. What is the total length of thread used to make a square yard of such linen?

4. I wish to ascertain how many sheets of paper there are in a bundle of paper weighing 13 lbs. I find that 20 sheets weigh between 0·73 and 0·74 lb. Within what limits does the number of sheets in the bundle lie?

5. A house which costs Rs. 1,500 lets for Rs. 65 a year; the repairs, etc., cost $\frac{1}{4}$ per cent. on the cost. What rate of interest does it pay?

XVIII.

1. A boy had to multiply £5·786495 by 317; but he disregarded all the figures in the multiplicand after the third decimal place. Find to the nearest farthing what the consequent error was.

2. The adjacent sides of a rectangular are measured and found to contain 1·534 m. and 0·662 m. respectively; find the area. In

each measurement there may be an error of 2 mm., what is the greatest possible error in the area of the rectangle as calculated from the above measurements ?

3. ‘To calculate the diameter of a circle, from $\frac{1}{3}$ of the circumference subtract one-hundredth and also one two-hundredth part.’ Show that this rule is approximately correct ; and apply it to find the diameter of a circle whose circumference is 382·7″.

4. A sells an article to B at a profit of 20 per cent. B sells it to C at a profit of 5 per cent. If C pays 70*s*., what did it cost A to the nearest shilling ?

5. The population of two towns are 107,509, and 189,160 ; their birth-rates per thousand are 27·9 and 25·7 respectively. Find to the same degree of exactness the brith-rate for the two towns together.

XIX.

1. Give reasons to show that the following statements cannot be correct :—

(1) $3{\cdot}456 \times 0{\cdot}987 = 3{\cdot}652\ldots\ldots\ldots$

(2) $3{\cdot}456 \div 0{\cdot}987 = 3{\cdot}348\ldots\ldots\ldots$

(3) $\dfrac{4\frac{1}{8} \times 32^2}{256} = 12{\cdot}5\ldots\ldots\ldots$

2. Coal is sent to a railway station at 21*s*. 10*d*. per ton ; and there is a futher charge for delivery of 1*s*. per ton per mile. Find the cost of 6 tons 19 cwt. delivered at a place 3 miles distant from the railway.

3. In a school there are five classes ; and the average number in each class is exactly 29. If the average for the 1st, 3rd and 5th classes is 30, find the total number of students in the 2nd and 4th classes. What modification will your answer need, if the above average of 29 is true only to the nearest integer ?

4. Six houses are sold for £2,500. They each let at £35 a year. Four of them are occupied the whole year ; two of them are empty for a quarter. The owner has to spend £60 a year on repairs during the year. What per cent. does he get on his money ?

5. In 1902 there were—

Primary Schools.	England.	Scotland.	Ireland.
No. on the registers	5,881,278	768,598	737,086
Average attendance	4,890,237	646,501	487,098

Express the attendance as a percentage of the number on the registers for each of the three countries, and also for the three countries together. Answer to the nearest integer.

XX.

1. The revenue of India for the year 1903–4 was Rs. 1,145,331,000. Express this in crores to one place of decimal, and in lakhs to the nearest ten lakhs.

2. The district rate is 3*s*. 6*d*. in the £ on the rateable value of a house, and the house-rent is $\frac{6}{5}$ of the rateable value. What is the district rate on a house rented at £90.

3. Standard gold consists of 11 parts of pure gold to 1 part of alloy. If 1 oz. Troy of standard gold is worth £3-17-10½, how many grains, to the nearest integer, of pure gold are there in a sovereign? 1 oz. Troy = 480 grains.

4. The surface of a sphere of radius $r = 4\ \pi\ r^2$ where $\pi = 3{\cdot}14159$, if π is taken instead to be equal to $3\frac{1}{7}$, find roughly in square miles the difference that this will make in the calculated area in the earth's surface, the earth being a sphere of 4,000 miles radius.

5. A man borrows Rs. 250 for 9 months at 3 per cent. per annum, and by immediately lending it out at a higher rate of interest he makes a profit of Rs. 2 13 as. What rate of interest does he charge?

ANSWERS

CHAPTER I.

Examples.

i. **1.** 17,075. **2.** 999,090. **3.** 909,900. **4.** 2,200,002.
5. 90,090,009. **6.** 800,080,080. **7.** 300,003,003. **8.** 8,008,018,018.
9. 1,15,015. **10.** 8,09,098. **11.** 15,15,015. **12.** 9,99,09,009.

ii. **1.** Rs. 50,204. **2.** Rs. 3,738. **3.** Rs. 41,325. **4.** 178,750,000.

iii. **1.** Furn Rs. 3,250 ; Land Rs. 7,625 ; House Rs. 9,625
2. Rs 100. **3.** 6. **4.** Coat Rs. 12 ; Waist-coat [Rs. 6 ; Trousers Rs. 18.

CHAPTER II.

Examples.

i. **1.** 5,625. **2.** 12,876. **3.** Rs. 2,26,600. **4.** 3,097,600.

ii. **1.** Rs. 354. **2.** Rs. 15,025. **3.** 336 yds. **4.** 140,371.

iii. **1.** (1) 4,024,422. (2) 3,207,624. (3) 670,595. (4) 3,882,564.
2. (1) 740249947815. (2) 813118878408. (3) 316849151416.
(4) 1141920618282.
3. (1) 4,307,040. (2) 33,573,330. (3) 54,687,807. (4) 59,200,080.
4. (1) 596,725,248. (2) 1,308,380,360. (3) 2,023,238,610.
(4) 196,265,225,592.

CHAPTER III.

Examples.

i. **1.** 345. **2.** 7,487. **3.** 2,024. **4.** 2,547.
5. (1) 5 ; 5. (2) 3 ; 29. (3) 3 ; 17. (4) 2 ; 41.
6. (1) 9 ; 7. (2) 5 ; 22. (3) 3 ; 43. (4) 2 ; 64.
7. (1) 78 ; 28. (2) 66 ; 53. (3) 60 ; 125. (4) 52 ; 29.
8 (1) 101 ; 92. (2) 85 ; 72. (3) 83 ; 73. (4) 55 ; 137.
9. (1) 3,145 ; 73. (2) 2,383 ; 31. (3) 1,532 ; 20. (4) 1,429 ; 163.

ii. **1.** 103 ; 65. **2.** 42 ; 263. **3.** 47 ; 296. **4.** 2,593 ; 149.
5. 5,399 ; 636. **6.** 32,465 ; 836. **7.** 15,564 ; 1,745. **8.** 40,276 ; 750
9. 789,075. **10.** 307,021. **11.** 786,250. **12.** 7,089,764.
13. 37. . . . **14.** 33. **15.** 589. **16.** 6,956.

iii. I. **1.** $2^4 \times 13$. **2.** $2^6 \times 11$. **3.** $2^2 \times 13^2$ **4.** $2 \times 5 \times 17^2$.
5. $5 \times 11 \times 31$. **6.** $2^4 \times 3^2 \times 5 \times 11$. **7.** $3^6 \times 2 \times 11^2$. **8.** $3^4 \times 5^3$.
II. $469 = 7 \times 67$; $867 = 3 \times 17^2$; $1365 = 3 \times 5 \times 7 \times 13$; $2127 = 3 \times 709$.
[$2197 = 13^3$; $8172 = 3^2 \times 2^3 \times 11^2$.

iv. **1.** $3^2 \times 7^2$; 21. **2.** $2^2 \times 11^2$; 22. **3.** $3^2 \times 8^2$; 24. **4.** $3^2 \times 4^2 \times 5$.
5. $5^2 \times 7^2$; 35. **6.** $4^2 \times 4^2 \times 5$. **7.** $4^2 \times 11^2$; 44. **8.** $2^{10} \times 3$.
9. $7^2 \times 11^2$; 77. **10.** $2^2 \times 3^6 \times 3$. **11.** $3^4 \times 13^2$; 117. **12.** $2 \times 2^6 \times 3^4$.
13. 3. **14.** 2. **15.** 3. **16.** 11.
17. 2. **18.** 5. **19.** 3. **20.** 3×7.

v. **1.** A, 44,559 ; B, 43,311. **2.** Rs. 47,161 ; Rs. 52,839.
3. Rs. 34,735 ; Rs. 34,249. **4.** 502,624 ; 497,376.

vi. **1.** 126. **2.** 259. **3.** 16. **4.** 312,500.

vii. **1.** 20,237,225. **2.** 344. **3.** 336. **4.** 24.
5. 840. **6.** Rs. 550. **7.** Rs. 1,36,925.
8. In the 1st 304 ; in the second 436. **9.** 63,779,352.
10. 657083603757. **11.** 108761733. **12.** 987654319753086.
13. 499,800. **14.** 26 yrs. **15.** 30 yrs.
16. 12,247. **17.** Rs. 9,410. **18.** 235,014.
19. 79 runs. **20.** 2,338 mi.; 1030 ft. **21.** Rs. 174.
22. Rs. 99,975. **23.** 12 times. **24.** 38 times. **25.** £215.
26. Rs. 540 ; Rs. 660. **27.** Husband, 55 meas. ; wife, 29 meas.
28. 20 yds. **29.** 21. **30.** 12 cows. **31.** He copied 80,976 for 89,076.
32. He copied 598 for 578. **33.** 2,637 ; 131.
34. Dividend, 167,594 ; quotient, 321. **35.** 11 ; 5667 ; 708.
36. 25,793 ; 3684 ; 409. **37.** A, 45 ; B, 55 ; C, 50.
38. A, Rs. 42,000 ; B, 58,000 ; C, Rs. 33,000. **39.** 74. **40.** Rs. 35.

Examples.

CHAPTER IV.

i. **1.** (1) 495152. (2) 1682928. (3) 876970. (4) 1915453.
2. (1) 9003. (2) 18766. (3) 45428. (4) 40698.
3. (1) 499428. (2) 55646. (3) 632111. (4) 150129.
4. (1) Rs. 5,024-15-1. (2) Rs. 4,109-6-10. (3) Rs. 5,266-4-3.
(4) Rs. 9,976-5-1.
5. (1) Rs. 13,922-6-9. (2) Rs. 15,624-15-9. (3) Rs. 188-3-$4\frac{1}{2}$.
(4) Rs. 401-4-$10\frac{1}{2}$.
6. (1) 14470+1a. 6 p. (2) 18960+2 as. 8 p. (3) 21326+2as. 3 p.
(4) 44213+10 p.

ii. **1.** (1) 52,765. (2) 91,845. (3) 358,075. (4) 688,196.
2. (1) 112,695. (2) 258,565. (3) 379,195. (4) 758,201.
3. (1) 28,399. (2) 103,510. (3) 286,537. (4) 841,859.
4. (1) 14,039. (2) 18,146. (3) 1,489. (4) 3,353.
5. (1) 174. (2) 433. (3) 211. (4) 257.
6. (1) £416-13-3. (2) £841-19-9. (3) £81-0-$4\frac{1}{4}$. (4) £527-12-$8\frac{1}{3}$
7. (1) 849. (2) 4,356. (3) 8,765 + 2*s*. 3*d*. (4) 16,175 + 1*s*. 2*d*.

iii. I.	Rs.	A.	P.	II.	£	*s.*	*d.*
1.	9,891	14	3	**1.**	989	18	$9\frac{3}{4}$
2.	11,495	13	3	**2.**	6,091	7	$1\frac{3}{4}$
3.	9,002	4	8	**3.**	10,064	16	$5\frac{3}{4}$
4.	14,082	4	7	**4.**	12,222	4	$0\frac{3}{4}$
5.	18,064	15	0	**5.**	10,389	15	$7\frac{1}{2}$
6	5,138	4	10	**6.**	1,371	13	$4\frac{1}{4}$
7.	5,531	13	8	**7.**	2,930	2	11
8.	9,603	13	0	**8**	5,340	8	$6\frac{1}{2}$
9.	2,052	1	0	**9.**	3,977	18	$11\frac{3}{4}$
10.	4,081	5	3	**10.**	5,358	7	$1\frac{1}{4}$

iv. **1.** Rs 6788·82 c. **2.** Rs. 602·30 c. **3.** Rs. 9,214-7-3.
4. Rs. 2,25,081-13-8. **5.** £47-17-$1\frac{3}{4}$. **6.** £145-15-$10\frac{1}{4}$.

7. Rs. 4,99,867-14-8. **8.** Rs. 7,39,456-15-10. **9.** £8,769-15-10¾.
10. £23,456-15-10½. **11.** Rs. 1,112-1-4. **12.** Lost £14,071-12-2½.

v. **1.** 5 as 4 p. **2.** £1,770-7-1¾. **3.** A has 13*s*. 7*d*. more. **4.** £6-9-0¼.

vi. **1.** Rs. 828. **2.** £761 5*s*. **3.** Rs. 6,524 6 as. **4.** Rs. 6,33,254-2-3.
5. Rs. 641-12-8. **6.** £20-9-1½. **7.** Rs. 27-6 as. **8.** £64-5-3.
9. Rs. 216. **10.** Rs. 600 10 as. **11.** Rs. 1,817 10 as. **12.** Rs. 3,308.

vii. **1.** (1) Rs. 690. 19 c. Rem. 67 c. (2) Rs. 795. 17 c. Rem. 93 c.
(3) Rs. 1846·99 c. ,, 2 c. (4) Rs. 4415·66 c. ,, 66 c.
2. £83-15-3¼. **3.** £3-4-7. **4.** 3 as. **5.** £34-8-2½.

viii. **1.** 24. **2.** Rs. 453. **3.** 31. **4.** 105. **5.** 21. **6.** 3. **7.** 68. **8.** 315.
ix. **1.** 160. **2.** 68. **3.** 54,576. **4.** 16 mds. 7 viss.

CHAPTER V.

Examples.

i. **1.** 81,797. **2.** 294,065. **3.** 5,769,900. **4.** 685,014.
5. 119 can. 5 mds. 3 srs **6.** 109 can. 9 mds. 2 srs. 4 pals.
7. 351 can. 18 mds. 2v. 2 srs. 7 pals. **8.** 228 can. 5 mds. 2v. 3 srs. 1 pal.

ii. **1.** 62720. **2.** 2194. **3.** 18277. **4.** 182420.
5. 1 qr. 10 lbs. 4 oz. 8 drs. **6.** 20 tons 17 cwt. 3 qrs. 1 lb.
7. 20 tons 11 cwt. 1 qr. 1 lb. 14 oz. **8.** 25 tons 7 cwt. 10 lbs. 2 oz.

iii. **1.** 255. **2.** 137. **3.** 2. **4.** 5,382. **5.** 10585200. **6.** 2937650.
7. 8 yrs. 252 days 3 hrs. 33 min. **8.** 18 yrs. 2 days 19 hrs. 13 m. 20 sec.

iv. **9.** 190080 in. **10.** 14580 ft. **11.** 59064 in. **12.** 27024 ft.
13. 78 mi. 7 fur. 68 yds. 2 ft. 8 in. **14.** 168 mi. 6 fur. 3 yds. 2 ft.
15. 37 mi. 37 yds. 2 ft. 2 in. **16.** 8601 mi. 2 fur. 103 yds. 1 ft.

v. **1.** 117228. **2.** 1235031. **3.** 480262. **4.** 3083371.
5. 980 kal. 7 mar. 1 mea. 7 oll.

	Gar.	par.	mar.	mea.	ol.		Gar.	par.	mar.	mea.	ol.
6.	17	59	3	5	0	**7.**	34	19	0	7	6.
8.	48	18	0	0	7.						

vi. **1.** 78. **2.** 3,348. **3.** 312. **4.** 588.
5. 1 bar. 64 gals. **6.** 8 bar. 6 gals. 2 qts.
7. 1,972 qrs. 5 bush. 2 pks. **8.** 114 qrs. 5 bush. 2 pks.

vii. **1.** 7,128 sheets. **2.** 27,096 sheets.
3. 167 bales 5 reams 19 quires 12 sheets.
4. 118 bales 3 reams 2 quires 2 sheets.

viii.

I.	Mds.	viss.	pals.	tolas		Mds.	viss.	pals.	tolas.
1.	96	6	12	1	**2.**	92	6	6	2
3.	152	6	33	2	**4.**	88	1	33	0
5.	71	2	7	0	**6.**	113	3	18	1
7.	61	3	29	2	**8.**	71	5	8	0
9.	56	2	35	2	**10.**	56	3	7	0

II.	Tons.	cwt.	qrs.	lbs.	oz.		Tons.	cwt.	qrs.	lbs.	oz.
1.	2	3	1	18	8	**2.**	3	12	2	12	0
3	2	11	0	0	0	**4.**	72	5	1	0	0

	Tons.	cwt.	qrs.	lbs.	oz.		Tons.	cwt.	qrs.	lbs.	oz.
5.	10	12	3	2	12	**6.**	19	11	3	20	7
7.	9	15	1	19	9	**8.**	7	12	2	15	10
9.	17	0	0	23	8	**10.**	15	19	1	4	10

III.	Mi.	fur.	yds.	ft.	in.		Mi.	fur.	yds.	ft.	in.
1.	3	3	26	2	6	**5.**	16	0	209	0	10
2.	19	—	584	2	10	**6.**	14	1	180	2	1
3.	18	—	611	0	2	**7.**	16	6	145	0	11
4.	20	—	504	1	2	**8.**	14	1	91	2	10

ix. **1.** 1419 can. 16 md. 5 v. 4 sr. 7 pals.
2. 4656 can. 15 md. 7 v. 1 sr. 2 pals.
3. 361 tons 17 cwt. 2 qr. 15 lbs.
4. 432 tons 7 cwt. 3 qr. 23 lbs. 12 oz.
5. 7 cwt. 2 qr. 21 lbs. 12 oz. 13 dr. **6.** 5 mi. 7 fur. 8 ch. 73 l.
7. 28 mi. 6 fur. 8 ch. 75 l. **8.** 268 mi. 5 fur. 108 yds. 2 ft.
9. 326 gar. 77 par. 4 mar. 6 m. 7 oll.
10. 6843 gar. 13 par. 3 mar. 5 m. 6 oll.
11. 26 ba. 7 rea. 16 qui. 22 sht. **12.** 55 ba. 7 rea. 12 qui. 18 sht.

x.	Can.	mds.	vis.	srs.	pals.
1.	8267	8	1	3	1
2.	218307	15	5	—	37

		Tons	cwt.	qrs.	lbs.	oz.
3.	(1)	56	5	1	13	4
	(2)	96	7	1	11	12

		Tons.	cwt.	qrs.	lbs.	oz.
4.	(1)	45388	5	2	24	0
	(2)	39515	13	2	8	0
5.	(1)	320152	4	3	24	0
	(2)	265373	6	3	20	0

		Mi.	fur.	yds.	ft.	in.
6.	(1)	12	7	99	1	4
	(2)	15	0	206	0	4
7.	(1)	155	2	197	0	8
	(2)	390	4	68	1	4

8. 1386 days 4 hrs. 11 min. 50 sec.
9. 6273wks. 6da. 14 hrs. 4 min. 20 sec.

	Gar.	par.	mar.	mes.	oll.
10.	8926	27	1	2	0
11.	12430 kal.	8	4	4	
12.	182768 kal.	11	7	6	

xi. **1.** 2 mi. 4 fur. **2.** 1 mi. 1166 yds. 2 ft.
3. 1 mi. 146 yds. 2 ft. **4.** 2 mi. 5 fur.

xii. **1.** 54 tons 3 cwt. 2 qrs. 24 lbs. 9 oz. **2.** 37 tons 3 cwt. 3 qrs. 20 lbs. 2 oz. 11 dr.
3. 2 cwt. 3 qrs. 21 lbs. **4.** 4 tons 5 cwt. 27 lbs. 1 oz. 10 dr.
5. 2 tons 4 cwt. 1 qr. 21 lbs. **6.** 19 mds. 6 v. 4 srs. 3 pals.
7. 11 can. 4 mds. 7 v. 31 pals. **8.** 6 par. 4 mar. 7 meas. 7 oll.
9. 9 par. 3 mar. 6 meas. 4 oll. **10.** 2 days 22 hrs. 7 min. 42 sec.
11. 3 yrs. 230 days 20 hrs. 32 min. 38 sec. **12.** 54 yds. 9 in.

xiii. **1.** 6 yds. **2.** 4 yds. **3.** 2 yds. 2 ft. 3 in. **4.** 13 ft.

xiv. **1.** 83. **2.** 20102. **5.** 7040. **6.** 5940.
7. 221760. **8.** 1980.

Examples.

CHAPTER VI.

i. **1.** £5,986-0-8. **2.** Rs. 12-4-4.
3. Carriage Rs. 1,500 ; horse Rs. 300. **4.** 865 lbs.

ii. **1.** 11. **2.** 140. **3.** 135. **4.** 106.

iii. **1.** 16 ft. 6 in. **2.** 10 ft. **3.** 11 ft. **4.** 5 ft. 4 in.

iv. **1.** Rs. 9-6 as. **2.** £120-6-3. **3.** 2 pies. **4.** 1*s*. 3*d*.

v. **1.** Rs. 33-5-4. **2.** £7-16-1¼. **3.** Gain £17-13-4. **4.** 6 as. 7 p.
vi. **1.** Re. 1-8 as. **2.** Rs. 3-1 an. **3.** £37-11*s*. **4.** £1-5-5.
vii **1.** Rs. 7-8 as. **2.** Rs. 6-8 as. **3.** Rs. 20. **4.** Rs. 1-7-4.
viii. **1.** 160. **2.** 25. **3.** 216. **4.** 97.
ix. **1.** 10. **2.** 19. **3.** 5. **4.** 11.

Examples.

CHAPTER VIII.

	Kg.	g.	dg.	cg.	mg.		Km.	m.	dm.	cm.	mm.		fur.	ch.	links.
1.	—	38	7	3	2	**2.**	—	29	3	4	3	**3.**	44	0	80
4.	—	57	5	5	6	**5.**	—	80	9	5	8	**6.**	37	2	26
7.	301	0	2	1	6	**8.**	267	3	9	6	1				

9. Rs. 124. **10.** Rs. 354·625. **11.** Rs. 5210·96. **12.** Rs. 295·33.
13. £864·525. **14.** 3559·6725 cwt. **15.** 253·86 ft. **16.** 63·886 tons.
17. (1) 189·925. (2) 94·97025. (3) 565·9508. (4) 863·0507.
18. (1) 320·907. (2) 909·0873. (3) 1993·9954.
(4) 135·9799. (5) 1398·63902. (6) 1020·7404.
(7) 844·0715. (8) 3535·9475.

		M.	dm.	cm.	mm.		G.	dg.	cg.	mg.		Fur.	ch.	links
ii.	**1.**	4	8	7	9.	**2.**	3	9	8	9.	**3.**	1	7	85.
	4.	3	8	2	6.	**5.**	3	3	8	9.	**6.**	5	7	78.
	7.	5	1	9	7.	**8.**	6	8	5	8.				
	9.	3	8	7	8.	**10.**	3	8	0	8.	**11.**	1	7	88
						12.	17	4	9	7.				

13. (1) 7·419. (2) 224·663. (3) 75·2111. (4) 126·549322.
14. (1) 16·888235. (2) 1·24685. (3) 224·2235679. (4) 90·08035.
15. (1) 1·2088. (2) 0·2788. (3) 92·892. (4) 4·21798

iii. **1.** 27·9883. **2.** 103·2816. **3.** 8·1985. **4.** 2·24047.
5. 1949·578. **6.** 30·91356. **7.** 0·97696. **8.** 0·4587.

Examples.

CHAPTER IX.

		G.	dg.	cg.	mg.	grams.		M.	dm.	cm.	mm.	metres.
i.	**1.**	11	7	2	5	= 11·725.	**4.**	22	9	6	2	= 22·962.
	2.	71	2	5	6	= 71·256.	**5.**	48	6	2	4	= 48·624.
	3.	88	1	0	1	= 88·101.	**6.**	8	0	7	3	= 8·073.

7. 61 fur. 3 ch. 76 l. = 61·376 fur. 8. 88 fur. 74 l. = 88·074 fur.

ii. **1.** 68·95 ; 96·53 ; 551·6 ; 11032 yds.
2. Rs. 154·5 ; 231·75 ; 772·5 ; 10300.
3. £435·75 ; £581 ; 3631·25 ; 50837·5.
4. 100·0625 ; 160·1 ; 8005 ; 12007·5 tons.
5. 1383·36 ; 1844·48 ; 16139·2 ; 207504.
6. 763·343 ; 981·441 ; 6542 94 ; 87239·2.
7. 336·4734 ; 494·7101 ; 1803·945 ; 39255 23.
8. 594·579 ; 792·772 ; 6936·755 ; 89186·85.

iii. **1.** (1) 2·744. (2) 5·047. (3) 8·722. (4) 147·343.
2. (1) 2162. . . . (2) 471. (3) 118. (4) 38030.

3. (1) 0·02958. (2) 0·0435. (3) 0·81606. (4) 0·697292.
4. (1) 0·00018. (2) 25543·69. (3) 21941289·736. (4) 107477·37.

iv. **1.** (1) 238·0025. (2) ·02412. (3) 58·59375. (4) 47·680272.
(5) ·007245. (6) 1·7592. (7) ·010763. (8) 82·383276.
2. (1) 57·4294. (2) ·8336818. (3) 188·22713. (4) ·015625.
3. (*a*) 425·083606. (*b*) ·017567224. (*c*) 1000·2340881. (*d*) 1·2307809.
4. (*a*) ·00112896 ; 11·2896. (*b*) 3254·76. (*c*) 2·368125.
(*d*) ·00062625.
5. (1) 1396·367424. (2) 5457·515625. (3) 39124·217375.
(4) 16915·218263.

Examples. CHAPTER X

i. **1.** £1·493 ; 3·233m. **2.** Rs. 10·157 ; 4·601 gs. **3.** 10·712 ; 5·356.
4. 0·02403 ; 0·00801. **5.** 0·6255. **6.** 0·1375. **7.** 0·1907125.
8. 26·55. **9.** 2·18. **10.** 0·00038. **11.** 0·0053. **12.** 0·5075 ;
[0·03625 ; 0·126875.

ii. **1.** 3·00105. **2.** 4·0034. **3.** 7·0004. **4.** 0·239.
5. 0·969. **6.** 0·00003009. **7.** 0·093. **8.** 0·07003.

iii. **1.** 0·0000528. **2.** 0·0000609. **3.** 0·000239. **4.** 0·000321.
5. 0·026375. **6.** 0·0002525. **7.** 0·0006475. **8.** 0·000067625.
9. 22·28125. **10.** 0·15625. **11.** 59·9125. **12.** 375·31168.

iv. **1.** 91·7. **2.** 4914·01. **3.** 12·1. **4.** 5·28. **5.** 225.
6. 793·47. **7.** 70·031. **8.** 60·56.

v. **1.** 0·21. **2.** 0·0072. **3.** 0·168. **4.** 0·97.
5. 793·47. **6.** 0·82175. **7.** 0·0031. **8.** 5700.

vi. **1.** 1679. **2.** 57. **3.** 7853. **4.** 32.
5. 576. **6.** 6056. **7.** 543. **8.** 9487096.

vii. **1.** 3290. **2.** 73730. **3.** 22500. **4.** 30700.
5. 570000. **6.** 700310. **7.** 29900. **8.** 56390.

viii. **1.** 3·74. **2.** 120·712. **3.** 31·25. **4.** 67·36.
5. 1181·5625. **6.** 1·6. **7.** 0·51472. **8.** 1·25.

ix. **1.** 1 2827. **2.** 0·0001. **3.** 0·7103. **4.** 0·0003.
5. 10·5138. **6.** 12·1760. **7.** 2·6116. **8.** 12644·0425.

x. **1.** 2. **2.** 4. **3.** 2·3. **4.** 2·62. **5.** 0·12.
6. 2·301. **7.** 9·37. **8.** 1·395. **9.** 1·36. **10.** 15·3.
11. 0·5. **12.** 0·97. **13.** 0·0525. **14.** 0·4464. **15.** 4·75.
16. 0·05.

Examples. CHAPTER XI.

i. **1.** 989·628*d*. **2.** 1320·048 p. **3.** 1070·592 pals. **4.** 5215·2 f.
5. 5842·7432 lbs. **6.** 2685·76 yds. **7.** 308262·24 sec. **8.** 1320 oll.

ii. **1.** 6 as. **2.** 17*s*. 6*d*. **3.** Rs. 2-5 as.
4. £3-2-9. **5.** £14-3-9. **6.** 3 tons 16 cwt. 2 qrs.
7. 6 days 3 hrs. 25 min. 12 sec. **8.** 8 gar. 30 par.

iii. **1.** 18 cwt. 3 qrs. 19 lbs. 11 oz. 6·272 drs. **2.** 10 srs. 7 pals. $1\frac{1}{2}$ tol.
3. 16 mds. 5 v. 1·76 pals. **4.** Rs. 131 4 as.

5. 223 mi. 5 fur. 44 yds. **6.** Rs. 17-5-6. **7.** Rs. 59 8 as.
8. 7 mds. 17 pal. $1\frac{1}{2}$ tol.

iv. **1.** Rs. 64·296875. **2.** 6·25 mi. **3.** 0·009375 days.
4. 0·009375 tons. **5.** Rs. 180·015625. **6.** 36·65 can.
7. 15·640625 mi. **8.** £4·565625.

v. **1.** Rs. 5·71875. **2.** £10·6875. **3.** £4·78125. **4.** £1·615625.
5. Rs. 3·46875. **6.** 6·09375 tons. **7.** 10·33125 mi. **8.** 5·196875 mi.

vi. **1.** £1-3-$5\frac{1}{4}$. **2.** Rs. 2-15-6. **3.** Rs. 413-12-10. **4.** Rs. 410-6-6.
5. £34-5-$2\frac{1}{2}$. **6.** Rs. 164 15 as. **7.** 1 ton 5 cwt. 2 qrs. 17·16 lbs.
8. The 2nd by 0·01 of a rupee.

vii. **1.** $\frac{1}{4}$*d.* **2.** £41-11-$7\frac{1}{4}$. **3.** Rs. 3-8-$5\frac{1}{4}$. **4.** £2-5-$9\frac{3}{4}$.
5. £5-12-$9\frac{3}{4}$. **6.** Rs. 54-2-6. **7.** Rs. 6-14-$4\frac{1}{2}$. **8.** Rs. 11-7-8.

Examples.

CHAPTER XII.

	Value.	Value.		Mi.	fur.	ch.	yds.	ft.	in.
i. **1.**	(1) Rs. 6-9-3	**2.** (1) £87-13-5.	**3.** (1)	864	2	5	14	2	11
	(2) Rs. 65-12-6.	(2) £8-15-4.	(2)	8	5	1	10	0	2
	(3) Rs. 0-10-6.	(3) £876-14-0.	(3)	0	6	9	20	0	4
	(4) Rs. 657-3-2.	(4) £0-17-6.	(4)	0	0	0	15	0	8

ii. **1.** Rs. 771900. **2.** £1513000. **3.** 151·355 g. ; 151·351 g.
4. 34·05 and 34·09.

iii. **1** 15·809 m. **2.** 71·690 g. **3.** 470·564 ; 470·562. **4.** 3·87 and 3·89.

iv. **1.** (1) 5130500. (2) 6916000. (3) 248000000. (4) 43 millions.
2. (1) 164·75. (2) 0·04. (3) 685·22. (4) 3·16.
3. (1) 1059. (2) 4464. (3) 273. (4) 398001.
4. (1) 0·002873. (2) 0·0001445. (3) 277·3. (4) 2·888.
5. 24855 miles ; 0·063 mile to 3 places. **6.** 11895 million francs.
7. 46·18 miles. **8.** ± 0·0125 gram.
9. 2·864632 Km. to the nearest mm. **10.** 17·2 lbs. to the nearest [10th.
11. 62·445 ft. to the nearest 1000th. **12.** 70351 to the nearest ounce.

v. **1.** (1) 36·74. (2) 53·02. (3) 22·59. (4) 94·63.
2. (1) 1·113. (2) 4·181. (3) 2·178. (4) 11·11.
3. (1) 3·098. (2) 1·185. (3) 14·249. (4) 53·2042.
4. 7958 miles. **5.** 28·57 m. **6.** 4·19 cm. :—0·8 mm. **7.** 4·2 ; 160.

Examples.

CHAPTER XIII.

i. **1.** Rs. 14-3-4. **2.** 84·4. **3.** (1) 66·85. (2) 60·92. (3) 50·75. (4) 65·68.
4. James 15·10, Dharma 16·25, Doss 31·91, Chelliah 17·51.
5. Rs. 1118511 13 as. ; Rs. 37077 11 as. 10 p.
6. 29·715 ; 0·030— ; 0·045 + ; 0·041 + .
7. 1619565 to the nearest integer. **8.** 6 as. 1 p. **9.** 3·76.
10. 11 st. 0·75 ± ·05 lbs. **11.** 0·026 + . **12.** Rs. 5591541 6 as. 5 p.

ii. **1.** 15 years. **2.** Rs. 2 7 as. 7 p. to the nearest pie.
3. 440. **4.** 18·25.

iii. **1.** 3·35. **2.** 4·4. **3.** 220. **4.** 22·5.

iv **1.** 30·5 in. **2.** 11 st. 8 oz. **3** £103-6-8 **4.** 72·1.

One-Hour Papers : First Series.

i. 1. Rs. 4 6 as. 2. 65. 3. £127-4-1¼. 4. 0·31875 tons. 5. 4·5 litres.
ii. 1. 20. 2. Rs. 61 4 as. 3. 790. 4. 44·56 g. 5. Rs. 750
iii 1. 17*s*. 4*d*. 2. 27 mi. 3. 31 min. 15·5 sec. 4. £ 90-11-0¾.
5. 2 yds. 1 ft. 10·3 in.
iv. 1. £127-3-9. 2. Rs. 1-8-1. 3. 7 yds. 1 ft. 7 in 4. 63·36 in 5. 3*s*. 1*d*.
v. 1. £66-12-4¼. 2. 16. 3. £6048-18-6½. 4. 43·4 oz. 5. 9 cm.
vi. 1. 31. 2. 165. 3. 25·15 ± 0·02. 4. 2·04 m. 5. 3069 ft. ; 935·4 m.
vii. 1. 9. 2. 8·3. 3. 1·8 in. 4. 5*s*. 4½*d*. 5. 11303 yds.
viii. 1. 215. 2. Rs 3,828. 3. 15·063 Km. 4. Rs. 111-5-2.
5. 1567·3 lb. 720 Kg.
ix. 1. £1-8-6. 2. 28. 3. 0·157. 4. 0·08 in. ; 19·5 in. 5. 2·041 mm.
x. 1. £2142-5-4. 2. 11·743. 3. 5149 ; 0·048375. 4. £18-1-8¼. 5. 364 in.
xi. 1. Rs. 9-6 as. 2. 725 ; 0·111 in. 3. 473·34. 4. 0·03 ; ½*d*. 5. 6955 mi.
xii. 1. 0·3575 m. ; 1 ± ·0005 m. 2. 0·000315. 3. 43.
4. 4·28 ; 0·004 ; 0·093 ; 6 as. 4 p. 5. 7·23 ft. lbs.

Examples.

CHAPTER XIV.

i. 1. 180. 2. 165. 3. 33. 4. 13
5. 9. 6. 434. 7. 287. 8. 303.
9. 3 × 3. 10. 5 × 3 × 7. 11. 3 × 3 × 11. 12. 3 × 2 × 2.
ii. 1. 17. 2. 91. 3. 285714. 4. 29. 5. 851. 6. 523.
7. 69. 8. 23. 9. 19. 10. 4. 11. 142857. 12. 76923.
iii. 1. 11. 2. 23. 3. 44. 4. 3. 5. 23. 6. 9. 7. 13. 8. 29.
iv. 1. 74. 2. 57. 3. 117. 4. 36. 5. 153. 6. 2003.
7. 75. 8. 442.
v. 1. 1 an. 1 p. 2. 1*s*. 3*d*. 3. Rs. 1-13-1. 4. 15*s*. 4*d*.
5. 1 pal. 6. 3 sec. 7. Rs. 5-6-3. 8. 1*s*. 7*d*.
vi. 1. 11 in. 2. £1-3-3. 3. 4 as.
4. Rs. 2 8 as. ; 11, 17 and 30 sheep respectively.
vii. 1. 124. 2. Re. 1 14 as. 3. 46 ; 509. 4. 1 pal. 5. 8 cm. 6. 379.
7. Rs. 4 12 as. 8. 3 ft. square.
9. Every fifth division of the former corresponds to the fourth in the latter. 10. 41 ; Rs. 8 5 as. 11. 5 Kg. = 11 lbs.
12. 60 Doll. = £7.

Examples.

CHAPTER XV.

i. 1. 1008. 2. 4752. 3. 8316. 4. 20064. 5. 1076400. 6. 78540.
7. 3465. 8 25410.
ii. 1. 2520. 2. 840. 3. 52360. 4. 3150. 5. 7200. 6. 1008.
7. 120120. 8. 10080. 9. 16800. 10. 720. 11. 1080. 12. 964.
iii. 1. 498672. 2. 848102. 3. 13440. 4. 477764.
5. 584598. 6. 3946110. 7. 4324320. 8. 255255.

iv. **1.** 7200. **2.** Rs. 1,400. **3.** 720. **4.** 14549535.
5. 2522. **6.** 247. **7.** 509. **8.** 93.
9. Rs. 161. **10.** 499. **11.** 890. **12.** Rs. 3,919

v. **1.** Rs. 4-9-4. **2.** £12-13-9. **3.** 42 mds. **4.** Rs. 2-15-6.
5. £9-18-4. **6.** 26 yds. 9 in. **7.** 7 cwt. 2 qrs. **8.** Rs. 1,360.

vi. **1.** Rs. 1,878-3 as. **2.** Rs. 1-10-3. **3.** 205 tons 13 cwt. 1 qr. 17 lbs.
4. 40 yds. 2 ft. 6 in.

vii. **1.** 10257. **2.** 1 hr. 24 min.; 630, 504, 420, 360, 315, 280.
3. 12 yds. 1 ft. 6 in. **4.** 26 yds. 2 ft. **5.** 3,600.
6. 1553 mi.; 1353 Knots; 2508 Km. **7.** Rs. 127 9 as. **8.** Rs. 3,961.
9. 59. **10.** 494. **11.** Rs. 1,440. **12.** Least sum Rs. 596.
Largest sum Rs. 996.

Examples.

CHAPTER XVI.

i. **1.** 7 as. 4 p. **2.** 3*s*. $1\frac{1}{2}$*d*. **3.** 18*s*. $4\frac{1}{2}$*d*. **4.** Rs. 2 3 as.
5. 13 as. $1\frac{1}{2}$ p. **6.** 1 qr. 7 lbs. **7.** 20 lbs. 4 oz. $12\frac{4}{5}$ drs.
8. 2 fur. 7 po. 1 yd. 1 foot 6 in. **9.** Rs. 15 3 as. 9 p.
10. Rs. 80 8 as. 4 p. **11.** Rs. 51 11 as. 1 p. **12.** £65 16*s*. 7*d*.
13. £106 17*s*. 4*d*. **14.** 6 tons 1 cwt. 1 qr. $9\frac{1}{3}$ lbs.
15. £10 5*s*. 5*d*. **16.** 2 cwt. 21 lbs.

ii. **1.** £13. **2.** £5. **3.** 2 cwt. **4.** Rs. 4. **5.** 2 days. **6.** 7 maunds.
7. Rs. 6. **8.** 4 mi. 7 fur. 143 yds. 0 ft. $7\frac{1}{5}$ in.

iii. **1.** (1) $\frac{399}{19}, \frac{608}{19}, \frac{1425}{19}, \frac{1539}{19}$. (2) $\frac{651}{31}, \frac{992}{31}, \frac{2325}{31}, \frac{2511}{31}$.
2. (1) $\frac{88}{8}, \frac{152}{8}, \frac{184}{8}, \frac{248}{8}$. (2) $\frac{121}{11}, \frac{209}{11}, \frac{253}{11}, \frac{341}{11}$.
3. (*a*) (1) $\frac{9}{5} = \frac{135}{75} = \frac{225}{125} = \frac{270}{150} = \frac{1620}{900}$.
(2) $\frac{7}{25} = \frac{21}{75} = \frac{35}{125} = \frac{42}{150} = \frac{252}{900}$.
(*b*) (1) $\frac{70}{126}$. (2) $\frac{72}{126}$. (3) $\frac{63}{126}$. (4) $\frac{40}{126}$.
4. (1) $\frac{6}{17} = \frac{18}{51} = \frac{72}{204} = \frac{90}{255} = \frac{108}{306}$.
(2) $\frac{9}{20} = \frac{18}{40} = \frac{72}{160} = \frac{90}{200} = \frac{108}{240}$.

iv. **1.** £17-1-8. **2.** 214 grams. **3.** £823-17-3. **4.** Rs. 683-2-3.
5. Rs. 934-0-5. **6.** 14 tons 4 cwt. 3 qrs. 13 lbs.
7. 116 days 13 hrs. 6 min. 40 sec. **8.** 63 can. 11 mds. $15\frac{1}{2}$ pals.
9. (1) $\frac{2453}{45}$ (2) $\frac{6667}{85}$. (3) $\frac{2988}{101}$. (4) $\frac{30425}{403}$.
(5) $\frac{432272}{817}$. (6) $\frac{614093}{905}$. (7) $\frac{767463}{987}$. (8) $\frac{805847}{919}$.

v. **1.** $18\frac{59}{127}$. **2.** $19\frac{169}{173}$. **3.** $33\frac{268}{295}$. **4.** $98\frac{714}{901}$.
5. $243\frac{919}{3705}$. **6.** $399\frac{2299}{2468}$. **7.** $192\frac{2459}{4567}$. **8.** 60.

vi. **1.** $\frac{1}{3}$. **2.** $\frac{71}{101}$. **3.** $\frac{7}{8}$. **4.** $\frac{36}{55}$. **5.** $\frac{71}{101}$. **6.** $\frac{36}{85}$.
7. $\frac{57}{83}$. **8.** $\frac{5}{7}$. **9.** $\frac{128}{127}$. **10.** $\frac{23}{33}$. **11.** $\frac{47}{149}$. **12.** $\frac{29}{53}$.
13. $\frac{538}{993}$. **14.** $\frac{85}{87}$. **15.** $\frac{111}{632}$. **16.** $\frac{563}{767}$. **17.** $32\frac{23}{25}$. **18.** $80\frac{93}{94}$.
19. $74\frac{19}{85}$. **20.** $74\frac{1}{5}$. **21.** $81\frac{11}{93}$. **22.** $84\frac{6}{7}$. **23.** $138\frac{1}{2}$. **24.** $86\frac{1}{9}$.

vii. **1.** $\dfrac{140, 105, 84, 70, 60}{420}$. **2.** $\dfrac{297, 352, 396, 432, 462}{792}$.
3. $\dfrac{105, 108, 110, 72}{120}$. **4.** $\dfrac{63, 88, 102, 76}{144}$. **5.** $\dfrac{7, 6, 6}{9}$.
6. $\dfrac{48, 55, 45}{60}$. **7.** $\dfrac{11, 7, 2}{14}$. **8.** $\dfrac{36, 36, 35}{60}$.

viii. **1.** (1) $\frac{5}{9}$. (2) $\frac{23}{25}$. (3) $\frac{17}{18}$. (4) $\frac{19}{32}$.
2. (*a*) $\frac{8}{20}, \frac{10}{21}, \frac{7}{8}, \frac{9}{7}$. (*b*) $\frac{12}{21}, \frac{9}{12}, \frac{5}{6}, \frac{13}{10}$. (*c*) $\frac{7}{26}, \frac{4}{13}, \frac{11}{12}, \frac{12}{10}$. (*d*) $\frac{5}{7}$, $\frac{7}{8}, \frac{20}{21}, \frac{13}{12}$.
3. (*a*) $\frac{11}{12}, \frac{7}{24}, \frac{3}{11}; \frac{12}{121}$. (*b*) $\frac{7}{23}, \frac{8}{30}, \frac{6}{45}, \frac{5}{69}$. (*c*) $\frac{12}{13}, \frac{10}{11}, \frac{9}{26}, \frac{23}{39}$
(*d*) $\frac{13}{14}, \frac{75}{100}$, or $\frac{21}{28}, \frac{5}{8}$.
4. (1) $\frac{21}{22}$ is the greatest ; $\frac{7}{10}$ is the least.
(2) $\frac{303}{410}$,, ; $\frac{21}{100}$,,
(3) $\frac{8}{9}$,, ; $\frac{6}{11}$,,
(4) $\frac{4}{5}$,, ; $\frac{6}{13}$,,

x. **1.** $\frac{95}{96}$. **2.** $\frac{793}{960}$. **3.** $\frac{51}{80}$. **4.** $\frac{11}{224}$. **5.** $\frac{943}{2000}$. **6.** $\frac{3}{32}$. **7.** $\frac{311}{64}$. **8.** $\frac{777}{320}$.

xi. **1.** $\frac{14}{25}$. **2.** $\frac{13}{125}$. **3.** $1\frac{13}{125}$. **4.** $1\frac{149}{4000}$. **5.** $5\frac{147}{400}$. **6.** $\frac{7}{32}$.
7. $30\frac{113}{400}$. **8.** $105\frac{11}{16}$ **9.** $\frac{603}{8000}$. **10.** $\frac{5}{128}$. **11.** $\frac{25}{512}$. **12.** $1\frac{951}{1024}$.

xii. **1.** 0·9375 , 0·3475 ; 0·265625 ; 0·0078125.
2. 0·01171875 ; 0·21875 ; 0·21875 ; 0·0390625.
3. 0·2375 ; 0·048828125 ; 0·0075264 ; 1·9287109375.
4. 7·47 ; 0·27 ; 0·74 ; 3·37 (each correct to two places).

Examples.

CHAPTER XVII.

i. **1.** Rs. $2\frac{7}{48}$. **2.** Rs. $1\frac{29}{48}$. **3.** £$1\frac{11}{16}$. **4.** £$1\frac{9}{32}$. **5.** R. $\frac{7}{24}$. **6.** £$\frac{109}{225}$.
7. $\frac{136}{363}$ mi. **8.** £$\frac{131}{7800}$. **9.** $2\frac{11}{112}$. **10.** $\frac{2696}{2835}$. **11.** $2\frac{1155}{2016}$. **12.** $3\frac{19}{96}$.
13. $3\frac{31}{68}$. **14.** $2\frac{197}{900}$. **15.** $2\frac{77}{204}$. **16.** $1\frac{443}{525}$. **17.** $\frac{23}{204}$. **18.** $\frac{17}{138}$.
19. $\frac{101}{504}$. **20.** $\frac{25}{864}$. **21.** $1\frac{38}{39}$. **22.** $2\frac{59}{140}$. **23.** $2\frac{11}{84}$. **24.** $2\frac{1}{60}$.

ii. **1.** $31\frac{17}{147}$. **2.** $17\frac{491}{576}$. **3.** $16\frac{4}{45}$. **4.** $22\frac{85}{96}$. **5.** $3\frac{17}{36}$. **6.** $5\frac{17}{144}$. **7.** $\frac{7}{90}$.
8. $4\frac{23}{105}$. **9.** 105·6555. **10.** 30·2825. **11.** 28. **12.** 10·00325.

iii. **1.** $3\frac{11}{12}$. **2.** $\frac{5}{12}$. **3.** $12\frac{113}{504}$. **4.** $16\frac{12}{35}$. **5.** $4\frac{281}{288}$. **6.** $1\frac{4}{125}$. **7.** $1\frac{541}{576}$. **8.** $\frac{38}{63}$.

iv. **1.** Rs. 3-5-2. **2.** £3-13-0. **3.** 185 yds. 1 ft. **4.** 309 g. 2 dg. 5 cg. $9\frac{7}{8}$ mg. **5.** $\frac{7}{48}$. **6.** $\frac{1}{8}$. **7.** $\frac{5}{48}$. **8.** $\frac{11}{30}$.
9. The former by $\frac{14}{27}$. **10.** $\frac{62}{231}$. **11.** $\frac{8}{25}$. **12.** $\frac{3691}{4095}$.

Examples.

CHAPTER XVIII.

i. **1.** (1) $10\frac{11}{89}$. (2) $12\frac{45}{89}$. (3) $17\frac{77}{89}$. (4) $27\frac{88}{89}$.
2. (1) $4\frac{2}{5}$. (2) $5\frac{13}{15}$. (3) $7\frac{1}{3}$. (4) $8\frac{4}{5}$.
3. (1) $14\frac{1}{4}$. (2) $17\frac{1}{10}$. (3) $22\frac{4}{5}$. (4) $28\frac{1}{2}$.
4. (1) $9\frac{2}{3}$. (2) $14\frac{1}{2}$. (3) $19\frac{1}{3}$. (4) $43\frac{1}{2}$.

ii. **1.** (1) $63\frac{3}{4}$. (2) 85. (3) $159\frac{3}{8}$. (4) 170.
2. (1) $79\frac{1}{2}$. (2) $132\frac{1}{2}$. (3) $238\frac{1}{2}$. (4) 318.
3. $1346\frac{5}{12}$. **4.** $1029\frac{3}{11}$. **5.** $4307\frac{7}{8}$. **6.** $42024\frac{1}{2}$.

iii. **1.** (1) $\frac{2}{21}$. (2) $\frac{4}{63}$. (3) $\frac{8}{189}$. (4) $\frac{3}{63}$.
2. (1) $\frac{8}{9}$. (2) $\frac{5}{9}$. (3) $\frac{4}{9}$. (4) $\frac{8}{15}$.
3. (1) $2\frac{1}{10}$. (2) $1\frac{17}{35}$. (3) $1\frac{1}{3}$. (4) $\frac{2}{5}$.
4. (1) $4\frac{11}{13}$. (2) $3\frac{6}{13}$. (3) $1\frac{11}{13}$ (4) $1\frac{19}{26}$.

iv. **1.** $\frac{170}{741}$. **2.** $\frac{260}{567}$. **3.** $\frac{2678}{3885}$. **4.** $\frac{1573}{10761}$. **5.** $\frac{40}{93}$. **6.** $\frac{8}{27}$. **7.** $\frac{1}{7}$. **8.** $\frac{3}{10}$.

v. **1.** 14 **2.** $9\frac{12}{13}$. **3.** $16\frac{1}{2}$. **4.** $58\frac{1}{2}$. **5.** 24. **6.** 93. **7.** 77. **8.** 80.

vi. **1.** $22\frac{1}{2}$. **2.** $15\frac{49}{64}$. **3.** 63. **4.** $\frac{1}{14}$. **5.** $\frac{4}{21}$. **6.** $\frac{1}{5}$.
7. 100. **8.** $1\frac{2}{7}$. **9.** 3 as. $1\frac{1}{2}$ p. **10.** 7 mds. 3 viss, 9 pals. $\frac{1}{2}$ tola.
11. £2-13-9. **12.** £1-3-2.

vii. **1.** Rs. 36-9-4. **2.** Rs. 9-6-6. **3.** 12*s*. 4*d*. nearly. **4.** £48-2-0.
5. Rs. 4. **6.** Rs. 1,600-1-4. **7.** 4 miles. **8.** Rs. 60.
9. (1) $1\frac{1}{2}$. (2) $1\frac{13}{20}$. (3) $1\frac{1}{2}$. (4) $\frac{4}{5}$.
10. (1) 81. (2) $3\frac{9}{14}$. (3) $42\frac{2}{3}$. (4) 16.

viii. **1.** 4 as. 6 p. to the nearest pie. **2.** 2 can. 15 mds. 7 viss 3 tolas to the nearest tola. **3.** 4*s*. **4.** Rs. 32-10-8.

ix. **1.** $1\frac{7}{16}$. **2.** $4\frac{13}{28}$. **3.** $2\frac{23}{50}$. **4.** 60. **5.** $8\frac{88}{153}$. **6.** $\frac{5}{96}$. **7.** 1. **8.** 4.

x. **1.** 8 hrs. 20 min. **2.** 5 hrs. **3.** Rs. 60. **4.** 105.
G.C.M. **5.** $\frac{1}{198}$. **6.** $\frac{3}{152}$. **7.** $\frac{1}{12}$. **8.** $\frac{1}{720}$.
L.C.M. **5.** 504. **6.** $\frac{24531}{88}$. **7.** $\frac{8008}{3}$. **8.** $\frac{87961}{3}$.

Examples.

CHAPTER XIX.

i. I. **1.** ·6125. **2.** ·035. **3.** ·00091875. **4.** ·303.
5. ·096. **6.** ·00084375. **7.** ·57. **8.** ·375.
II. **1.** $\frac{7}{875}$. **2.** $\frac{48}{101}$. **3.** $\frac{2}{9}$. **4.** $\frac{43}{59}$.
III. **1.** ·3047. **2.** ·1223 **3.** ·03671875. **4.** ·3489.

ii. **1.** ·75. **2.** $\frac{233}{20}$ **3.** ·2265625. **4.** $\frac{9}{25}$.

iii. **1.** $\frac{1}{4}$. **2.** $\frac{4}{5}$. **3.** $\frac{2}{3}$. **4.** $\frac{13}{16}$. **5.** 0·225. **6.** 0·6875.
7. 0·2875. **8.** 0·00125. **9.** $\frac{11}{128}$. **10.** $\frac{7}{48}$. **11.** $\frac{1}{6}$. **12.** $\frac{5}{8}$.

iv. **1.** $\frac{1}{42240}$. **2.** $\frac{1}{63360}$. **3.** $\frac{1}{1584}$. **4.** $\frac{1}{1588}$. **5.** $\frac{1}{1267200}$.
6. $\frac{1}{66000}$. **7.** 7·04 yds. **8.** 1 ft. 6 in. **9.** $1\frac{1}{2}$ miles nearly.
10. 60 chains. **11.** $9\frac{1}{3}$ miles nearly. **12.** 50 cm.

v. **1.** ·521. **2.** ·619. **3.** ·761. **4.** ·805. **5.** ·683. **6.** ·877.
7. 1·579. **8.** 2·541 = 2·540625. **9.** 5·584 = 5·584375.
10. 15·481 = 15·48125. **11.** 29·199. **12.** 76·994 = 76·99375.

vi. I. **1.** ·141. **2.** ·339. **3.** ·297. **4.** ·479.
5. ·615. **6.** ·682. **7.** 1·422. **8.** 3·354.
9. 9·833. **10.** 6·911. **11.** 13·927. **12.** 15·995.
II. **1.** 14 as. **2.** 5 as. 9 p. **3.** 7 as. 7 p. **4.** 9 as. 7 p. **5.** 12 as. 6 p.
6. 9 as. 5 p. **7.** Rs. 1-7-5. **8.** Rs. 3-12-0. **9.** Rs. 5-5-1.
10. Rs. 8-12-3. **11.** Rs. 10-13-5. **12.** Rs. 13-15-7.

Examples.

CHAPTER XX.

i. 1. 6 pies. 2. 66 ft. 3. 21. 4. Rs. 2-3-0. 5. 16. 6. Rs. 4-2-0.
7. Rs. 10-2-8. 8. 85 Kg. 9. $18\frac{4}{5}$ inches. 10. $7\frac{3}{4}$ miles.
11. 9·61 m. 12. 28 Km. 13. 1527·27 . . Kg. 14. 4·57 . . litres.
15. ·72 in.; 17·5. 16. Rs. 72-6-0; Rs. 54-4-6.
17. Rs. 1,866-10-8. 18. $7\frac{7}{10}$ mi. 19. Rs. 4,200. 20. 15*s*.

ii. 1. 3 hrs.; 2 hrs. 2. 58 men. 3. 9 days. 4. (1) 25 days; (2) 10 days.
5. $8\frac{4}{5}$ hrs. 6. Rs. 9-13-6. 7. Sooner by $1\frac{1}{2}$ yrs. 8. 9 min.

iii. 1. 1000; 200; 250. 2. 315; 70, 63, 15. 3. 972. 4. £8,000.

iv. 1. $\frac{1}{6}$. 2. A, Rs. 25,000; B, Rs. 45,000.
3. Rs. 186-10-8. 4. 8 as. 4 p.

v. 1. £307; £230-5-0; £76-15-0.
2. Rs. 10,000; sons Rs. 3,750 each; widow Rs. 2,500.
3. Rs. 50,000; daughters Rs. 16,000 each; son Rs. 18,000.
4. Rs. 4,200.

vi. 1. 32 lakhs. 2. 300 gals. 3. Rs. 120. 4. $87\frac{1}{10}$ yds.
5. Rs. 1,728. 6. 63 votes. 7. Rs. 60,000. 8. 569·68125 ac.

Examples.

CHAPTER XXI.

i. 1. Rs. 1,947-14-8. 2. Rs. 286-7-4; Rs. 4,984-6-0. 3. 6 p. 4. $5\frac{3}{4}$*d*.

ii. 1. Rs. 1,140. 2. Rs. 138 nearly. 3. £9,375. 4. $293\frac{1}{3}$ lakhs.
5. Rs. 1,152. 6. Rs. 6,875. 7. Rs. 10,800. 8. £3,375.

iii. 1. Rs. 30,000. 2. £1,666-13-4. 3. Rs. 5,888. 4. Rs. 1,086.

iv. 1. $7\frac{1}{2}$*d*. 2. £540. 3. Rs. 11,687-8-0. 4. £250.
5. 5*d*. 6. $\frac{1}{2}$ p. 7. £600. 8. Rs. 1,440.
9. Rs. 1,440. 10. £960. 11. £2,686-13-4. 12. £500.

One-Hour Papers: Second Series

i. 1. $2^5 \times 3 \times 11 \times 31$; $31 \times 32 \times 33$. 2. $46\frac{23}{40}$; £4-4-$11\frac{1}{2}$ nearly.
3. $1\frac{23}{33}$ mi. 4. Rs. 979-2-8. 5. Rs. 3-12-8.

ii. 1. 10·81 gals. 2. 5 hrs. $37\frac{1}{2}$ min. 3. 1,58,400; Rs. 4,950.
4. 308. 5. 24 : 45.

iii. 1. 0·4g. 2. £146·9 fl. 6 c. 5 mi. = £146-19-$3\frac{1}{2}$.
3. 2670. 4. $\frac{1}{7}$. 5. 100 ft.

iv. 1. $2^4 \times 3^2 \times 5^2 \times 7$; $2^{14} \times 5^2$; $2^4 \times 5 \times 7 \times 11^2$: H. C. F. $2^4 \times 5$; L.C.M. $= 2^7 \times 3^2 \times 5^2 \times 7 \times 11^2$. 2. 22·125. 3. 89.
4. Rs. 625. 5. A, 20; B, 30 days.

v. 1. 27 ft. 6 in.; 10; 11. 2. £37-10-0. 3. £125. 4. $\frac{1}{3}$. 5. 81 days.

vi. 1. £4-7-$9\frac{1}{2}$. 2. 19 mi. 3. $\frac{1}{12}$. 4. 256. 5. 160.

vii. 1. $36\frac{2}{3}$ ft.; 2·27. 2. 288; 432. 3. Rs. 1-0-8. 4. 150 tons. 5. 52.

viii. 1. 8*d*. 2. $31\frac{1}{2}$ mi. 3. Rs. 6-9-0. 4. 26·49 sec.
21 yds. 2 ft. 9·8 in.

ix. **1.** 7922 mi. **2.** 45 gals. **3.** £23-15-0. **4.** 10·03125 yrs.
5. £9 12*s*. ; £21 12*s*.

x. **1.** According to the rule 2·205 lbs. = 1 Kg. 2g. **2.** £10-15-0$\frac{3}{4}$.
3. 5*d*. **4.** 32. **5.** 128 ; 96 ; 72 ; 54.

xi. **1.** 0·001. **2.** 1·25 Km. ; 0·507. **3.** Rs. 4-10-1. **4.** 5*s*. 1$\frac{3}{4}$*d*. **5.** 25.

xii. **1.** £75-7-9$\frac{3}{4}$. **2.** Rs. 2-13-0. **3.** Rs. 3,000. **4.** 10 ft. 10$\frac{1}{2}$ in.
5. 2 tons 4 cwt. 10$\frac{1}{4}$ lbs.

xiii. **1.** Rs. 129-8-0 ; 3 cwt. 9$\frac{1}{3}$ lbs. **2.** 10$\frac{5}{6}$ millions. **3.** £72.
4. A, 4 more. **5.** £3,200.

xiv. **1.** At the end of 5 years. **2.** Debts, £15,000 ; assets, £5,000.
3. Rs. 9,000. **4.** 115·2 Km. **5.** 51 miles.

xv **1.** 4·901. **2.** C. **3.** 0·4. **4.** Rs. 54,000. **5.** 564 ; 560.

CHAPTER XXIII.

Examples.

i. **1.** Rs. 13-5-4. **2.** £6-0-1. **3.** £346-5-4. **4.** £1,283-16-0.
5. Rs. 1,016-15-10$\frac{1}{2}$. **6.** Rs. 1,840-0-1$\frac{1}{2}$. **7.** £1,085-0-6.
8. £1,907-7-1.

ii. **1.** £1,168-18-9. **2.** £1,645. **3.** Rs. 20,405-2-8. **4.** £1,345-16-8.
5. £7,818-8-4. **6.** Rs. 46,976-11-3. **7.** Rs. 10,414-7-7.
8. £24,931-19-9.

iii. **1.** Rs. 6,86,140-14-4$\frac{1}{2}$. **2.** £37,95,651-2-4. **3.** £43,649-0-11$\frac{1}{4}$.
4. Rs. 7,49,791-6-0.

iv. **1.** Rs. 10,678-9-7. **2.** £848-18-2$\frac{1}{2}$. **3.** £1,637-18-8$\frac{3}{4}$.
4. £9,362-11-5$\frac{1}{2}$. **5.** Rs. 9,99,500-0-9. **6.** £450-14-0$\frac{1}{2}$.
7. Rs. 10,357-6-0. **8.** Rs. 13,386-14-2.

v. **1.** Rs. 165-13-4. **2.** £367-10-0. **3.** £916-13-4. **4.** £345-6-8.
5. £10,195-4-0. **6.** Rs. 19,057-8-0. **7.** Rs. 2,800. **8.** £5,910-15-0.

vi. **1.** £822-13-4. **2.** £666-12-0. **3.** £23,135-4-5. **4.** £156-3-5$\frac{1}{4}$.

vii. **1.** Rs. 13,603-8-8. **2.** Rs. 4,306-10-8. **3.** £1,289-15-4$\frac{3}{4}$.
4. £14,966-5-1$\frac{3}{4}$.

viii. **1.** Rs. 59-6-0. **2.** Rs. 125-5-0. **3.** Rs. 16-4-0. **4.** £17-10-11.
5. Rs. 3-14-1$\frac{1}{2}$. **6.** £118-17-7. **7.** Rs. 36-11-9. **8.** Rs. 76-10-2.

ix. **1.** Rs. 30-9-11. **2.** Rs. 41-9-9. **3.** Rs. 241-8-8. **4.** Rs. 1,239-13-4.
5. £20-2-2$\frac{1}{4}$. **6.** Rs. 48-14-9. **7.** Rs. 2,900-1-6. **8.** Rs. 5,267-11-7.

x. **1.** Rs. 14,465-1-2. **2.** Rs. 950-15-0. **3.** Rs. 886-0-9.
4. £10-9-8$\frac{1}{4}$. **5.** £72,410-1-6$\frac{3}{4}$. **6.** £1,810-13-6.
7. Rs. 3,27,450-8-4. **8.** Rs. 67,567-9-7.

xi. **1.** £140-7-4$\frac{3}{4}$. **2.** Rs. 32,427-8-0. **3.** Rs. 935-15-7.
4. Rs. 37,236-5-4. **5.** Rs. 8,989-0-3. **6.** £2,066-8-6$\frac{3}{4}$.
7. Rs. 58,965-12-11. **8.** Rs. 37,695-4-10.

xii. **1.** £48-1-0$\frac{1}{2}$. **2.** £603-9-5. **3.** £512-9-1.
4. £197-11-11$\frac{1}{2}$. **5.** Rs. 2,03,317-12-10. **6.** Rs. 61 6-10.
7. Rs. 1,450-8-4. **8.** 62 lbs. 3 oz. **9.** £14,297-9-11$\frac{1}{2}$.
10. Rs. 152-14-8. **11.** £332-12-10. **12.** Rs. 81,75,441-3-4.

Examples.

CHAPTER XXIV.

i. **1.** 783 sq. ft. **2.** 1120 sq. in.
3. 578 sq. yds. **4.** 553 sq. yds.
5. 16 sq. yds. 7 ft. 126 in. **6.** 30 sq. yds. 1 foot 90 in.
7. 70 sq. yds. 7 ft. 32 in. **8.** 17 sq. yds. 4 ft. 128 in.
9. 10 sq. yds. 5 ft. 126 in. **10.** 5 sq. yds. 3 ft. 126 in.
11. 1 sq. yd. 7 ft. **12.** 12 sq. yds. 1 foot 54 in.

ii. **1.** 47 sq. yds. 7 ft. 81 in. **2.** 234 sq. yds. 2 ft. 49 in.
3. 1 sq. Km. 1 Ha. 25 ca. **4.** 41 ac. 9 sq. ch. 4304 sq. l.
5. $62\frac{1}{2}$ ac. **6.** 10·50625 ac. **7.** 5·4390625 ac.
8. 101·6064 Ha. **9.** 7·5. **10.** 2·42055 ac. **11.** 1. **12.** 1120.

iii. **2.** ·96 acres. **3.** $333\frac{1}{3}$ ac. **4.** 4250 sq. mi.

iv. **1.** 17 sq. yds. 7 ft. 18 in. **2.** 27 sq. yds. 2 ft. 120 in.
3. 6 sq. yds. 6 ft. **4.** 22 sq. yds. 7 ft. **5.** Rs. 113-1-7.
6. £85-1-$6\frac{3}{4}$. **7.** 701·25 fr. **8.** 502 fr. to the nearest franc.

v. **1.** 77 sq. yds. 7 ft. **2.** 6·45 sq. cm. **3.** 33·77 sq. ft. **4.** 1·08 ac.
5. 3096 sq. yds. 6 sq. ft. **6.** (1) 2534 ; (2) 1267 ; (3) 634.
7. 69 sq. yds. 4 ft. **8.** 26 sq. yds. 1 foot 108 in.
9. (1) 16·8 sq. in. (2) 123 sq. cm. (3) 706 sq. cm. (4) 17·8 sq. in.
10. (1) 76980·75 & 76419·75 sq. m. (2) 152286·75 & 151113·75 sq. m.
(3) 937·8325 and 931·6125 sq. cm. (4) 1569·1975 and 1561·2075 sq. cm.

vi. **1.** 106 sq. yds. **2.** Rs. 44-5-4. **3.** Rs. 28-10-4. **4.** 29 sq. yds. 2 ft. 96 in.

vii. **1.** 5 ft. 9 in. **2.** 11 yds. **3.** 45 yds. **4.** 89 yds.

Practical Exercise—1. $33\frac{1}{3}$ yds. **2.** $41\frac{1}{2}$ yds. **3.** 22 yds.
4. 360 m. **5.** 24 yds. **6.** 50·16 m.

viii. **1.** Rs. 192. **2.** Rs. 527-6-$1\frac{1}{2}$. **3.** Rs. 6-9-0. **4.** Rs. 343-12-0.

ix. **1.** 30 yds. **2.** 36 yds. **3.** 9 yds. **4.** 5 yds. 1 foot $10\frac{1}{2}$ in.

Practical Exercise—1. 120. **2.** 540. **3.** 800. **4.** 720 ; Rs. 18.
5. 288 ; Rs. 60. **6.** 1014.

x. **1.** 1369. **2.** 2944. **3.** 14 yds. ; 2562. **4.** $4\frac{13}{14}$ yds. ; 207.

Practical Exercise—2. (1) 400 sq. ft. (2) 1140 sq. ft.
(3) 848 sq. ft. ; Rs. 53. (4) Rs. 4.

xi. **1.** 125 sq. yds. **2.** 93 sq. yds. 4 ft. 72 in. **3.** £300. **4.** Rs. 9-7-1.

xii. **1.** Rs. 62-8-0. **2.** Rs. 26-9-0. **3.** Rs. 339-3-9. **4.** Rs. 75-4-$7\frac{1}{2}$.

xiii. **1.** 272 yds. **2.** 16. **3.** Rs. 19 8 as. **4.** 168.

xiv. **1.** 24 ft. ; 16 ft. **2.** $23\frac{7}{16}$ sq. yds. **3.** $14\frac{1}{2}$ ft. **4.** 20 ft. ; 18 ft.

xv. **1.** 1345 sq. ft. 119 in. **2.** £1-15-0. **3.** 1 ac. 1 ro. 29·5808 po.
4. $8\frac{1}{4}$ yds. **5.** 7 as. 6 p. **6.** 1 sq. foot 64 in. **7.** 33 ; 5 sq. ft.
8. Place the length of the board along the breadth of the platform ; 670. **9.** 4800 ; 7 : 500. **10.** 432 sq. mi. **11.** 4 in.

12. 239 ac. to the nearest integer. **13.** 2·4 sq. in. nearly. **14.** $1\frac{1}{2}$ ft.
15. 4480 ; $130\frac{2}{3}$ yds. **16.** $3\frac{7}{16}$ sq. yds. **17.** 89 sq. ft. **18.** 11700 sq. yds.
19. $104\frac{8}{9}$ sq. yds. **20.** Rs. 5-10-0. **21.** 8 pieces ; $110\frac{1}{4}$ sq. ft.
22. 93 sq. in. **23** $46\frac{5}{8}$ lbs. **24.** 10 ft.

Examples.

CHAPTER XXV.

i. **1.** 750 c.ft. **2.** $145\frac{5}{6}$ c. yds. **3.** $22\frac{1}{2}$ c. ft. **4.** 60,000 litres.
5. Rs. 46-12-0. **6.** Rs. 14-9-4. **7.** 720 Kg. **8.** 100 lbs.
9. (1) $15\frac{5}{8}$ c. ft. (2) $37\frac{1}{27}$ c. ft. (3) 157·464 c. cm. (4) 91·125 c. cm.

ii. **1.** 30 ft. **2.** 18 yds. **3.** 12 ft. **4.** 0·043 in. **5.** 2 m.
6. 0·0019 sq. ft. **7.** $1\frac{1}{3}$ in. **8.** 20 sq. ft

iii. **1.** 36,000. **2.** 28,336. **3.** 3,240. **4.** 61,440. **5.** 6,912. **6.** 864.
7. 155. **8.** 31 : 24.

iv. **1.** 1155 c. in. **2.** 3392 c. cm. ; 4·928 litres. **3.** $3\frac{65}{144}$ c. ft.
4. 5 tons 4 cwt. 2 qrs. 8 oz.

v. **1.** 101 tons 5 cwt. 2 qrs. 19 lbs. **2.** 3 tons 4 cwt. 3 qrs. 4 lbs. 13 oz.
3. £64. **4.** 1 ton 3 cwt. 1 qr. $2\frac{2}{3}$ oz. **5.** $21\frac{31}{64}$ tons. **6.** 14,479,674.
7. 64,800. **8.** 9 in. **9.** Rs. 75. **10.** Rs. 2-10-0. **11.** Rs. 8-2-5.
12. £1-6-0$\frac{3}{4}$. **13.** $202\frac{1}{2}$ Kg. **14.** $13\frac{1}{3}$ cm. **15.** Rs. 108-8-5.
16 31·6 ft. **17.** $757\frac{1}{32}$ gals. **18.** 16 tons 6 cwt. 3 qr. 25 lbs. $1\frac{1}{2}$ oz.
19. Rs. 77. **20.** 5 in. **21.** 2 tons 6 cwt. 16 lbs. 12 oz. **22.** 6450 Kg.
23. $72\frac{11}{12}$ gals. **24.** 691 hrs. 12 min.

Examples.

CHAPTER XXVI.

i. **1.** 25 ac. **2.** 108 kalams. **3.** 200 sq. yds. **4.** Rs. 150.

ii. **1.** Rs. 9. **2.** Rs. 16,875. **3.** 42,240 c. ft. **4.** $110{,}491\frac{1}{14}$ tons.

iii. **1.** 176 yds. ; 2,464 sq. yds. **2.** 7,900. **3.** $687\frac{1}{2}$ c. ft.
4. $321\frac{17}{63}$ sq. yds. **5.** $1{,}240\frac{2}{63}$ sq. yds. **6.** 14 sq. yds.
7. Rs. 152 10 as. **8.** £36-6-4.

iv. **1.** 259,988. **2.** 81,500 sq. mi. more. **4.** $116\frac{13}{28}$ c. in
5. $66{,}377\frac{1}{7}$ c. in. **6.** 22 ft. 8·5 in. to the nearest tenth of an inch.
7. 163·363 sq. in. ; 15·708 sq. in. **8.** 16.

CHAPTER XXVII.

Practical Exercise—Page 286.

2. (*a*) £10. (*b*) £13-12-6. **5.** 3·4 mo. ; 14·6 wks. nearly.
7. (*a*) 5·05 cm. (*b*) 92·5 g. **8.** (*a*) 5·2 mm. ; 12·6 mm.

Practical Exercise—Page 291.

1. 30·5 ; 46 millions. **2.** (1) Rs. 38. (2) $27\frac{1}{2}$.
6. (1) $30\frac{3}{4}$ yrs. (2) $20\frac{1}{2}$ yrs. (3) $22\frac{1}{2}$. (4) $36\frac{1}{2}$.
7. 35°. **8.** (1) 10 as. 8 p. (2) 13 as.

Practical Exercise—Page 296.

1. 104 ; 131 ; 179·6 ; -4 ; $-1\frac{1}{9}$; 35 ; 80 ; $93\frac{1}{3}$.
2. (*a*) (1) 29·3. (2) 36·6. (3) 51·3. (4) 66 ft. per sec.
(*b*) (1) 41. (2) 54·6. (3) 30·7. (4) 34 1 miles per hour.

3. (1) Rs. $78\frac{3}{4}$. (2) £$6\frac{2}{3}$. **4.** (1) Rs. $56\frac{1}{4}$. (2) 60*s*.
5. (1) 14*s*. 2*d*. (2) 2 dol. 50 c. **8.** (1) 35 yds. = 32 m. ; 35 m. = 38·3 yds.
(2) 35 mi. = 56 Km. ; 35 Km. = 21·9 mi. (3) 35 gals. = 160 litres. 35 litres = 7·7 gals. (4) 35 lbs. = 15·9 Kg. ; 35 Kg. = 77 lbs.
(5) 35 sq. yds. = 29·2 sq. m. ; 35 sq. m. = 41·9 sq. yds.
(6) 35 ac. = 14·2 Ha. ; 35 Ha. = 86·4 ac.

Practical Exercise—Pages 299 and 300.

7. (1) £2. (2) £$3\frac{3}{4}$. **8.** (1) 42·5. (2) 31. **9.** (1) 456. (2) 310.
13. $7\frac{1}{2}$ days. **14.** $2\frac{2}{5}$ hrs. **15.** 15 hrs. **16.** 12 days.
17. 4·30 p.m. **18.** $11\frac{1}{4}$ hrs. **19.** After 5 hrs. **20.** 11·50 a.m.

CHAPTER XXVIII.

Examples.

i. **1.** (1) 4 : 5. (2) 14 : 13. (3) 1 : 7. (4) 285 : 184.
2. (1) 50 : 1. (2) 2 : 21. (3) 16 : 9. (4) 48 : 91.
3. (1) The 2nd by 0·01. (2) The 1st by $\frac{121}{11300000}$.
(3) The 1st by $\frac{1}{14}$. (4) The 2nd by $\frac{17}{42}$.
4. 87 : 350. **5.** 20 : 21. **6.** 200 : 183. **7.** 1 : 240. **8.** The latter.
9. 14 : 15. **10.** 1 : 9. **11.** 3 : 4.

ii. **1.** (1) Rs. 58. (2) Rs. $37\frac{1}{2}$. (3) $1\frac{1}{2}$ hrs. (4) 10 fr. (5) 9. (6) Rs. $4\frac{1}{2}$.
(7) £3-13-4. (8) 54. **2.** 17 cwt. **3.** Rs. 875.
4. Rs. 7,29,015. **5.** 5*s*. $0\frac{1}{4}$*d*.

iii. **1.** 3 and 4 form proportion.

iv. **1.** Rs. 2 13 as. ; $53\frac{1}{3}$ yds. **2.** Rs. 9 9 as. ; 24. **3.** $50\frac{10}{11}$ Kg. ; $1018\frac{2}{11}$ Kg. ; 100·1 lbs. **4.** 2*s*. $2\frac{1}{4}$*d*. **5.** Rs. 297 8 as.
6. £225-2-3. **7.** 7 days. **8.** 117 boys. **9.** £37-12-6.
10. 60 days. **11.** 14 lbs. **12.** 6 cwt. 3 lbs. **13.** $91\frac{31}{59}$ m.
14. $71\frac{1}{9}$ pp. ; $84\frac{3}{8}$. **15.** $78\frac{2}{5}$ lbs. ; 12·5. **16.** $3\frac{3}{4}$ in. ; 27 mi.

v. **1.** 4·1 lb. **2.** 12·7 Kg. **3.** 10·9 gals. **4.** 40 Kg.
5. 3·0 in. **6.** 29·92 in. **7.** 613·2 g. **8.** 8·75 lbs.

vi. **1.** 1·64 g. **2.** 80·4375 m. **3.** $117\frac{1}{2}$. **4.** 52·5 litres.
5. Yes. **6.** 9 tons 6 cwt. 2 qrs. $18\frac{2}{3}$ lbs. **7.** 224 sq. ft.
8. 42 tons. **9.** 46,400 c. in. **10.** 19*s*. $2\frac{1}{2}$*d*. **11.** 110 ft.
12. 2 hrs. 24 min.

vii. **1.** $286\frac{2}{13}$ mi. **2.** $12\frac{1}{2}$ days. **3.** 1*s*. $10\frac{1}{2}$*d*. **4.** Rs. 31,25,000.
5. $\frac{3}{4}$ pal. **6.** 1 ton 16 cwt. **7.** 640 times. **8.** 4,800.
9. 300 men. **10.** 5 men. **11.** 18 hrs. **12.** 8 sec.

viii. **1.** 2·16 Kg. **2.** 214. **3.** 83·3 c. cm. **4.** 320 sq. ft.

ix. **1.** 80 days. **2.** $6\frac{2}{3}$ days. **3.** 26. **4.** 180.

x. **1.** 210 men. **2.** 40·4 Km. **3.** 852 Kg. **4.** 192.
5. 15·8 mi. to the inch ; 55·3 mi. **6.** 324 in. **7.** 74·4 Kg.
8. 288 hrs. **9.** 168. **10.** $22\frac{1}{2}$ min. **11.** $2\frac{6}{7}$ mo.
12. 40 men. **13.** $10\frac{1}{2}$ days. **14.** £187 10*s*. **15.** 52 women.
16. 390 oz. **17.** 18 days. **18.** 32902·5 gals. **19.** 6 tons. **20.** 52 days.

Examples.

CHAPTER XXIX.

i. **1.** Rs. 46 4 as. **2.** Rs. 512-2-4. **3.** Rs. 200. **4.** 15 cwt.
5. $2\frac{7}{8}$ yds. **6.** $9\frac{3}{8}$ hrs. **7.** Rs. 14. **8.** 48 days.

ii. **1.** 8 wks. **2.** $5\frac{5}{8}$ oll. **4.** 49·3 lbs.

iii. **1.** Rs. 121. **2.** £180. **3.** £20. **4.** £$131\frac{1}{4}$ days.

iv. **1.** 6 men. **2.** 90 men. **3.** 9 men. **4.** 17 men.

v. **1.** 7 tons 4 cwt. **2.** 110 reams. **3.** $16\frac{1}{2}$ min. **4.** $6\frac{1}{25}$ min.
5. $21\frac{1}{44}$ pp. **6.** £177 13*s*. **7.** 150 men. **8.** £17,300.
9. $\frac{1}{2}$ hour. **10.** $51\frac{877}{128}$ sq. yds. **11.** $\frac{15}{16}$ in. **12.** 2·5 in.
13. $358\frac{2}{5}$ pp. **14.** $7\frac{1}{2}$ hrs. **15.** $16\frac{5}{8}$ sheets. **16.** Rs. 18.
17. £12 9*s*. **18.** 180 knots. **19.** $6\frac{2}{3}$ pp. **20.** $35\frac{5}{8}$ tons.

Examples.

CHAPTER XXX.

1. 9 mds. 36 srs. ; 10 mds. 29 srs. **2.** A, £1,282 10*s*. ; B, £25 13*s*.
3. 400 ; 360 ; 240. **4.** A, Rs. 300 ; B, Rs. 200 ; C, Rs. 150.
5. 152 men, 76 women, 38 children.
6. A, B, C, each Rs. 615 ; D, Rs. 410
7. Rs. 100, Rs. 120, Rs. 160, Rs. 200. **8.** 90 ; 130 ; 210 ; 320.
9. 113001. **10.** 750 ; 150 ; 100.
11. A, Rs. 13 8 as. ; B, Rs. 9 ; C, Rs. 6 12 as.
12. Rs. 2 15 as. **13.** A, Rs. 326-10-8 ; B, Rs. 153-5-4.
14. A, Rs. 264 ; B, Rs. 224 ; C, Rs. 112. **15.** 15 lbs.
16. $63\frac{1}{3}$ cwt. cement ; $5\frac{5}{18}$ c. yd. sand ; $10\frac{5}{9}$ c. yd. stone.

ii. **1.** 36 boys. **2.** 16 women. **3.** 13 sheep ; 52 hens.
4. 17 pagodas ; 85 half-rupees

iii. **1.** A man, Rs. 20 ; A woman, Rs. 4.
2. A man, Rs. 25 ; A woman, Rs. 8-5-4.
3. A boy, £1-0-6 ; A girl, 6*s*. 10*d*. **4.** A goat, Rs. 6 ; A sheep, Rs. 4.
5. Rs. 12 ; Rs. 6 ; Rs. 2. **6.** £1 13*s*. ; 16*s*. 6*d*. ; 5*s*. 6*d*.
7. £1,000 : B, £333-6-8 ; C, £66-13-4.
8. A, Rs. 120 ; B, Rs. 160 ; C, Rs. 288.
9. Rs. 30-0-5 ; Rs. 24-0-4 ; Rs. 18-0-3. **10.** 3*s*. ; 2*s*. ; 1*s*. 4*d*
11. Rs. 4 ; Rs. 3 ; Rs. 2. **12.** A, £1-1-3 ; B, £1-5-6 ; C, £3-10-$1\frac{1}{2}$.

iv. **1.** Rs. 25 ; 75 half-rs. ; 125 qr.-rs. **2.** Rs. 20 ; 40 qr.-rs. ; 60 two-as.
3. 40 half-sov. ; 20 half-guin. ; 80 half-crowns. **4.** 40 ; 20 ; 30.

v. **1.** Rs. 12-10-8. **2.** £1-4-9. **3.** 6 meas. **4.** 10 meas.

Examples.

CHAPTER XXXI.

i. **1.** Rs. 2-4-0. **2.** 2 as. **3.** Rs. 6-4-0. **4.** 4 lbs.

ii. **1.** 20. **2.** 315 lbs. **3.** Rs. 1,715. **4.** 121.

iii. **1.** 40. **2.** 14 : Rs. 3. **3.** Rs. 17. **4.** 1,200 ; 1 cwt. $1\frac{1}{8}$ qrs.

iv. **1.** 5. **2.** 16. **3.** 24. **4.** Rs. 1-0-2.

v. **1.** 600 men ; 400 women. **2.** 60 boys ; 40 girls.
3. 56 Euc. ; 64 Alg. **4.** $49\frac{1}{2}$ mi.

One-Hour Papers: Third Series

Note.—Answers in money are given to the nearest pie or penny.

i. **1.** $7\frac{1}{2}$ pals. **2.** Mango, 1 an. $6\frac{1}{2}$ p.; guava, $2\frac{1}{2}$ p.
3. 192. **4.** 54 days. **5.** 54 lbs.

ii. **1.** 37 women. **2.** 338 sq. ft. **3.** 1869 : 1733 nearly. **4.** 17. **5.** 196.

iii. **1.** 4,000 gals. **2.** 600. **3.** 8 meas. **4.** 113 grs. **5.** $18\frac{3}{4}$ ac.

iv. **1.** Tea, 2*s*. 8*d*.; coffee, 1*s*. 8*d*.; sugar, $3\frac{1}{2}$*d*. **2.** A, Rs. 12; B, Rs. 8.
3. 135 days. **4.** $12\frac{1}{2}$ tons. **5.** 32 mats; Rs. 21-6-0.

v. **1.** 5 as. 2 p. **2.** 72 guin. **3.** 13 as. 4 p.
4. A, Rs. 1,380; B, Rs. 3,220. **5.** 9·6 in.

vi. **1.** Rs. 2,343-12-4. **2.** Rs. 13-6-6. **3.** $491\frac{1}{160}$ yds.
4. Rs. 3-12-0. **5.** 11 ft. 6·2 in.

vii. **1.** Mango, 2 as. 3 p.; Orange, 1 an. **2.** 21 days.
3. 45 days. **4.** A, Rs. 320; B, Rs. 293-5-4; C, Rs. 110;
D, Rs. 201-10-8. **5.** 0·068 Kg.

viii. **1.** 0·61. **2.** Rs. 33-12-0. **3.** Rs. 6,300. **4.** 9. **5.** 24 sq. mi.

ix. **1.** 210 viss; 280 viss. **2.** £77-14-$7\frac{1}{4}$. **3.** 51 days. **4.** Rs. 704.
5. $1\frac{7}{8}$ in.

x. **1.** Rs. 285-7-9. **2.** 64. **3.** 12 women. **4.** 3,600 litres. **5.** £14.

xi. **1.** 5*s*. **2.** Gains, Rs. 24. **3.** 20. **4.** (1) 80 sq. ft. (2) 960 c. ft.
5. Rs. 150.

xii. **1.** 17 ft. $1\frac{1}{2}$ in. **2.** 9 qrs. **3.** Rs. 1-14-0. **4.** 60 men.
5. 513 sq. ft. and 609 sq. ft.

xiii. **1.** Rs. 15,697-13-0. **2.** 62·09 miles. **3.** $5\frac{1}{2}$ ac. **4.** (1) 29·66.
(2) Rs. 295-7-3. (3) £30-5-$5\frac{1}{2}$. 5. 990 ft.

xiv. **1.** Rs. 740-2-4. **2.** 120 men and 120 boys. **3.** 9 ac.; 16 ac.
4. 90 men. **5.** 264 yds.

xv. **1.** 132480 gals. **2.** 403313·9049. **3.** 26*s*.; 35*s*. 6*d*.
4. 0·003. **5.** 30 days.

Examples.

CHAPTER XXXII.

i. **1.** 21. **2.** 90. **3.** 204. **4.** 35. **5.** (1) $\frac{7}{40}$. (2) $\frac{2}{3}$. (3) $\frac{2}{3}$. (4) $\frac{19}{35}$.
6. (1) 0·065. (2) 0·1225. (3) 0·1375. (4) 0·17125. (5) 0·166.
(6) 0·007. (7) 0·231. (8) 0·061.
7. (1) 13 as. (2) £106-1-0. (3) 1 fur. $82\frac{1}{2}$ yds. (4) 635 grams.
8. (1) Rs. 118-5-7. (2) Rs. 1,830-7-8. (3) Rs. 4,418-6-5.
(4) Rs. 38,029-12-10.
9. (1) £21-2-$6\frac{3}{4}$. (3) Rs. 43-3-1. (3) Rs. 65-6-8. (4) £4-4-$1\frac{1}{4}$.

ii. **1.** 4. **2.** 48. **3.** $16\frac{2}{3}$. **4.** 72. **5.** 6·15. **6.** 0·034375. **7.** 11.
8. 9. **9.** $8\frac{1}{3}$. **10.** 25. **11.** $14\frac{1}{6}$. **12.** $31\frac{11}{24}$. **13.** 3. **14.** 15.
15. 60. **16.** $18\frac{3}{4}$; $81\frac{1}{4}$.

iii. **1.** Rs. 50. **2.** Rs. 300. **3.** 250 yds. **4.** 120 mi. **5.** £1. **6.** 80.
7. 300 tons. **8.** 5 days. **9.** 500. **10.** 135. **11.** £180,000. **12.** £5,000.

iv. **1.** (1) $56\frac{1}{4}$. (2) $83\frac{1}{3}$. (3) $46\frac{7}{8}$. **2.** (1) $65\frac{5}{8}$. (2) (*a*) $71\frac{3}{7}$. (*b*) $46\frac{7}{8}$.
3. (1) $14\frac{22}{47}$; $14\frac{2}{13}$. (2) $14\frac{2}{7}$. **4.** Increase 16.

v. **1.** 1,000. **2.** 252. **3.** Rs. 160. **4.** Rs. 75. **5.** 20,000. **6.** Rs. 2,00,100.
7. Rs. 1,60,000. **8.** 18,909 ; 21,273 to the nearest integer.

vi. **1.** (*a*) 1·6. (*b*) 2·3. (*c*) 0·21. (*d*) 1·1. **2.** 92·5 : 7·5. **3.** 0·00015.
4. 19·85. **5·** 0·75. **6.** Rs. 2,250. **7.** $56\frac{1}{4}$.
8. $91\frac{2}{3}$; $8\frac{1}{3}$. **9.** $10\frac{1}{4}$. **10.** 7625 ; 12600 ; 4775. **11.** $70\frac{5}{16}$.
12. 500 lbs. ; $62\frac{1}{2}$ lbs. **13.** 4. **14.** 21 ; 27 ; 42 ; 10.
15. England and Wales ; 12·33 ; 11·09. **16.** 9 ; 15 ; 11.

Examples.

CHAPTER XXXIII.

i. **1.** Rs. 70 5 as. **2.** £92-15-0. **3.** Rs. 709 14 as. **4.** £182-16-3.
5. Rs. 43-9-1. **6.** £32-13-4. **7.** Rs. 27-9-2. **8.** £11-14-$3\frac{1}{4}$.

ii. [illegible] Rs. 207. **2.** £93 15*s*. **3.** Rs. 382-8-0. **4.** £267-19-$4\frac{1}{2}$.
5. £308 10*s*. **6.** £234-10-9. **7.** Rs. 353-6-6. **8.** Rs. 296-11-$8\frac{1}{2}$.

iii. **1.** £284-1-$9\frac{1}{4}$. **2.** Rs. 221-7-2. **3.** Rs. 11,626-15-$1\frac{1}{2}$. **4.** Rs. 814-5-7.
5 Rs. 269-4-7. **6.** Rs. 5,957-9-10. **7.** Rs. 157-9-4. **8.** £72,406-1-$7\frac{1}{2}$.

iv. **1.** Rs. 93 12 as. ; Rs. 718 12 as. **2.** Rs. 286 14 as. ; Rs. 1,136 14 as.
3. Rs. 354 6 as. Rs. 2,716 14 as. **4.** £302-14-9 ; £1,648-4-9.
5. £2,785-10-2 **6** £3,114-6-$10\frac{1}{2}$. **7.** £3,086-13-4. **8.** Rs. 1,482-1-2.

v. **1.** (1) £703-2-6. (2) Rs. 153 3 as. (3) £30-3-9. (4) £34-11-0.
2. (1) £2,381 5*s*. (2) £1,001-4-$4\frac{1}{2}$. (3) Rs. 634-6-8. (4) Rs. 2,309-1-6.

vi. **1.** £75 12*s*. **2.** £54. **3.** Rs. 30-15-8. **4.** £2-6-$2\frac{1}{2}$.
5. Rs. 119 7 as. **6.** £24 10*s*. **7.** Rs. 103-1-3. **8.** £1,534-10-$6\frac{1}{2}$.

vii. **1.** Rs. 14-0-6. **2** £35-16-$9\frac{1}{4}$. **3.** Rs. 1,610-15-4. **4.** Rs. 4,610-13-2.

viii. **1.** Rs. 700. **2.** Rs. 200. **3.** Rs. 45. **4.** Rs. 90.

ix. **1.** £956. **2.** Rs. 4,143 12 as. **3.** £1,860. **4.** Rs 450.

x. **1.** 4 yrs. **2.** $3\frac{1}{4}$ yrs. **3.** $9\frac{3}{5}$ mo. **4.** $8\frac{1}{3}$ yrs.
5. 2 yrs. **6.** 20 yrs. **7.** $3\frac{1}{3}$ yrs. **8.** $3\frac{1}{2}$ yrs.

xi. **1.** 75. **2.** 4. **3.** $23\frac{1}{2}$. **4.** 3. **5.** 4. **6.** $2\frac{1}{2}$. **7.** $17\frac{1}{2}$. **8.** 5.

xii. **1.** Rs. 16,875. **2.** Rs. 6,275-6-3. **3.** Rs. 3,250. **4.** 12.
5. $28\frac{4}{7}$ yrs. **6.** Rs. 562 8 as. ; 75%. **7.** Rs. 262 8 as. **8.** £190.
9. Rs. 4,169-9-11 to the nearest pie ; $29\frac{1}{41}$ yrs. **10.** $9\frac{4}{5}$%.
11. Rs. 28,659 6 as. **12.** Rs. 510. **13.** 14 yrs. **14.** Rs. 421 ; 10%.
15. Rs. 7,896 ; 4%. **16.** Rs. 32,000.

Examples.

CHAPTER XXXIV.

i. **1.** 20. **2.** $33\frac{1}{3}$. **3.** 24. **4.** 10. **5.** 4.
6. $4\frac{1}{6}$. **7.** $18\frac{8}{19}$. **8.** 8. **9.** 20% gain. **10.** 5% loss.

ii. **1.** Rs. 12. **2.** £11-12-$2\frac{3}{4}$. **3.** 10*s*. 10*d*. **4.** Rs. 187 8 as.
5. Rs. 3,600. **6.** 1*s*. $3\frac{1}{4}$*d*. to the nearest penny. **7.** 10 p.
8. 3*s*. $4\frac{3}{4}$ *d*. to the nearest penny.

iii. **1.** £6-17-$9\frac{3}{4}$. **2.** Rs. 1,190 **3.** Rs. 5-10-0. **4.** Rs. 1,260.

iv. **1.** £450. **2.** £39. **3.** Rs. 36. **4.** Rs. 1,840.

v. **1.** 12% gain. **2.** $16\frac{2}{3}$% gain. **3.** $12\frac{1}{2}$. **4.** 0·8% gain.

vi. **1.** Rs. 10-8-0. **2.** 5 as. 4 p. **3.** Rs. 5 12 as. **4.** £56.

vii. **1.** Rs. 20. **2.** Rs. 50. **3.** 12*s*. 6*d*. **4.** £1,001-2-6.

viii. **1.** £1-16-6. **2.** £55. **3.** Rs. 21,666-10-8. **4.** Rs. 11-1-9.
5. Rs. 62-2-6. **6.** Gains Rs. 703-8-0. **7.** 33% nearly. **8.** 7*s*. $11\frac{1}{2}$*d*.
9. Gain $18\frac{3}{4}$%. **10.** Rs. 4-2-8. **11.** $11\frac{2}{3}$. **12.** Loses 80 %.

One-Hour Papers: Fourth Series.

i. **1.** 17 ft. **2.** 1 cawni = $\frac{160}{121}$ ac.; $15\frac{1}{5}$ ac. **3.** Rs. $29\frac{13}{64}$ (1 lb.=12 oz.).
4. 132. **5.** $27\frac{169}{453}$.

ii. **1.** Rs. 64,315-4-6; Rs. 881-0-6. **2.** 15 sq. in. **3.** $11\frac{1}{5}$ days.
4. 30 per cent. **5.** 90 miles.

iii. **1.** 10 ft. **2.** $4\frac{1}{2}$ days. **3.** $62\frac{1}{2}$ cawnies. **4.** £1,953-1-8. **5.** 700.

iv. **1.** 136 yds. 2 ft. 8 in. **2.** 2,400 men. **3.** A, £2,900-13-9;
B, £2,331-19-5; C, £1,610-1-3. **4.** 25% per annum.
Rs. 140 13 as. **5.** 59%.

v. **1.** 8 wks. **2.** 15 in.; $12\frac{1}{2}$ in.; 30 sq. ft. **3.** 114 days.
4. 50%; 16%; 13%. **5.** £475.

vi. **1.** 15*s*. 1*d*. **2.** The train by $2\frac{4}{15}$ ft. per sec. **3.** 31 oz. **4.** 9.
5. A, Rs. 500; B, Rs. 520; C, Rs. 1,000; D, Rs. 1,500.

vii. **1.** 1123. **2.** 10·56 in. **3.** Rs. 450. **4.** 360 days. **5.** 35 yds.; $1\frac{1}{4}$ yds.

viii. **1.** A, 2 as. 6 p.; B, 8 as. 9 p.; C, 6 as. 3p. **2.** 2,757 tiles; $23\frac{2}{9}$
sq. ft. **3.** 1·5 in. **4.** 32 days. **6.** Rs. 33-5-4.

ix. **1.** £560-6-8; 474·1 c. ft. **2.** 3·8 yrs. **4.** 187·5 gals.; 1869·63 lbs.
5. $\frac{9}{10}$ md.

x. **1.** 1100. **2.** £3,745; $2\frac{1}{2}$%. **3.** Gained $5\frac{6}{19}$%. **4.** 3,000 men.
5. 25 mi. an hr.

xi. **1.** 3,490 mds. 7 v. 3 srs. **2.** 36 hrs. **3.** 6 yrs. **4.** 6 mds. **5.** 20.

xii. **1.** 38Kg. **2.** £55-11-$1\frac{1}{4}$. **3.** $3\frac{1}{8}$%. **4.** 30 boys.
5. $15\frac{15}{19}$ mi. after starting.

xiii. **1.** £3-12-$2\frac{1}{4}$; 0·2375. **2.** £1,706-14-7; Rs. 25,600 15 as. **3.** 59·265
grs. **4.** 4·51 fr. **5.** 1 foot 9 in.

xiv. **1.** £187-5-7. **2.** 4%. **3.** £1-13-4. **4.** £44 Kg. 298 g. **5.** 20 yrs.

xv. **1.** 25·7 and 27·3 mi. **3.** 1 foot 2 in. **4.** 25%. **5.** 2·115 c. in.

xvi. **1.** 1·0162 : 1. **2.** 0·006; 0·6%. **3.** By rule 0·1838″ and by formula
0·1836″. **4.** 1 in. to 22 mi. **5.** Rs. 750.

xvii. **1.** 211 Kg. **2.** 25 in. to the mile; 62·5 sq. in. **3.** 7 mi. 640 yds.
4. 352 and 356. **5.** $3\frac{1}{12}$%.

xviii. **1.** 3*s*. $1\frac{3}{4}$*d*. **2.** 1·015 sq. m.; 0·005 sq. m. **3.** By rule and by
formula, diameter = 121·8″. **4.** £2 16*s*. **5.** 26·5.

xix. **2.** £8-12-7. **3.** 55; 53 to 57. **4.** 5·3%. **5.** 83, 84, 66, 81%.

xx. **1.** 114·5; 11450. **2.** £13-2-6. **3.** 113 grs. **4.** 81,097 sq. mi.
more. **5.** $4\frac{1}{2}$% per annum.

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